ESTIMATION OF SIGNAL-TO-NOISE RATIO IN SIGNALS CONTAMINATED BY WHITE NOISE

Pablo Laguna*, Hervé Rix#, Raimon Jané* and Pere Caminal*

* Institut de Cibernètica (UPC-CSIC), Diagonal 647, 08028 Barcelona, Spain
# Laboratoire de Signaux et Systèmes. Université de Nice - Sophia Antipolis. France.

ABSTRACT

A method to estimate the SNR is very useful since it allows a knowledge of the signal contamination. This estimation can be specially useful in signal averaging process of recurrent transient signals, because the error in the occurrence time estimation of each recurrence has been shown to be a function of the SNR. We propose a method to estimate the SNR in case of random noise. This method is based on a time delay estimate between two noisy transient signals. A simulation study, using a deterministic signal corrupted by additive white zero-mean gaussian noise, has shown that the proposed method is appropriate to estimate the SNR.

INTRODUCTION

Many signals are composed of a deterministic component of interest and a usually called noise component. This noise can be deterministic or random.

The accuracy and performance of most signal processing procedures are limited by the signal-to-noise ratio (SNR) of the analyzed signal. This occurs, for example, in determining fiducial points through time delay estimation techniques in signal averaging [1], or in estimating occurrence times of characteristic events (e.g. ECG waves [2]), and, in general, in any processing devoted to information recovery from noisy signals.

A method to estimate SNR is very useful since it allows a knowledge of signal contamination. One case where this estimation can be specially useful is in signal averaging of an ensemble of recurrent transient signals. This process is usually used to recover the deterministic component of the ensemble. The errors in the determination of the fiducial point, used in the averaging process, lead to a low-pass effect in the recovered signal. The cutoff frequency (f_c) of this process can be estimated if we know the standard deviation (σ) of the fiducial point estimation [3]. Different time delay estimates have been shown to result in a standard deviation σ of the estimate which is a function of SNR [3, 4]. Then the use of a time delay estimate to align the signal recurrences with a previous knowledge of SNR leads to an estimation of σ that allows the estimation of f_c in the averaging process. This will be important in cases like late potentials detection in high-resolution electrocardiography, where in this way we could know if absence of late potentials in processed signal can be due to filtering effect ($f > f_c$) or to real absence in the original deterministic component ($f < f_c$).

The knowledge of the SNR is important, even if we do not know the deterministic and noise components of the signal. When signal and noise are both deterministic, we cannot distinguish between them, nor esstimate SNR, if we do not have additional information. When the noise is random it is possible to estimate the SNR, and in this work we propose a method to estimate the SNR in this case. The method is based on a time delay estimate between two noisy transient signals [5].

We will consider that the signal duration is included in a finite interval and the time delay estimate is applied to two different records of the signal, with the same deterministic component and different noise components. The estimation given by the time delay estimate is biased when it works with the squared signal, and the bias is a function of SNR. This dependence will allow us to estimate the SNR of the signal with the assumption of random noise.

*This work was supported by grant TIC88-0204, from CICYT (Spain).

TIME DELAY ESTIMATION METHOD

This method is based on the normalized integrals of two delayed signals [5]. Let $s(t)$ be a signal defined in the interval $[-a, a]$, and we will assume parameter a large enough to satisfy that:

$$s(t) = 0 \quad t \notin [-a + D_{max}, a - D_{max}] , \quad D_{max} > 0 . \quad (1)$$

Let $S(t)$ be its normalized integral

$$S(t) = \frac{\int_{-a}^{t} s(t')\, dt'}{A_0} , \quad where \quad A_0 = \int_{-a}^{a} s(t)\, dt \neq 0 . \quad (2)$$

If $v(t)$ is another signal of the form $v(t) = s(t - D)$, whose normalized integral is $V(t)$, the delay D between the two signals $s(t)$ and $v(t)$ $(D < D_{max})$ can be computed by

$$D = \int_{-a}^{a} (S(t) - V(t))\, dt . \quad (3)$$

This relation defines the time delay estimation method. Same expression can be derived for the estimate considering $s^2(t)$ instead of $s(t)$. When signal is contaminated by noise, equation (3) leads to an estimate $(\hat{D})$ of the real value D, that will be denoted by $\hat{D}_{s^2}$ when working with the squared signal $s^2(t)$. We considered in this study the signal corrupted by an additive white zero-mean Gaussian noise $n(t)$. In this case $s(t)$ and $v(t)$ become $s'(t) = s(t) + n_1(t)$ and $v'(t) = v(t) + n_2(t)$, respectively. $n_1(t)$ and $n_2(t)$ are supposed to be uncorrelated with the same standard deviation (σ_n). Under these assumptions we study the estimate $\hat{D}_{s^2}$.

From (3) we can express $\hat{D}_{s^2}$ as

$$\hat{D}_{s^2} = \frac{\int_{-a}^{a} \int_{-a}^{T} (s(t) + n_1(t))^2\, dt\, dT}{\int_{-a}^{a} (s(t) + n_1(t))^2\, dt} - \frac{\int_{-a}^{a} \int_{-a}^{T} (v(t) + n_2(t))^2\, dt\, dT}{\int_{-a}^{a} (v(t) + n_2(t))^2\, dt} . \quad (4)$$

With the assumptions on noise considered above we can approximate $\int_{-a}^{a} 2s(t)n_i(t)\, dt = 0$, $\int_{-a}^{a} \int_{-a}^{T} 2s(t)n_i(t)\, dt\, dT = 0$ $(i = 1,2)$ and $\int_{-a}^{a} \int_{-a}^{T} (n_1^2(t) - n_2^2(t)) = 0$. Then the estimate $\hat{D}_{s^2}$, in this first order approximation, results in

$$\hat{D}_{s^2} = \frac{\int_{-a}^{a} \int_{-a}^{T} (s^2(t) - v^2(t))\, dt\, dT}{A + B} = D \frac{A}{A + B} \quad (5)$$

CH3068-4/91/0000-0363 $01.00

where

$$A = \int_{-a}^{a} s^2(t)\,dt\,, \quad and \quad B = \int_{-a}^{a} n_i^2(t)\,dt = 2a\sigma_n \quad (i = 1,2). \quad (6)$$

Defining the signal-to-noise ratio (SNR) as

$$SNR = \frac{\int_{-a}^{a} s^2(t)\,dt}{\int_{-a}^{a} n_i^2(t)\,dt} = \frac{A}{B}\,, \qquad (7)$$

$\hat{D}_{s^2}$ can be expresed as

$$\hat{D}_{s^2} = D\frac{SNR}{1+SNR}\,. \qquad (8)$$

The estimate $\hat{D}_{s^2}$ is biased. This bias depends on the SNR and the exact delay D.

SNR ESTIMATION

Using this dependence of $\hat{D}_{s^2}$ with SNR (8) we propose a method to estimate the SNR. To do that, we can recalculate $\hat{D}_{s^2}$ by shifting $v'(t)$ to $v'(t-\tau_i)$ obtaining several $\hat{D}_{s^2 i}$ for different τ_i values. Then we have a linear function $\hat{D}_{s^2 i}(\tau_i)$

$$\hat{D}_{s^2 i} = \alpha\tau_i + \beta. \qquad (9)$$

In this case (8) becomes

$$\hat{D}_{s^2 i} = (D+\tau_i)\frac{SNR}{1+SNR} \qquad (10)$$

and comparing (9) and (10) we can see that

$$\alpha = \frac{SNR}{1+SNR} \quad \beta = D\frac{SNR}{1+SNR}. \qquad (11)$$

We can have a SNR estimation given that α depends of SNR, and we obtain

$$SNR = \frac{\alpha}{1-\alpha} \qquad (12)$$

This estimation of SNR is also valid if we had considered $v'(t) = s'(t)$. In this case $D = 0$ and if noise is white same approximations can be done leading to

$$\hat{D}_{s^2 i} = \tau_i\frac{SNR}{1+SNR} \qquad (13)$$

and then the calculation of the linear function slope will lead to the estimation of SNR even if we have only one record of the signal under study.

SIMULATION STUDY

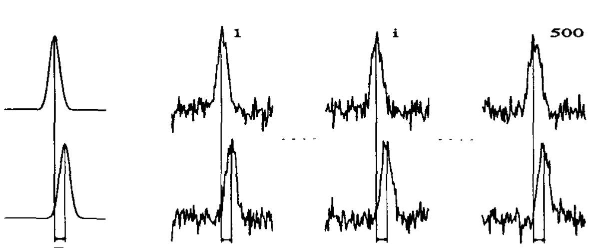

Figure 1: Simulated signals

The simulation study has considered a deterministic signal corrupted by computer generated noise for different SNR. For each SNR we have generated two ensembles of signals by adding white zero-mean Gaussian noise to the deterministic signal, and to the same deterministic signal delayed a time distance $D = 10$ ms. The deterministic signal used is a computer generated Gaussian signal ($\sigma = 5$ ms) that extends 100 ms. Each ensemble of signals has 500 records, and the sampling frequency is 1 kHz. Fig. 1 shows the original delayed Gaussian signals and the two delayed ensembles generated for SNR=10. We have computed the expected value of the estimated delay $\hat{D}_{s^2}$ applied to the 500 pairs of signals and the α values for SNR of 1, 5, and 10. Table 1 shows the expected value of the delays for different SNR, obtained theorically and experimentally. We see that the expression reached for the bias of $\hat{D}_{s^2}$ (8) agrees with simulation results.

Table 1. Expected values of the delays $\hat{D}_{s^2}$ (ms)

SNR	1	5	10	10^2	10^3
Theoretical	5.00	8.33	9.09	9.90	9.99
Experimental	5.03	8.36	9.11	9.91	9.99

Table 2 shows the mean values ($\overline{\alpha}$) and standard deviation of α measured in the different pairs of signal realizations with SNR of 1, 5 and 10. SNR' is the SNR estimated through equation (12) and the obtained $\overline{\alpha}$ values. These results show that the proposed method to estimate the SNR gives values that allow a good approximation to the real SNR.

Table 2. SNR estimation

SNR	1	5	10
$\overline{\alpha}$	0.52 ± 0.18	0.82±0.07	0.89±0.04
SNR'	1.08± 0.78	4.55±2.16	8.09±3.30

CONCLUSIONS

We have proposed a method to estimate SNR of a signal with the assumption that noise is white and zero-mean. This method is based on a time delay estimate that calculates the normalized integrals of the squared signals. The behaviour of this estimate is biased with a bias that is a function of the SNR. This property is used to estimate SNR. This result is very important given that can be applied to any signal, even if this signal is not repetitive. In this case we can consider $s'(t) = v'(t)$, and the same derivation for SNR estimation remains valid. The SNR information obtained can be used to analize the signal quality or to know the limits of subsequent signal processing as in case of signal averaging.

REFERENCES

[1] Rompelman O. and Ros H.H. (1986), "Coherent averaging technique: A tutorial review. Part 1: Noise reduction and the equivalent filter. Part 2: Trigger jitter, overlapping responses and non-periodic stimulation", *J. Biomed. Eng.*, vol. 8, pp. 24-35.

[2] Koeleman A.S.M., van den Akker T.J., Ros H.H., Janssen R.J., and Rompelman O. (1984), "Estimation accuracy of P-wave and QRS complex occurrence times in the ECG: The accuracy for simplified theoretical and computer simulated waveforms", *Signal Processing*, vol. 7, n. 4, pp. 389-405.

[3] R. Jané, H. Rix, P. Caminal, P. Laguna. Alignment methods for averaging of high resolution cardiac signals: a comparative study of performance. *IEEE Trans. Biomed. Eng.* (to appear). June 1991.

[4] Laguna P. (1990). Nuevas técnicas de procesado de señales electrocardiográficas: aplicación a registros de larga duración. *Doctoral Thesis (in spanish)* Faculty of Science, University of Zaragoza, Spain.

[5] H. Rix, S. Jesus. Estimation du retard entre signaux de même forme. *C.R. Acad. Sc. Paris.*, t. 299, Serie II, n 8, pp. 399-404. 1984.

CH3068-4/91/0000-0364 $01.00 © 1991 IEEE