PERFORMANCE ANALYSIS OF A TIME DELAY ESTIMATE BETWEEN TWO NOISY TRANSIENT SIGNALS

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ABSTRACT

Signal averaging technique is a classical method to recover deterministic signal components. Time delay estimation is critical in signal averaging. We present the performance study of a new method for time delay estimation based on coincidence of normalized integrals. The estimate is considered working with the signal ($\hat{D}_s$) and the squared signal ($\hat{D}_{s^2}$). In both cases the bias and the standard deviation of the estimate have been evaluated when the signal is contamined by noise. $\hat{D}_s$ is not biased but the standard deviation is higher than working with $\hat{D}_{s^2}$. In this case there is a bias in the estimate that must be corrected. Analytical expressions for the bias and a procedure to correct it are presented. Simulation results with noisy ensembles of gaussian signals and QRS complexes from a real ECG, agree the theory.

INTRODUCTION

The signal averaging technique is a classical method to estimate a deterministic component that is presented in an ensemble of signals corrupted by noise. This technique has been broadly used in different fields like evoked potentials or high-resolution electrocardiography (His bundle activity and ventricular late potentials).

An alignment jitter around the so called Fiducial Point (F.P.), for each signal of the ensemble, produces a filtering effect in the estimated signal. If the jitter is normally distributed (standard deviation of σ (ms)), the cutoff frequency (f_c) of the filter at -3 dB is $f_c = 132.3/\sigma$ [4]. Then the precision in the definition of the F.P. depends on the alignment method performance.

Different methods have been proposed to define the F.P. We have proposed a simple one [1] that works with the time domain signal. It has been applied on ECG signals [2,3]. We have studied the performance of this method in comparison with matched filtering and double level method [3,4]. We have found that this method works better than the double level and is comparable to matched filtering. When noise is not white, this method reaches better performance than matched filtering for the same signal-to-noise ratio (SNR) [4].

In this work we have studied the bias and the standard deviation of the estimate, when noise is present, working with the signal ($\hat{D}_s$) and with the squared signal ($\hat{D}_{s^2}$), respectively. Signals are supposed to be contaminated by an additive white zero-mean gaussian noise. We have achieved a mathematical expression for the bias of $\hat{D}_{s^2}$, and a technique to obtain the unbiased time delay (D) has been proposed. We have tested the mathematical expression by computer simulation using ensembles of delayed gaussians and actual ECGs as deterministic signals.

THE TIME DELAY ESTIMATION METHOD

This method is based on the normalized integrals of two delayed signals [1]. Let $s(t)$ be a transient signal whose duration is included in the interval $[-a, a]$, and we will assume parameter a large enough so that:

$$s(t) = 0 \quad t \notin [-a + D_{max}, a - D_{max}]\,, \quad D_{max} > 0. \tag{1}$$

Let $S(t)$ be its normalized integral

$$S(t) = \frac{\int_{-a}^{t} s(t')\,dt'}{A}\,, \quad where \quad A = \int_{-a}^{a} s(t)\,dt \neq 0\,. \tag{2}$$

If $v(t)$ is another signal of the form v(t) = s(t-D), whose normalized integral is $V(t)$. The delay D between the two signals $s(t)$ and $v(t)$ ($D < D_{max}$) can be computed by

$$D = \int_{-a}^{a} (S(t) - V(t))\,dt\,. \tag{3}$$

This relation defines the time delay estimation method. Same expression can be derived for the estimate considering $s^2(t)$ instead of $s(t)$. When signal is contaminated by noise, (3) leads to an estimate ($\hat{D}$) of the real value D, that presents different behaviors when working with the signal $s(t)$ (estimate $\hat{D}_s$) or with the squared signal $s^2(t)$ (estimate $\hat{D}_{s^2}$). We considered the signal corrupted by an additive white zero-mean gaussian noise $n(t)$. In this case $s(t)$ and $v(t)$ become $s'(t) = s(t) + n_1(t)$ and $v'(t) = v(t) + n_2(t)$, respectively. $n_1(t)$ and $n_2(t)$ are supposed to be uncorrelated with same standard deviation (σ_n). Under these assumptions we study the estimates $\hat{D}_s$ and $\hat{D}_{s^2}$.

ESTIMATE $\hat{D}_s$

From (3) we can express $\hat{D}_s$ as

$$\hat{D}_s = \frac{\int_{-a}^{a}\int_{-a}^{T}(s(t) + n_1(t))\,dt\,dT}{\int_{-a}^{a}(s(t) + n_1(t))\,dt} - \frac{\int_{-a}^{a}\int_{-a}^{T}(v(t) + n_2(t))\,dt\,dT}{\int_{-a}^{a}(v(t) + n_2(t))\,dt}. \tag{4}$$

With the noise assumptions we can approximate $\int_{-a}^{a} n_i(t)\,dt = 0$ $(i = 1,2)$ and $\int_{-a}^{a}\int_{-a}^{T} n_i(t)\,dt\,dT = 0$ $(i = 1,2)$. The estimate ($\hat{D}_s$) becomes

$$\hat{D}_s = \frac{\int_{-a}^{a}\int_{-a}^{T}(s(t) - v(t))\,dt\,dT}{A} = D. \tag{5}$$

Then, with this first order approximation, $\hat{D}_s$ is an unbiased estimate of D.

ESTIMATE $\hat{D}_{s^2}$

From (3) we can express $\hat{D}_{s^2}$ as

$$\hat{D}_{s^2} = \frac{\int_{-a}^{a}\int_{-a}^{T}(s(t) + n_1(t))^2\,dt\,dT}{\int_{-a}^{a}(s(t) + n_1(t))^2\,dt} - \frac{\int_{-a}^{a}\int_{-a}^{T}(v(t) + n_2(t))^2\,dt\,dT}{\int_{-a}^{a}(v(t) + n_2(t))^2\,dt}. \tag{6}$$

With the same noise assumptions above considered we can approximate

$\int_{-a}^{a} 2s(t)n_i(t)\,dt = 0$, $\int_{-a}^{a}\int_{-a}^{T} 2s(t)n_i(t)\,dt\,dT = 0$ $(i = 1,2)$ and $\int_{-a}^{a}\int_{-a}^{T}(n_1^2(t) - n_2^2(t)) = 0$. Then the estimate $\hat{D}_{s^2}$, in this first order approximation, results in

*This work was supported by grant TIC88-0204, from CICYT (Spain), and "Acción integrada Hispano-Francesa" HF-174 (Spain and France).

CH2936-3/90/0000-0877 $01.00

$$\hat{D}_{s^2} = \frac{\int_{-a}^{a}\int_{-a}^{T}(s^2(t) - v^2(t))\,dt\,dT}{A' + B} = D\frac{A'}{A' + B}. \quad (7)$$

where

$$A' = \int_{-a}^{a} s^2(t)\,dt\,, \quad and \quad B = \int_{-a}^{a} n_i^2(t)\,dt = 2a\sigma_n \quad (i = 1,2). \quad (8)$$

Defining signal-to-noise ratio (SNR) as

$$SNR = \frac{\int_{-a}^{a} s^2(t)\,dt}{\int_{-a}^{a} n_i^2(t)\,dt} = \frac{A'}{B}, \quad (9)$$

$\hat{D}_{s^2}$ can be expresed as

$$\hat{D}_{s^2} = D\frac{SNR}{1 + SNR}. \quad (10)$$

The estimate $\hat{D}_{s^2}$ is biased. This bias depends on the SNR and D. The estimate $\hat{D}_s$ seems better than $\hat{D}_{s^2}$, but in the simulation results we will see that standard deviation of $\hat{D}_s$ is higher than in the case of $\hat{D}_{s^2}$. Then a bias correction for $\hat{D}_{s^2}$ can lead us to an unbiased estimate with smaller standard deviation than $\hat{D}_s$.

CORRECTED ESTIMATE $\hat{D}'_{s^2}$

We can recalculate $\hat{D}_{s^2}$ by shifting $v'(t)$ to $v'(t - \tau_i)$ obtaining several $\hat{D}_{s^2 i}$ for different τ_i values. Then we have a linear function $\hat{D}_{s^2 i}(\tau_i)$.

$$\hat{D}_{s^2 i} = \alpha\tau_i + \beta. \quad (11)$$

In this case (10) becomes

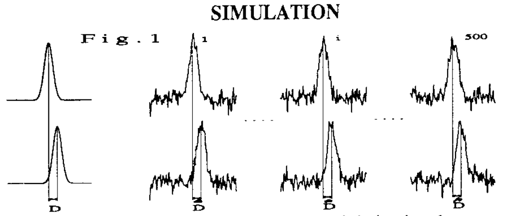
$$\hat{D}_{s^2 i} = (D + \tau_i)\frac{SNR}{1 + SNR} \quad (12)$$

and comparing (11) and (12) we can see that

$$\alpha = \frac{SNR}{1 + SNR} \quad \beta = D\frac{SNR}{1 + SNR}. \quad (13)$$

Then we define a new estimate $\hat{D}'_{s^2} = \frac{\beta}{\alpha} = D$ that is unbiased. Same expressions can be achieved for these estimates, when $v(t)$ is affected by a scale factor ($v(t) = k \cdot s(t - D)$).

SIMULATION



We have taken two *a priori* known deterministic signals: a computer generated gaussian signal (σ = 5 ms) and a QRS complex from a real ECG, both extend 100 ms. With each one of these signals we have generated two ensembles, adding white zero-mean gaussian noise, and delayed a time distance D = 10 ms. Each ensemble of signals has 500 records, and the sampling frequency is 1 kHz. Fig. 1 shows the original delayed gaussian signals and the two delayed ensembles generated for SNR=10. We have computed the delay in the 500 pairs (1,..,500, fig.1) of two delayed records by the considered estimates ($\hat{D}_s$, $\hat{D}_{s^2}$, $\hat{D}'_{s^2}$) for SNR of 1, 5, 10, 10^2, 10^3, 10^4 and 10^5. The standard deviation of the 500 delays has also been calculated. D is the artificially generated delay and each $\hat{D}$ is the delay estimate of D in each pair of records. Fig. 2 shows the generated signals for the QRS complex. $\hat{D}'_{s^2}$ has been evaluated through the parameter α and β obtained by the regression line that approximates (11).

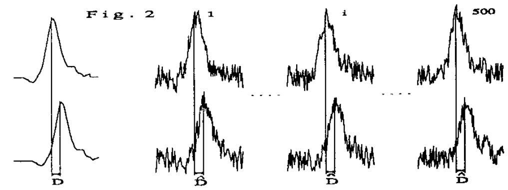


Table 1. Expected values of the delays $\hat{D}$

	SNR	1	5	10	10^2	10^3	10^4	10^5
$\hat{D}_s$ (ms)	The.	10.00	10.00	10.00	10.00	10.00	10.00	10.00
Gau.	Exp.	10.16	10.07	10.05	10.01	10.00	10.00	10.00
QRS	Exp.	11.35	10.51	10.35	10.10	10.03	10.01	10.00
$\hat{D}_{s^2}$ (ms)	The.	5.00	8.33	9.09	9.90	9.99	10.00	10.00
Gau.	Exp.	5.03	8.36	9.11	9.91	9.99	10.00	10.00
QRS	Exp.	5.30	8.50	9.21	9.92	9.99	10.00	10.00
$\hat{D}'_{s^2}$ (ms)	The.	10.00	10.00	10.00	10.00	10.00	10.00	10.00
Gau.	Exp.	9.61	9.84	9.90	9.89	9.89	9.89	9.89
QRS	Exp.	8.67	10.52	10.49	10.45	10.43	10.42	10.42

Table 1 shows the expected value of the delays for different SNRs, theorical (The.) and experimental (Exp.), with gaussian (Gau.) and QRS signals. We see that the expresion reached for the bias of $\hat{D}_{s^2}$ (10) agrees with simulation results.

Table 2. Standard deviation $\sigma(\hat{D})$

SNR	1	5	10	10^2	10^3	10^4	10^5
Gau. $\sigma(\hat{D}_s)$ (ms)	9.55	4.27	3.02	0.96	0.30	0.10	0.03
QRS $\sigma(\hat{D}_s)$ (ms)	7.67	3.29	2.31	0.73	0.23	0.07	0.02
Gau. $\sigma(\hat{D}_{s^2})$ (ms)	2.85	0.99	0.57	0.11	0.03	0.01	0.00
QRS $\sigma(\hat{D}_{s^2})$ (ms)	1.98	0.91	0.63	0.19	0.06	0.02	0.01
Gau. $\sigma(\hat{D}'_{s^2})$ (ms)	5.87	2.15	1.05	0.15	0.03	0.01	0.00
QRS $\sigma(\hat{D}'_{s^2})$ (ms)	5.24	1.91	1.28	0.37	0.12	0.04	0.01

Table 2 shows the standard deviation $\sigma(\hat{D})$ for the three estimates considered. We can see that $\sigma(\hat{D}'_{s^2})$ is higher than $\sigma(\hat{D}_{s^2})$ but lower than $\sigma(\hat{D}_s)$. Then we can conclude that $\sigma(\hat{D}'_{s^2})$ is the best estimate because it presents the lower standard deviation of the unbiased estimates.

CONCLUSIONS

We have studied a new time delay estimate method, when signals are contamined by noise. First order approximation of the estimates has been theoretically analysed and tested by simulation. We show that the estimate $\hat{D}_s$, that works with the signal, is unbiased in this approximation when noise is present. The estimate $\hat{D}_{s^2}$, that works with the squared signal, becomes biased, but with lower standard deviation than $\hat{D}_s$. A new estimate $\hat{D}'_{s^2}$ that corrects the bias is proposed. This estimate preserves the standard deviation lower than $\hat{D}_s$ without bias. Then this method is concluded to perform better for signal averaging with $\hat{D}'_{s^2}$, and the deterministic signal recovered with this estimate will preserve higher frequency components. This point is very important in high-resolution ECG applications.

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