# ON THE COMPLEXITY-PERFORMANCE TRADEOFF OF TWO ACTIVE NOISE CONTROL SYSTEMS FOR VEHICLES

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## ABSTRACT

The aim of this paper is to show the experimental results achieved in the attenuation of periodic disturbances inside a vehicle with two Active Noise Control algorithms implemented on the TMS320C6701 DSP and to compare the computational complexity of both strategies:

(1) Modified FxGAL: Modified filtered-x gradient adaptive lattice algorithm. This technique is based on the signal orthogonalization carried out by an adaptive lattice predictor in a previous stage.

(2) *Gμ*-FxSLMS: Filtered-x sequential least mean square algorithm with step-size gain. This strategy is based on partial updates of the weights of an adaptive filter as well as on the controlled increase in step-size of the algorithm.

This work illustrates by means of two different algorithms the tradeoff established among computational costs, convergence rate, stability and mean-square error excess when DSP-based strategies are used in control systems focused on the attenuation of acoustic disturbances.


## PROPOSED ALGORITHMS

**Modified Filtered-x Gradient Adaptive Lattice (FxGAL) algorithm.**

The FxGAL algorithm [1] can be seen as a version of the gradient adaptive lattice (GAL) algorithm [2] suitable to be used in the context of active control. The aim of FxGAL and Modified FxGAL algorithms is to obtain faster and much less signal dependent convergence than that of FxLMS systems, while maintaining the numerical stability of stochastic gradient algorithms. Also, better tracking capabilities can be expected in non-stationary environments with the FxGAL algorithms. The price of these improvements is an increase in computational complexity, which can be easily lessened by reducing conveniently the order of the adaptive lattice orthogonalizer.

The modified version of the FxGAL algorithm makes use of the same idea that leads from the standard FxLMS to the Modified FxLMS algorithm [3]: an estimation of the primary noise is used to properly swap the order between secondary path and adaptive control filter and a simultaneous copy of this control filter is used with the reference signal. In this way, the limitations imposed on the step-size for the standard version of the algorithm are overcome in the modified one.

The key system in the FxGAL algorithms is an Adaptive Lattice Predictor (ALP), which realizes an approximate time-domain orthogonalization of its input data, without loss of information. The combination of this orthogonalization with independent step-sizes for each filter weight in the Desired Response Estimator (DRE) makes possible the expected increase in convergence speed.

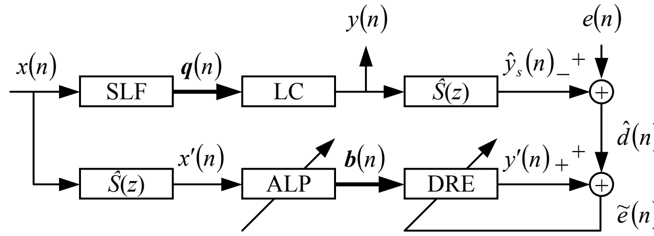


*Figure 1. Modified FxGAL block diagram.*

Figure 1 shows a block diagram of the modified FxGAL algorithm. As can be seen, the filtered reference signal is the input to the ALP-DRE combination, while the reference signal goes through a Slave Lattice Filter (SLF) and a Linear Combiner (LC) to yield the control signal $y(n)$. The ALP and DRE blocks are the adaptive ones, while the coefficients of the SLF and LC systems are simply copied from the ALP and DRE, respectively.

An iteration of the Modified FxGAL algorithm is given by:

$t_0(n) = q_0(n) = x(n)$ /* *Slave Lattice Filter (SLF )* */

*for* $l = 1$ *to* $Lw - 1$ *do*

$$t_l(n) = t_{l-1}(n) + k_l(n) \cdot q_{l-1}(n-1)$$

$$q_l(n) = q_{l-1}(n-1) + k_l(n) \cdot t_{l-1}(n)$$

*end of for*

$y(n) = \underline{w}^T(n) \cdot \underline{q}(n)$ /* *Linear Combiner (LC) filter* */

$\hat{y}_s(n) = \underline{\hat{s}}^T(n) \cdot \underline{y}(n)$ /* *Estimate of the primary noise* */

$\hat{d}(n) = e(n) - \hat{y}_s(n)$

$x'(n) = \underline{\hat{s}}^T(n) \cdot \underline{x}(n)$ /* *Filtering of the reference* */

$f_0(n) = b_0(n) = x'(n)$ /* *Adaptive Lattice Predictor (ALP)* */

*for* $l = 1$ *to* $Lw - 1$ *do*

$$f_l(n) = f_{l-1}(n) + k_l(n) \cdot b_{l-1}(n-1)$$

$$b_l(n) = b_{l-1}(n-1) + k_l(n) \cdot f_{l-1}(n)$$

$$\hat{P}_l(n) = \beta_{ALP} \cdot \hat{P}_l(n-1) + (1 - \beta_{ALP}) \cdot \left(f_{l-1}^2(n) + b_{l-1}^2(n-1)\right)$$

$$k_l(n+1) = k_l(n) - \frac{\alpha_{ALP}}{\hat{P}_l(n)} \cdot \left(f_{l-1}(n) \cdot b_l(n) + b_{l-1}(n-1) \cdot f_l(n)\right)$$

*end of for*

$y'(n) = \underline{w}^T(n) \cdot \underline{b}(n)$ /* *Desired Response Estimator (DRE)*

$\tilde{e}(n) = \hat{d}(n) + y'(n)$ *filtering and update* */

$$
\begin{aligned}
&for\ \ i = 0\ to\ Lw-1\ \ do \\
&\quad \hat{P}_{bl}(n) = \beta_{DRE}\cdot \hat{P}_{bl}(n-1) + \left(1-\beta_{DRE}\right)\cdot b_l^2(n) \\
&\quad w_l(n+1) = w_l(n) - \frac{\alpha_{DRE}}{\max\left\{\hat{P}_{bl}(n), P_{\min}\right\}}\cdot b_l(n)\cdot \tilde{e}(n) \\
&end\ of\ for
\end{aligned}
$$

**Filtered-x Sequential Least Mean Square algorithm with Step-Size Gain (*Gμ*-FxSLMS)**

Partial updates algorithms [4] update only a portion of the filter at each time instant in order to reduce their computational complexity. These algorithms suffer from one drawback: their convergence speed are also reduced in the same proportion.

The *Gμ*-FxSLMS algorithm [5] is aimed at reducing the computational costs of the control strategy without either incrementing the final misadjustment or slowing down the convergence speed.

A single iteration of the *Gμ*-FxSLMS algorithm can be expressed as follows:

$$
\begin{aligned}
&y(n) = \underline{w}^T(n)\ \cdot \underline{x}(n) && \text{/* Generation of antinoise */} \\
&if\ \ n \bmod N == 0 && \\
&\qquad x'(n) = \underline{\hat{s}}^T(n)\cdot \underline{x}(n) && \text{/* Filtering with the estimate} \\
&end\ of\ if && \text{of the secondary path */} \\
&e(n) = d(n) - y(n) && \text{/* Obtaining error signal */} \\
&for\ \ i = 1\ to\ Lw\ \ do && \\
&\quad if\ \ (n\text{-}i+1) \bmod N == 0 && \text{/* Filter partial updates */} \\
&\qquad w_i(n+1) = w_i(n) + \mu\cdot G_\mu(N, Lw, Fs)\cdot e(n)\cdot x'(n+1-i) && \\
&\quad end\ of\ if && \\
&end\ of\ for &&
\end{aligned}
$$

where the step-size gain *Gμ* is defined as the ratio between the bounds on the step-sizes in two cases: firstly, when the adaptive algorithm uses partial updates and, secondly, when every coefficient is updated at each iteration. In so doing, we obtain the factor by which the step-size can be multiplied when the adaptive algorithm uses partial updates.

The theoretical analysis of the strategy prevents from the use of certain frequencies corresponding to notches which appear in the gain in the step-size of the adaptive algorithm. Their width and exact location depend on the length of the adaptive filter, the decimating term and the sampling frequency. Figure 2 shows the step-size gain for different values of the length of the adaptive filter ($Lw$) and the decimating factor ($N$). It can be easily derived from the examples given that the number of notches appearing in the gain is $N$-1. Besides, the more the weights of the adaptive filter, the narrower the notch will be, that is, the narrower the bandwidth at which the gain in step-size cannot be applied at its full strength.

Summarizing, the step-size gain can be equal to $N$ as long as the regressor signal has no components at the notch frequencies

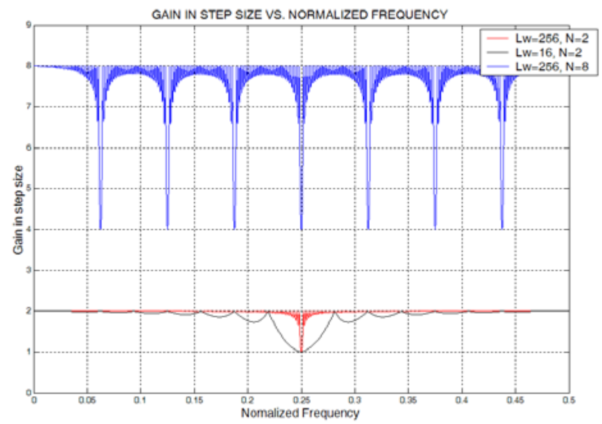


*Figure 2. Gain in step-size for different values of the length of the adaptive filter Lw and the decimating factor N.*

## COMPUTATIONAL COSTS

Table 1 and Table 2 show, respectively, the number of operations required per a single iteration of the Modified FxGAL and *Gμ*-FxSLMS algorithms.

| | |
|---|---|
| # Additions | $2\cdot(12\cdot Lw + 2\cdot Ls - 10)$ |
| # Multiplications | $2\cdot(17\cdot Lw + 2\cdot Ls - 10)$ |
| # Divisions | $2\cdot(2\cdot Lw - 1)$ |

*Table 1. Computational complexity of the Modified FxGAL algorithm in terms of the average number of additions, multiplies and divisions per iteration.*

| | |
|---|---|
| # Additions | $2\cdot\left(1+\frac{1}{N}\right)\cdot Lw + 2\cdot\left(\frac{Ls-1}{N}\right)$ |
| # Multiplications | $2\cdot\left(1+\frac{1}{N}\right)\cdot Lw + 2 + 2\cdot\left(\frac{Ls}{N}\right)$ |
| # Divisions | None |

*Table 2. Computational complexity of the Gμ-FxSLMS algorithm in terms of the average number of additions and multiplies per iteration.*

where $Lw$ is the length of the adaptive filter, $Ls$ is the length of the off-line estimate of the secondary path and $N$ is the decimating factor used in the partial updates of the second algorithm proposed.

It should be noticed that the sampling frequencies chosen for the practical implementation of both strategies were not the same. While for the Modified FxGAL a sampling frequency of 1000 samples/s was considered to be enough to deal with low frequency noise, in the case of the *Gμ*-FxSLMS algorithm, the sampling frequency was set to a value 8 times higher, that is, 8000 samples/s is order to broaden the bandwidth free of notches in the step-size gain.

As a result of that, the comparison between both strategies should be carried out on the basis of the number of operations required per second, instead of the number of operations per iteration.

## EXPERIMENTAL RESULTS

**Laboratory test set-up**

Figure 3 shows the physical arrangement of the electro-acoustic elements used in the implementation of the 1x2x2 Active Noise Control system placed at the front seats of a Nissan Vanette.

The main Digital Signal Processor board employed to develop both strategies is the PCI/C6600, based on the DSP TMS320C6701. The Input/Output board is the PMCQ20DS that disposes of 4 A/D and 4 D/A converters.

The control strategy implemented was either the Modified FxGAL algorithm or the $G\mu$-FxSLMS algorithm.

In order to carry out a performance comparison of different control strategies it is essential to repeat the experiment in the same conditions. So as to avoid fluctuations in level and frequency of the undesired disturbance, instead of starting the engine, we have previously recorded a signal consisting of two harmonics (150 and 450 Hz). The omnidirectional source Brüel & Kjaer Omnipower 4296 placed inside the van is fed with this signal and acts as the source of the primary noise.

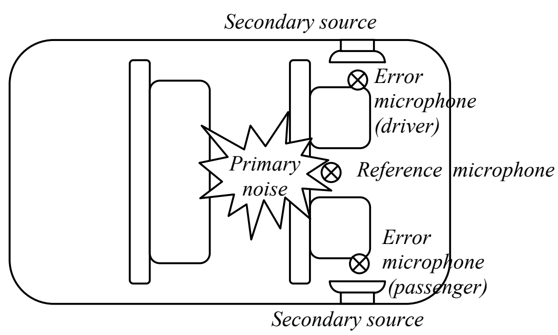


*Figure 3. Physical disposal of the electro-acoustic elements inside the van.*

Figure 4 shows the error microphone and the loudspeaker acting as secondary source for the ANC system implemented at the right front seat of the van. As far as the electroacoustic elements are concerned, low cost microphones and loudspeakers have been chosen.

*Figure 4. Detail of error microphone and secondary source.*

**Set of parameters chosen**

In order to obtain similar results with both algorithms in the attenuation of the undesired disturbance, the parameters are set to the following values:

<u>Modified FxGAL algorithm</u>

- Number of weights of the adaptive filter, $Lw$=8.
- Number of weights of the off-line estimate of the secondary path, $Ls$=200.
- Sampling Frequency, $Fs$=1000.
- Normalized step-size for the ALP stages is set to 0.06.
- Forgetting factor, $\beta$=0.97.
- Normalized step-size for the FxLMS algorithm is set to 0.08.

<u>$G\mu$-FxSLMS algorithm</u>

- Number of weights of the adaptive filter, $Lw$=128.
- Number of weights of the off-line estimate of the secondary path, $Ls$=200.
- Sampling Frequency, $Fs$=8000.
- Decimating factor, $N$=8.
- Gain in step-size, $G\mu$=8;
- Step-size of the adaptive algorithm, $\mu$=0.1.

So as to carry out a comparison of the computational requirements, it is assumed that the DSP can deal with 1 MAC operation -multiplication & accumulation- per DSP cycle whereas needs 40 cycles to perform a division. According to the parameters chosen and taking into account the complexity expressed in Tables 1 and 2, the number of clock cycles required between two consecutive samples is 2252 for the Modified FxGAL algorithm and 340 for the $G\mu$-FxSLMS algorithm. Considering that the sampling frequency is 8 times higher in the latter case, the cycles required per millisecond are 2252 and 2720, respectively. Thus, not only the performance achieved but also the computational costs of both strategies are quite similar despite being based on opposite underlying ideas.

**Analysis in the Time Domain**

Figures 5 and 6 show, respectively, the learning curves of the Modified FxGAL and the $G\mu$-FxSLMS algorithms, when the error signals are measured at the microphones located at the head of the driver and the passenger.

In both cases the two-harmonic signal is effectively attenuated by more than 20 dB within relatively short time.

**Analysis in the Frequency Domain**

The frequency response functions measured at the error sensors located at the front seats of the van are shown in Figure 7 -Modified FxGAL algorithm- and Figure 8 -$G\mu$-FxSLMS algorithm-. The signal before control is shown in a dotted line whereas the signal after control is shown in a solid line. As far as the attenuation achieved is concerned, more than 25 dB of peak reduction are obtained at the main harmonics with both ANC algorithms. Nonetheless, very little off-peak reduction was obtained.

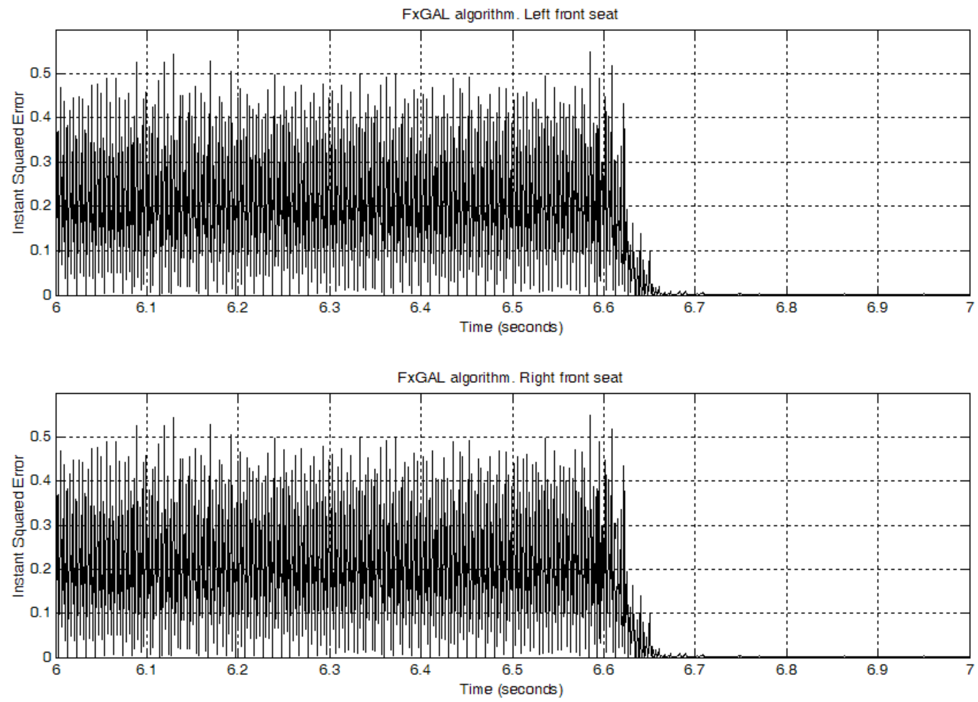


*Figure 5. Evolution of the squared error when the ANC system based on the Modified FxGAL algorithm is switched on. Upper: left front seat, Lower: right front seat.*

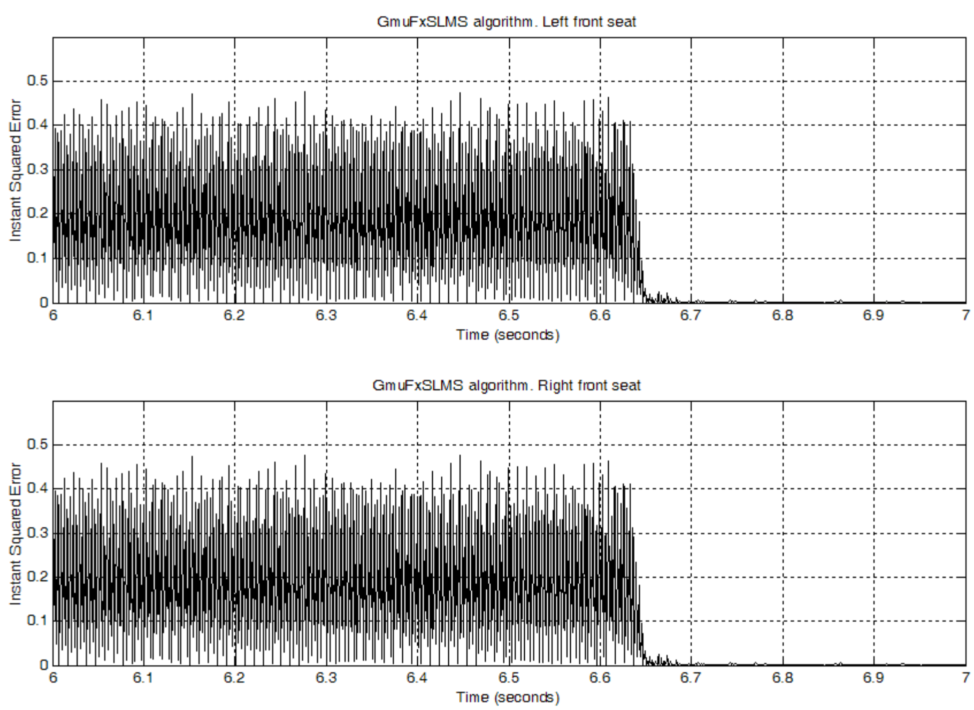


*Figure 6. Evolution of the squared error when the ANC system based on the Gμ-FxSLMS algorithm is switched on. Upper: left front seat, Lower: right front seat.*

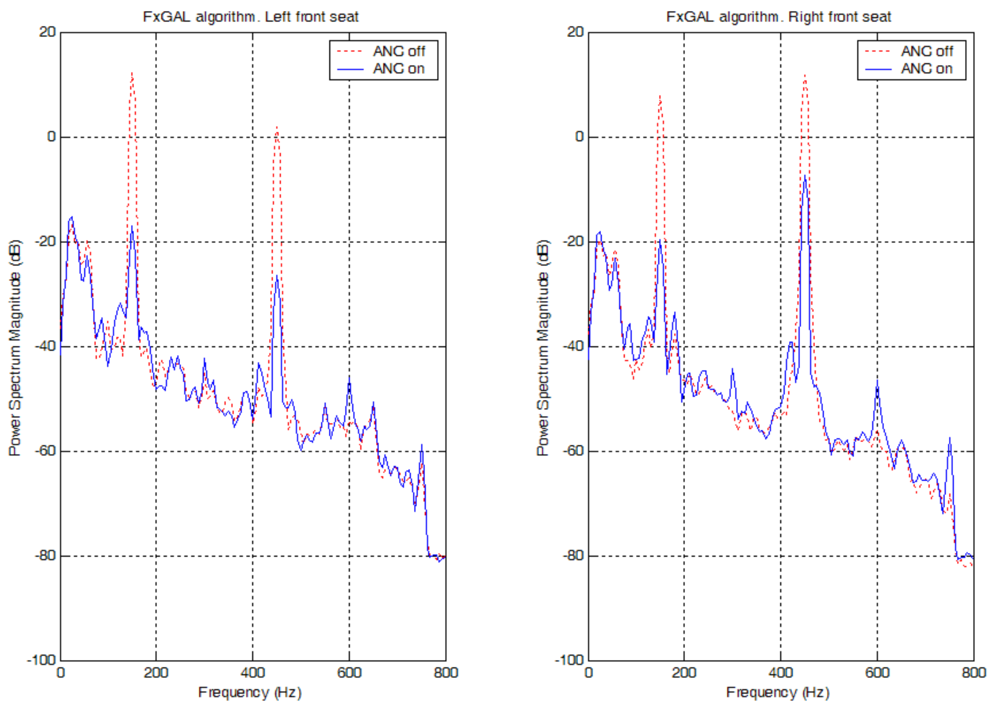


*Figure 7. Experimental control results for the ANC system based on the Modified FxGAL algorithm in the frequency domain. Left: left front seat, right: right front seat. Dotted (red): without control, solid (blue): with control.*

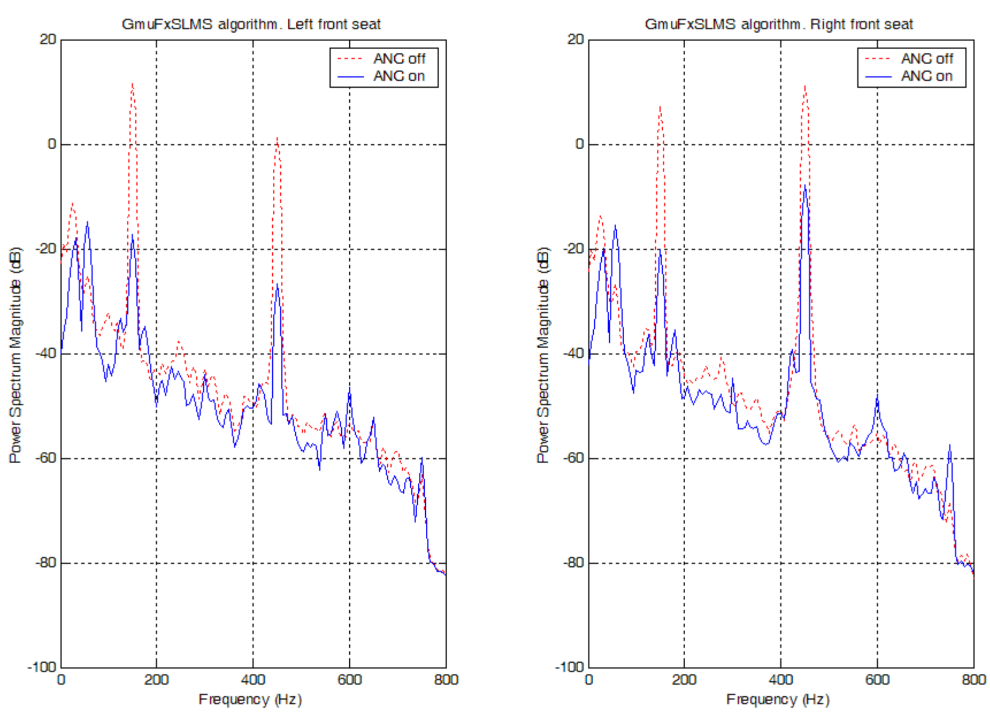


*Figure 8. Experimental control results for the ANC system based on the Gμ-FxSLMS algorithm in the frequency domain. Left: left front seat, right: right front seat. Dotted (red): without control, solid (blue): with control.*

Power spectral density of the undesired noise is depicted in Figures 7 and 8 where it can be seen that consists of two harmonics at 150 and 450 Hz. Looking carefully into the graphs, it can be noticed that an unexpected noise component appears at a narrow frequency band between 15 and 35 Hz. Provided that this component was not present in the two-harmonic signal when it was generated, we can conclude that it corresponds to a mode imposed by the geometry of the van. In fact, we have verified that this low frequency noise vanishes as soon as the microphone is located outside the van.

## CONCLUSIONS

This paper presents the results for applying two different control algorithms -Modified FxGAL and $G\mu$-FxSLMS- to actively attenuate periodic noise in a van.

The former strategy is aimed at speeding up the convergence rate at the expense of increasing the computational requirements whereas the latter puts forward a computationally less intensive solution.

In spite of the fact that the underlying proposals of both algorithms are based on opposite control strategies -higher complexity and faster convergence rate versus lower complexity-, the subsequent choice of the parameters permits of similar performance in terms of convergence speed, residual error and degree of attenuation with a computational complexity of the same order.

It has been experimentally shown that periodic noise may me substantially attenuated by Active Noise Control systems based on both algorithms.

With a sampling frequency of 1000 samples/s -for the Modified FxGAL algorithm- or 8000 samples/s -for the $G\mu$-FxSLMS-, the effective ANC system bandwidth is approximately 500 Hz, and usually produced more than 20 dB of reduction within about 0.05 seconds of the starting.


## ACKNOWLEDGEMENTS

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