Correspondence

A Time Delay Estimator Based on the Signal Integral: Theoretical Performance and Testing on ECG Signals

Pablo Laguna, Raimon Jané, and Pere Caminal

Abstract— **We present a theoretical and experimental performance study of a method for time delay estimation (TDE), based on the signal integral (TDE-SI). The TDE-SI method considers the delay between two transient signals as the difference between the center of mass of these signals. The method has three special cases: In the first, time is the mass coordinate and the signal $s(t)$ is the mass distribution (estimate $\hat{D}_s$); in the second case, the squared signal $s^2(t)$ is the mass distribution (estimate $\hat{D}_{s2}$); and the last is a variant of the second. The bias and the standard deviation (σ) of the estimate have been evaluated when the signal is contaminated by Gaussian white noise. $\hat{D}_s$ is not biased but the σ of the TDE is higher than that obtained when working with $\hat{D}_{s2}$. Moreover, the $\hat{D}_{s2}$ estimate is biased. The special case of a bias-corrected estimate ($\hat{D}'_{s2}$) is presented; this $\hat{D}'_{s2}$ yields a σ of its TDE lower than the estimate $\hat{D}_s$. Hence, $\hat{D}'_{s2}$ is the most suitable of the three TDE-SI options for TDE. Theoretical estimations are validated by simulation results with artificially generated signals and by real so-called QRS complex waves (ventricular activity) from an electrocardiographic (ECG) signal.**

I. Introduction

Time delay estimation (TDE) is a classical problem in signal processing [1]. This problem has been broadly studied for estimating the travel time of acoustic waveforms between two receiver sensors [2]. In biomedical signal processing, TDE is a technique used to estimate the relative alignment point between records of a repetitive signal in an averaging process [3]–[5]. TDE is also used to estimate time intervals between sequential waves in electrocardiographic (ECG) signals [5]–[7]. In particular, TDE has been used to align the QRS complex waves of ECG signals in averaging processes (these waves represent the ventricular activity of the heart). The performance of these TDE techniques is critical in recovering high frequency components like late potentials [3], since an erroneous alignment estimation produces a low-pass effect in the averaged estimate [8]. Several methods have been proposed for estimating the synchronization point in ECG signals. These methods can be classified into two groups: 1) those based on the special morphology of the QRS complexes [3], [5], [9]—generally more suitable for real-time estimation—and 2) those based on TDE techniques [3], [5], [10] that need further calculation but have better performance with low signal-to-noise ratios (SNR) [3].

TDE methods are typically based on generalized cross-correlation (GCC) techniques [11], [12]. In particular the maximum likelihood (ML) TDE has been shown to provide an unbiased time delay estimate with asymptotically minimum variance [13], [14] when the signals emanate from a single broadband source in the presence of noise uncorrelated from sensor to sensor. In this correspondence, we present a theoretical performance study and validation, using an ECG signal, of a TDE method based on the signal integral (TDE-SI). This method was first proposed in [15] and then applied to ECG signal averaging [3], [10] and to ECG time intervals estimation [6]; the TDE-SI method showed higher performance than matched filtering when signals have 50/60 Hz contamination [3]. The TDE-SI method studied in this correspondence has been experimentally proven to present time delay estimates of comparable variance to the ML or matched filter techniques [3]. The experimental part of this work follows similar evaluation procedures—mean and standard deviation—to those of the studies presented in [16] and [17] which consider the TDE methods based on GCC, rather than on the signal integral. The effect of finite signal observation interval is taken into account. Thus, the results can be compared to those presented in [18] for ML methods with short observation intervals.

The TDE-SI method considers the estimated delay as the difference between the two signal centers of gravity, considering time (t) as the coordinate of the gravity center and the signal value $s(t)$ as the mass distribution at coordinate t. An experimental study of the performance of the TDE-SI method when working with ECG signals has been presented in [19]. There, the variance of the estimate was experimentally studied in two different cases: When we consider the signal $s(t)$ to be the mass distribution (Estimate $\hat{D}_s$), and when we consider the squared signal $s^2(t)$ as the mass distribution (Estimate $\hat{D}_{s^2}$). In this correspondence, we present a theoretical study to predict the variance of the estimate as a function of the SNR and the signal shape $s(t)$, in the case of white, zero-mean and uncorrelated Gaussian noise. We verify that $\hat{D}_{s^2}$ is a SNR-dependent biased estimate and propose a bias-corrected estimate ($\hat{D}'_{s^2}$). We also present an indirect method to estimate the SNR based on this bias. Simulation results using Gaussian-shape and real so-called QRS complex signals are considered in order to corroborate theoretical predictions.

II. The Time Delay Estimation Method

The TDE-SI method [15] that we study is based on the normalized integrals of two delayed signals. Let $s(t)$ be a nonzero mean transient signal whose duration is included in the interval $[-a, a]$, and assume parameter a to be large enough (Fig. 1) so that:

$$s(t) = 0 \quad t \notin [-a + D_{\max}, a - D_{\max}], \quad D_{\max} > 0 \tag{1}$$

where $D_{\max}$ is the maximum delay that is supposed to appear between two records of the signal $s(t)$. In this way we ensure that all the finite-time records of the ensemble will contain the whole information present in $s(t)$. Let $S(t)$ be the normalized integral

$$S(t) = \frac{\int_{-a}^{t} s(t')dt'}{S_0}.$$

$$\text{where} \quad S_0 = \int_{-a}^{a} s(t)dt = \int_{-a}^{a} s(t-D)dt \neq 0 \tag{2}$$

and let $v(t)$ be another signal of the form $v(t) = s(t - D)$, whose normalized integral is $V(t)$. The delay D between the two signals $s(t)$ and $v(t)$ ($D < D_{\max}$) is

$$D = \int_{-a}^{a} (S(t) - V(t))dt = \frac{\int_{-a}^{a} t(s(t-D) - s(t))dt}{S_0}. \tag{3}$$

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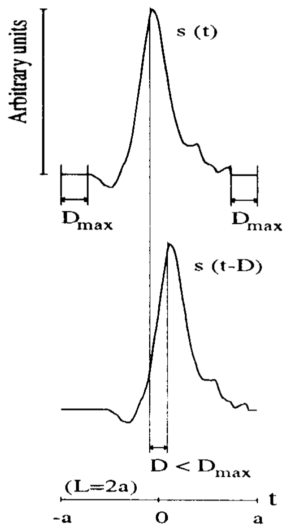

Fig. 1. Deterministic signal $s(t)$ and $s(t-D)$ at the observation interval $[-a,a]$, according to the assumptions referred to in (1). $D_{\max}$ is the maximum allowed signal delay (D) in the observation interval.

This relation defines the TDE-SI method. If $v(t)$ were changed by a constant gain factor k, then $v(t) = k \cdot s(t-D)$ equation (3) would remain valid. From this expression (3) it is evident that the TDE-SI method considers the delay D as the center of gravity, assuming that $s(t)$ is a mass distribution.

The same expression can be derived for the estimate, but considering $s^2(t)$ instead of $s(t)$. When noise is present, the TDE-SI behaves differently when working with the signal, $\hat{D}_s$, than when working with the squared signal, $\hat{D}_{s^2}$. To study the behavior of these estimates when noise is present, we considered the deterministic signal $s(t)$ corrupted by an additive band-limited white zero-mean Gaussian noise $n(t)$. Thus, $s(t)$ and $v(t)$ become $s'(t) = s(t) + n_1(t)$ and $v'(t) = v(t) + n_2(t)$, respectively. Noise records $n_1(t)$ and $n_2(t)$ are assumed to be uncorrelated and to have the same standard deviation (σ). Under these assumptions we study the estimates $\hat{D}_s$ and $\hat{D}_{s^2}$, calculating the expected value and the variance of the estimates.

A. Estimate $\hat{D}_s$

From (3), we can express $\hat{D}_s$ as

$$\hat{D}_s = \frac{\int_{-a}^{a}\int_{-a}^{T}(s(t)+n_1(t))\,dt\,dT}{\int_{-a}^{a}(s(t)+n_1(t))\,dt} - \frac{\int_{-a}^{a}\int_{-a}^{T}(v(t)+n_2(t))\,dt\,dT}{\int_{-a}^{a}(v(t)+n_2(t))\,dt}. \tag{4}$$

With the noise assumptions we can consider

$$\varepsilon_i = \int_{-a}^{a} n_i(t)\,dt \neq 0 \tag{5}$$

where ε_i satisfy $\varepsilon_i \ll S_0$. Then $\hat{D}_s$ can be rewritten as

$$\hat{D}_s = \frac{\int_{-a}^{a}\int_{-a}^{T}(s(t)+n_1(t))\,dt\,dT}{S_0+\varepsilon_1} - \frac{\int_{-a}^{a}\int_{-a}^{T}(s(t-D)+n_2(t))\,dt\,dT}{S_0+\varepsilon_2}. \tag{6}$$

Considering that $\frac{\varepsilon_i}{S_0} \ll 1$, we can approximate

$$\frac{1}{S_0+\varepsilon_i} = \frac{1}{S_0}\frac{1}{\left(1+\frac{\varepsilon_i}{S_0}\right)} \simeq \frac{1}{S_0}\left(1-\frac{\varepsilon_i}{S_0}+\frac{\varepsilon_i^2}{S_0^2}\right) \tag{7}$$

and by combining (7) and (6) we get the approximation to $\hat{D}_s$. To study the statistical behavior of the estimate, we consider different record pairs or realizations of $s(t) + n_{1j}(t)$ and $s(t-D) + n_{2j}(t)$, where j denotes the different realizations with uncorrelated noise.

Mean of $\hat{D}_s$: Computing the ensemble expected values of the estimates E[$\hat{D}_s$], and considering the noise assumptions, there appear terms of the form:

$$\begin{aligned} &E[\varepsilon_i] = 0\ \forall i: \quad E[\varepsilon_i n_j(t)] = \begin{cases}\sigma^2 T_0 & i=j\\ 0 & i\neq j\end{cases}\\ &E[\varepsilon_i\varepsilon_j] = \begin{cases}\sigma^2 L T_0 & i=j\\ 0 & i\neq j\end{cases}: \quad E[\varepsilon_i^2 n_j(t)] = 0\ \forall i,j \end{aligned} \tag{8}$$

where i, j take the integer values 1 or 2, T_0 is the sampling period, and L is the observation interval $[-a, a]$ $\left(a = \frac{L}{2}\right)$ (Fig. 1). Note that the results are expressed in terms of discrete time signals. Taking the expected value of (6), using the approximation in (7) and the results in (8), we obtain [21]

$$E[\hat{D}_s] \simeq D\left(1+\frac{T_0\sigma^2 L}{S_0^2}\right) = D\left(1+\frac{1}{N\,\mathrm{SNR}_s}\right) \tag{9}$$

where SNR_s is the particular SNR defined by

$$\mathrm{SNR}_s = \frac{\left(\int_{-a}^{a} s(t)\,dt\right)^2/L}{L\sigma^2} = \frac{S_0^2/L}{L\sigma^2} \tag{10}$$

which represents the ratio between the dc signal energy (S_0^2/L) and the noise energy ($L\sigma^2$). N is the number of samples $\left(N = \frac{L}{T_0}\right)$ of each signal record. We can see that $\hat{D}_s$ is biased, and the bias is inversely proportional to the number of samples in the interval and to the SNR expressed as in (10). Usually in ECG signals $\frac{1}{N\,\mathrm{SNR}_s} \ll 1$, in which case the estimate $\hat{D}_s$ can be considered unbiased. The observation interval L that minimizes the bias is, obviously, the minimum length interval satisfying the condition in (1). Note that from (10), signals with high energy could have low SNR_s if their mean values are near to zero (low dc component). In these cases the estimate $\hat{D}_s$ could be highly biased and is not reliable.

Variance of $\hat{D}_s$: To study the variance ($\sigma^2_{\hat{D}_s}$) of the estimate $\hat{D}_s$ we calculate the value of $\sigma^2_{\hat{D}_s} = E[\hat{D}_s^2] - (E[\hat{D}_s])^2$. If we substitute (6) to calculate $\sigma^2_{\hat{D}_s}$, with the approximations in (7), we neglect terms of order higher than ε_i^2, and use the derivations of (8), we obtain [21]

$$\sigma_{\hat{D}_s} = \left(\frac{1}{\mathrm{SNR}_s}\left[\frac{N}{6}+\frac{D_N^2}{N}\right]\right)^{\frac{1}{2}} \tag{11}$$

where $\sigma_{\hat{D}_s}$ is expressed in number of samples and D_N is the delay also expressed in number of samples $D_N = D/T_0$. We see that, as expected, $\sigma_{\hat{D}_s}$ decreases when SNR increases. For a constant SNR_s, the $\sigma_{\hat{D}_s}$ increases with the number of samples in the interval N and with the real delay D_N.

B. Estimate $\hat{D}_{s^2}$

In this section we study the performance of $\hat{D}_{s^2}$ in a way analogous to that of the $\hat{D}_s$ case, seen in the previous section. From (3) we can express $\hat{D}_{s^2}$ as

$$\hat{D}_{s^2} = \frac{\int_{-a}^{a}\int_{-a}^{T}\left(s^2(t)+n_1^2(t)+2s(t)n_1(t)\right)dt\,dT}{\int_{-a}^{a}(s^2(t)+n_1^2(t)+2s(t)n_1(t))\,dt} - \frac{\int_{-a}^{a}\int_{-a}^{T}\left(s^2(t-D)+n_2^2(t)+2s(t-D)n_2(t)\right)dt\,dT}{\int_{-a}^{a}(s^2(t-D)+n_2^2(t)+2s(t-D)n_2(t))\,dt}. \tag{12}$$

To study the standard deviation of $\hat{D}_{s^2}$, we consider

$$\varepsilon_1 = \int_{-a}^{a} 2s(t)n_1(t)dt + \left[\int_{-a}^{a} n_1^2(t)dt - N_0\right] \neq 0,$$
$$\varepsilon_2 = \int_{-a}^{a} 2s(t-D)n_2(t)dt + \left[\int_{-a}^{a} n_2^2(t)dt - N_0\right] \neq 0 \quad (13)$$

and denoting

$$S_0' = \int_{-a}^{a} s^2(t)dt = \int_{-a}^{a} s^2(t-D)dt;$$
$$N_0 = L\sigma^2 = \int_{-a}^{a} E[n_i^2(t)]dt \quad (i = 1, 2) \quad (14)$$

and

$$\mathrm{SNR}_{s^2} = \frac{\int_{-a}^{a} s^2(t)dt}{L\sigma^2} = \frac{S_0'}{N_0} \simeq \frac{\int_{-a}^{a} s^2(t)dt}{\int_{-a}^{a} n_i^2(t)dt} \quad (i = 1, 2) \quad (15)$$

we have that $\varepsilon_i \ll [S_0' + N_0]$ $(i = 1, 2)$. Then the estimate $\hat{D}_{s^2}$ is expressed as

$$\hat{D}_{s^2} = \frac{\int_{-a}^{a}\int_{-a}^{T}\left(s^2(t) + n_1^2 + 2s(t)n_1(t)\right)dtdT}{S_0' + N_0 + \varepsilon_1} - \frac{\int_{-a}^{a}\int_{-a}^{T}\left(s^2(t-D) + n_2^2 + 2s(t-D)n_2(t)\right)dtdT}{S_0' + N_0 + \varepsilon_2} \quad (16)$$

where, given that $\left(\frac{\varepsilon_i}{S_0'+N_0} \ll 1\right)$, we can approximate

$$\frac{1}{S_0' + N_0 + \varepsilon_i} \simeq \frac{1}{S_0' + N_0}\left(1 - \frac{\varepsilon_i}{N_0 + S_0'} + \frac{\varepsilon_i^2}{(N_0 + S_0')^2}\right) \quad (17)$$

and by combining (16) with (17) we have the approximate estimate $\hat{D}_{s^2}$. Next, we carry out a study of $\hat{D}_{s^2}$ statistical behavior, analogous to that of the $\hat{D}_s$ case.

Mean of $\hat{D}_{s^2}$*:* Taking the expected value in (16) with the approximation of (17) there appear terms that, with the noise assumptions, can be expressed as (18), which will be found at the bottom of the page.

Then, the expected value of $\hat{D}_{s^2}$ turns out to be [21]

$$E[\hat{D}_{s^2}] \simeq D\frac{\mathrm{SNR}_{s^2}}{1 + \mathrm{SNR}_{s^2}}\left(1 - \frac{2}{N\,(1 + \mathrm{SNR}_{s^2})^2}\right) \simeq D\frac{\mathrm{SNR}_{s^2}}{1 + \mathrm{SNR}_{s^2}} = D\left(1 - \frac{1}{1 + \mathrm{SNR}_{s^2}}\right). \quad (19)$$

These expressions show that the estimate $\hat{D}_{s^2}$ is biased with a bias that depends on both SNR and the delay D.

Variance of $\hat{D}_{s^2}$*:* Following the same procedure as that of the $\hat{D}_s$ estimate, we calculate the variance of $\hat{D}_{s^2}$: $(\sigma^2_{\hat{D}_{s^2}} = E[\hat{D}^2_{s^2}] - E[\hat{D}_{s^2}]^2)$. By substituting (16) in $\sigma^2_{\hat{D}_{s^2}}$, using the approximation of (17), neglecting terms of order higher than ε_i^2, and using the derivations of (18), we obtain [21]

$$\sigma_{\hat{D}_{s^2}} \simeq \frac{\mathrm{SNR}_{s^2}^{\frac{1}{2}}}{(1 + \mathrm{SNR}_{s^2})}\left[\frac{D_N\left(\frac{4D_N}{N} - 1 + \mathrm{SNR}_{s^2}\left(\frac{2D_N}{N} - 2\right)\right)}{(1 + \mathrm{SNR}_{s^2})^2} + \frac{N\left(\frac{7}{3}\mathrm{SNR}_{s^2}^2 - \frac{4}{3}\mathrm{SNR}_{s^2} + \frac{1}{3}\right)}{(1 + \mathrm{SNR}_{s^2})^2\mathrm{SNR}_{s^2}} + \frac{8F}{N}\right]^{\frac{1}{2}} \quad (20)$$

where F is a signal dependent factor defined as

$$F = \frac{\int_{-a}^{a} s^2(t)t^2 dt}{T_0^2 S_0'}. \quad (21)$$

Again, the $\sigma_{\hat{D}_{s^2}}$ depends on the number of samples N, the SNR, the delay D, and a signal dependent factor F. Comparing expression (20) directly with that of $\sigma_{\hat{D}_s}$ (Equation 11) is difficult because of our definition here of the SNR (SNR_s and SNR_{s^2}); also there appears the signal-dependent factor F that must be analyzed in each case. The two SNR definitions are also related by a signal-dependent relation:

$$\mathrm{SNR}_s = \frac{\mathrm{SNR}_{s^2}\left(\int_{-a}^{a} s(t)dt\right)^2}{L\int_{-a}^{a} s^2(t)dt}. \quad (22)$$

In the simulations that we performed, we verified that for a QRS complex signal and for Gaussian-shape signals $\sigma_{\hat{D}_{s^2}}$ is lower than $\sigma_{\hat{D}_s}$, which would make $\hat{D}_{s^2}$ more suitable for TDE except for the bias. In the next section, we propose a bias-corrected TDE-SI estimate ($\hat{D}'_{s^2}$), based on the squared signal, which preserves the standard deviation lower than in the case of $\hat{D}_s$.

TABLE I
SIMULATION RESULTS (EXPER.) WITH THE MEAN AND STANDARD DEVIATION OF THE ESTIMATES IN CASE OF A GAUSSIAN-SHAPE SIGNAL, TOGETHER WITH THE THEORETICALLY PREDICTED VALUES (THEOR.). ALL VALUES ARE EXPRESSED IN MS.

	SNR_{s^2} (dB)	0	7	10	20	30
	SNR_{s^2}	1	5	10	10^2	10^3
	SNR_s	0.18	0.88	1.77	17.72	177.2
$E[\hat{D}_s]$	Theor.	10.56	10.11	10.06	10.01	10.00
	Exper.	10.16	10.07	10.05	10.01	10.00
$\sigma_{\hat{D}_s}$	Theor.	9.90	4.48	3.15	0.99	0.31
	Exper.	9.55	4.27	3.02	0.96	0.30
$E[\hat{D}_{s^2}]$	Theor.	4.98	8.33	9.09	9.90	9.99
	Exper.	5.03	8.36	9.11	9.91	9.99
$\sigma_{\hat{D}_{s^2}}$	Theor.	2.66	1.91	0.79	0.17	0.03
	Exper.	2.85	0.99	0.57	0.11	0.03
$E[\hat{D}'_{s^2}]$	Theor.	10.00	10.00	10.00	10.00	10.00
	Exper.	10.03	10.05	10.03	10.00	10.00
$\sigma_{\hat{D}'_{s^2}}$	Exper.	7.78	1.12	0.60	0.11	0.03

$$E[\varepsilon_i] = 0, \qquad E[\varepsilon_i\varepsilon_j n_k(t)] = 0,$$
$$E[\varepsilon_i\varepsilon_j] = \begin{cases} 2\sigma^2 T_0(2S_0' + N_0) & i = j \\ 0 & i \neq j \end{cases} \qquad E[\varepsilon_i n_j(t) n_k(t')] = \begin{cases} 2\sigma^4 T_0 & i = j = k \\ 0 & \text{otherwise} \end{cases}$$
$$E[\varepsilon_i n_j(t)] = \begin{cases} 0 & i \neq j \\ 2\sigma^2\begin{Bmatrix} s(t) \\ s(t-D) \end{Bmatrix} T_0 & i = j \end{cases} \qquad E[n_i^2(t)\varepsilon_j^2] = 2\sigma^4 T_0(2S_0' + N_0) \quad (18)$$
$$E[n_i^4(t)] = 3\sigma^4.$$

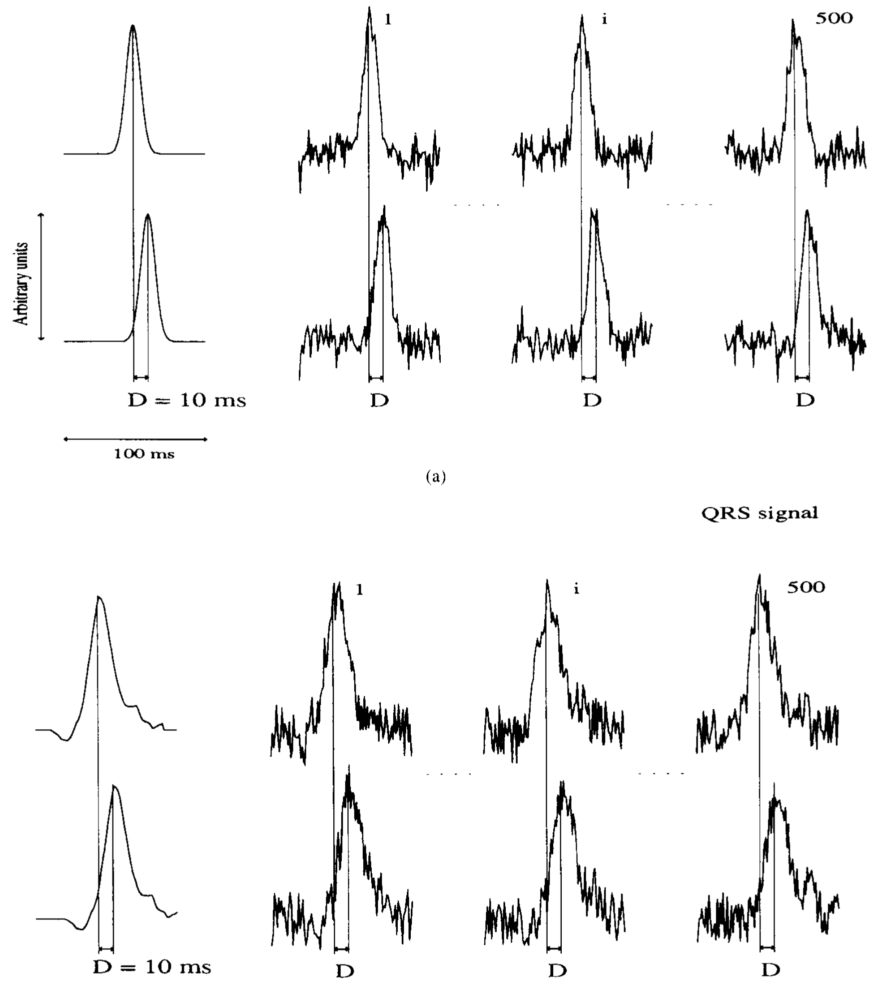

Fig. 2(a). Signals on the left are the Gaussian-shape signal, the uppermost is the original Gaussian-shape $s(t)$ ($\sigma = 5$ ms), and below is the delayed $s(t-D)$ with $D = 10$ ms. On the right there are some of the i ($i = 1, \ldots, 500$) computer-generated noisy pairs of delayed signals. Noise is generated to be white, zero mean and Gaussian, and the SNR in this case is $\mathrm{SNR}_{s2} = 10$; (b) the same two delayed artificially-generated ensembles with a real QRS complex as the deterministic signal $s(t)$.

C. Corrected Estimate $\hat{D}'_{s2}$

We propose a bias-corrected estimate that works in the following way. We recalculate $\hat{D}_{s2}$ by shifting $v'(t) = v(t)+n_2(t)$ to $v'(t-\tau_i)$, obtaining several $\hat{D}_{s2}(\tau_i)$ for different values of τ_i. From (19) we have that

$$E[\hat{D}_{s2}(\tau_i)] = (D+\tau_i)\frac{\mathrm{SNR}_{s2}}{1+\mathrm{SNR}_{s2}}. \tag{23}$$

Thus, with the pairs $(\hat{D}_{s2}(\tau_i), \tau_i)$ we can fit a linear function $\hat{D}_{s2}(\tau) = \alpha\tau + \beta$, where the calculated α and β values are related to the SNR_{s2}. Comparing (23) and the linear function we find that

$$E[\alpha] = \frac{\mathrm{SNR}_{s2}}{1+\mathrm{SNR}_{s2}} \text{ and } E[\beta] = D\frac{\mathrm{SNR}_{s2}}{1+\mathrm{SNR}_{s2}}. \tag{24}$$

After calculating α and β, we define a new estimate $\hat{D}'_{s2}$ as

$$\hat{D}'_{s2} = \frac{\beta}{\alpha}, \tag{25}$$

which, assuming α and β uncorrelated, is unbiased ($E[\hat{D}'_{s2}] = E[\beta/\alpha] = D$). Simulations will show that the variance of this estimate remains lower than that of $\hat{D}_s$. This makes $\hat{D}'_{s2}$ the more suitable estimate to calculate D using the TDE-SI method.

Estimation of Signal-to-Noise Ratio: Using the dependence of the TDE $\hat{D}_{s2}$ on the SNR_{s2} (19) and the calculation of the corrected estimate $\hat{D}'_{s2}$, we see [20] that the α value (24) is a function of SNR_{s2}. Thus, the value of α can be used to estimate the value of SNR_{s2}, assuming that noise remains white, zero-mean, and Gaussian. From (24) we have that $\mathrm{SNR}_{s2} = \frac{E[\alpha]}{1-E[\alpha]}$, and then we can estimate the SNR_{s2} ($\widehat{\mathrm{SNR}}_{s2}$) as

$$\widehat{\mathrm{SNR}}_{s2} = \frac{\alpha}{1-\alpha}. \tag{26}$$

This estimation of SNR_{s2} is very useful to estimate the performance of subsequent signal processing procedures. From the SNR_{s2} we can estimate the variance of the TDE calculated by using $\hat{D}_{s2}$ or by the methods related in [16] and [17]. The variance gives us an estimation of the accuracy of the measure. With this variance ($\sigma_{\hat{D}}$) we

TABLE II
SIMULATION RESULTS (EXPER.) WITH THE MEAN AND STANDARD DEVIATION OF THE ESTIMATES IN CASE OF A REAL QRS COMPLEX SIGNAL, TOGETHER WITH THE THEORETICALLY-PREDICTED VALUES (THEOR.). SAME NOTATION AS IN TABLE I, WITH VALUES EXPRESSED IN MS. THE LAST TWO ROWS SHOW THE $E[\alpha]$ AND σ_α ESTIMATED IN THE ENSEMBLE OF DELAYED QRS COMPLEX SIGNALS, AS WELL AS THE ESTIMATED $\widehat{SNR}_{s^2}$.

	SNR_{s^2} (dB)	0	7	10	20	30
	SNR_{s^2}	1	5	10	10^2	10^3
	SNR_s	0.31	1.54	3.08	30.81	308.1
$E[\hat{D}_s]$	Theor.	10.32	10.06	10.03	10.00	10.00
	Exper.	11.35	10.51	10.35	10.10	10.03
$\sigma_{\hat{D}_s}$	Theor.	7.56	3.38	2.40	0.75	0.24
	Exper.	7.67	3.29	2.31	0.73	0.23
$E[\hat{D}_{s^2}]$	Theor.	4.98	8.33	9.09	9.90	9.99
	Exper.	5.30	8.50	9.21	9.92	9.99
$\sigma_{\hat{D}_{s^2}}$	Theor.	2.80	2.05	0.95	0.25	0.07
	Exper.	1.98	0.91	0.63	0.19	0.06
$E[\hat{D}'_{s^2}]$	Theor.	10.00	10.00	10.00	10.00	10.00
	Exper.	11.28	10.29	10.17	10.03	10.00
$\sigma_{\hat{D}'_{s^2}}$	Exper.	4.06	1.06	0.68	0.19	0.06
QRS shape	$E[\alpha] \pm \sigma_\alpha$	0.51 ± 0.12	0.84±0.03	0.91±0.02	0.99±0.00	1.00±0.00
	$\widehat{SNR}_{s^2}$	1.04± 0.46	5.25±1.56	10.11±2.46	99.00±20.00	999±200

can estimate the cutoff frequency of a subsequent signal averaging process [3], [8] ($f_c = 132.3/\sigma_{\hat{D}}$), or quantify any other measure based on the TDE such as time interval variability in ECG signals (heart rate variability (HRV)) [6], [22].

III. SIMULATION RESULTS

To support the validity of the approximations used to derive the expected value and the standard deviation of the estimates, we calculate these values in a simulation study. We compare the experimental values with the theoretically predicted values.

We have taken two deterministic signals known *a priori*: a computer-generated Gaussian-shape signal ($\sigma = 5$ ms) and a QRS complex from a real ECG, both of duration 100 ms (Fig. 2). For each one of these signals we have generated two delayed ensembles with known delay $D = 10$ ms, adding zero-mean Gaussian band-limited white noise. Equation (1) is satisfied for $D_{\max} = 10$ ms. Each ensemble of signals has 500 records, and the sampling frequency is 1 kHz, which implies $N = 100$ samples. Fig. 2(a) shows the original delayed Gaussian-shape signals and the two delayed ensembles generated for $\mathrm{SNR}_{s^2} = 10$; Fig. 2(b) shows the generated signals from the QRS complex. We have computed the delay in the 500 pairs of delayed records using the three considered estimators ($\hat{D}_s$, $\hat{D}_{s^2}$, $\hat{D}'_{s^2}$) for SNR_{s^2} of 1, 5, 10, 10^2, and 10^3. The standard deviation of the 500 estimated delays has also been calculated.

To estimate $\hat{D}'_{s^2}$ we have shifted $v(t)$ to $v(t-\tau_i)$ for $\tau_i = i \cdot T_0 - 10$ where T_0 is the sampling interval, and i is an integer that goes from -10 to 10.

The theoretical and experimental results (mean and standard deviation of the estimates) in the case of the Gaussian-shape signal ($s(t) = Ae^{-t^2/50}$) are shown in Table I. In this case the F factor (21) takes the value $F = 12.5$ and from (22) the relation between SNR_s and SNR_{s^2} is given by $\mathrm{SNR}_s = \frac{\sqrt{\pi}}{10}\mathrm{SNR}_{s^2}$. From this Table I we verify that the theoretical predictions are in close agreement with the experimental data. This agreement improves as the SNR increases, as expected. We verify that the $\hat{D}_{s^2}$ is a biased estimate with the bias correctly predicted by expression (19). The behavior of $\hat{D}'_{s^2}$ is worth noting. From Table I we can see that this estimate corrects the bias of $\hat{D}_{s^2}$ causing, in turn, an increase in the standard deviation. This increase never reaches the level of the standard deviation $\sigma_{\hat{D}_s}$, which verifies that $\hat{D}'_{s^2}$ is the most suitable unbiased TDE-SI.

The results in the case of a QRS complex signal (Fig. 2(b) are shown in Table II. The only difference between this case and the previous one lies in the $s(t)$ signal, which now is a real QRS that extends 100 ms. For this particular $s(t)$ signal we have $F = 53.84$ and the relation between SNR_s and SNR_{s^2} is $\mathrm{SNR}_s = 0.31\ \mathrm{SNR}_{s^2}$. The same conclusions as from Table I are reached for a typical QRS complex wave shape, which is where the TDE is applied in practice. In these signals the delay D physically corresponds to the beat-to-beat difference of the QRS detector position relative to the real physiological origin of the heart beat. The differences between the "theoretical" values from Tables I and II are due to the differences in the signal-dependent parameter F and in the relation $\mathrm{SNR}_s(\mathrm{SNR}_{s^2})$. Note that depending on signal shape, for the same SNR_{s^2}, $\sigma_{\hat{D}_{s^2}}$ can have very large variations, mostly at low SNR (20).

To test the validity of the SNR estimation procedure presented in (26) we have considered the 500 pairs of delayed QRS complex signals in Fig. 2(b) and we have computed the α value in each pair. Then we have applied (26), using the mean value of all the calculated α in each signal pair ($E[\alpha]$). The last two rows of Table II show the mean values ($E[\alpha]$) and standard deviation (σ_α) of the estimated α for SNR_{s^2} of 1, 5, 10, 10^2, and 10^3. $\widehat{\mathrm{SNR}}_{s^2}$ is the SNR_{s^2} estimated through (26) using the obtained $E[\alpha]$ values. These results show that the proposed method to estimate the SNR_{s^2} gives values $\widehat{\mathrm{SNR}}_{s^2}$, that are a good approximation to the magnitude order of the real SNR_{s^2}.

IV. DISCUSSION AND CONCLUSION

The studied TDE-SI method has been considered in three cases: working with the signal ($\hat{D}_s$), working with the squared signal ($\hat{D}_{s^2}$) and a corrected version of the later ($\hat{D}'_{s^2}$). Approximations of the estimates have been theoretically analyzed to calculate the mean and standard deviation of the TDE as a function of the number of samples in the interval, the SNR, the delay D, and the deterministic transient signal $s(t)$. We have shown that the estimate $\hat{D}_s$, when noise is present, is practically unbiased for signals with a high dc component, however this is an important restriction for a general use of $\hat{D}_s$. The estimate $\hat{D}_{s2}$ becomes biased, but with lower standard deviation than $\hat{D}_s$ ($\sigma_{\hat{D}_{s2}} < \sigma_{\hat{D}_s}$). Then, a new estimate $\hat{D}'_{s2}$ that corrects this bias is proposed. This $\hat{D}'_{s2}$ estimate exhibits a standard deviation $\sigma_{\hat{D}'_{s2}}$ higher than $\sigma_{\hat{D}_{s2}}$ ($\sigma_{\hat{D}'_{s2}} > \sigma_{\hat{D}_{s2}}$) but lower than $\sigma_{\hat{D}_s}$ ($\sigma_{\hat{D}'_{s2}} < \sigma_{\hat{D}_s}$) without bias; this estimate is the one that achieves lower mean-squared error ($\text{MSE} = \sigma^2_{\hat{D}} + (\text{bias})^2$). Thus, we conclude that the TDE-SI method performs best with the estimate $\hat{D}'_{s2}$. We have also presented a method based on the TDE-SI to estimate the SNR of signals contaminated by white, zero-mean, and Gaussian noise; a realistic scenario with high-resolution ECG signals.

Simulation results show a close agreement between the theoretically approximated predictions and the experimental results. Likewise, we can estimate the TDE-SI performance with a previous knowledge of the SNR. This is very useful in further signal processing procedures that use the TDE. In signal averaging we can estimate the cutoff frequency of the process ($f_c = 132.3/\sigma_{\hat{D}}$) [3], [8], and in ECG time interval variability analysis we can interpret the variability as physiological or due to the TDE method [6], [22].

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A Least-Squares Algorithm for Multipath Time-Delay Estimation

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***Abstract*—We consider the problem of estimating the arrival times of overlapping ocean-acoustic signals from a noisy received waveform that consists of attenuated and delayed replicas of a known transient signal. We assume that the transmitted signal and the number of paths in the multipath environment are known and develop an algorithm that gives least-squares (LS) estimates of the amplitude and time delay of each path. Direct computation of the LS estimates would involve minimization of a highly oscillatory error function. By allowing the amplitudes to be complex valued, a much smoother error function that is easier to minimize using gradient-based techniques is obtained. Using this property and the knowledge (derived from the data) of the spacing between adjacent minima in the actual LS error function, an efficient algorithm is devised. The algorithm is a function of a data-dependent parameter, and we give rules for choosing this parameter. The algorithm is demonstrated on a broad-band signal, using simulated data. The proposed method is shown to achieve the Cramér-Rao lower bound over a wide range of SNR's. Comparisons are made with alternating projection (AP) and estimate maximize (EM) algorithms.**

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