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Sampling Rate and Time Alignment in the Estimation of QRS Variability

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Abstract— **In this work we study the sampling frequency effects in time alignment and averaging in the analysis of high resolution ECGs (HRECG). The resolution of time alignment is limited by noise and the sampling interval. We theoretically model the sampling rate effect, both in the estimated ensemble averaged (EA) signal and in the estimated variability with the ensemble deviation (ED) signal. We predict that for SNR of 30 dB the sampling frequency should be of around 4 kHz to get misalignment effect of the order of 10% the noise effect and then consider misalignment negligible. The results were quantifled in terms of ED signal power, and the power of the error signal. Experimental results corroborate the theoretical predictions and lead us to propose 4 kHz as the desired sampling rate for HRECG variability studies rather than 1 kHz that is routinely used in many places.**

I. Introduction

Averaging of time-aligned beats is a wide-spread technique used to improve the signal-to-noise ratio (SNR) of high resolution ECG signals. It makes use of the repetitive nature of signals and of the lack of correlation between noise and the ECG. This technique is applied to study ventricular late potentials (VLP), useful to identify patients at risk of ventricular tachycardia from those who had suffered myocardial infarction [1], to study His-Purkinje activity [2] or atrial late potentials [3]. Time alignment is also important when morphological beat-to-beat QRS variability is studied e.g. in patients with signs of infarction and ischemia [4]. In VCG loop analysis it is also necessary to use time-alignment techniques [5]. This studies make the assumption that the ensemble variability signal, measured as the variance of the ensemble, is intrinsic signal variability which may reflect variability in e.g. conduction pathways. It remains to be studied how the corrupting factors accompaning ECG signals affect the variability estimation. This work will address the influence in the variability estimation of misalignment (limited by sampling rate), and noise. The accuracy of high-frequency ECG analysis is limited by the presence of noise, time alignment precision and signal nonstationarity. The sampling rate is usually selected according to the expected frequency content of the ECG. However, the sampling rate also limits the alignment precision to a value proportional to the sampling period. We propose either a higher sampling rate or an interpolation procedure prior to alignment which reduces the dispersion of the alignment error. In the methods section we analytically model the effect of misalignment in both the estimated signal and the estimated signal variability.

II. Methods

The limitation of the alignment methods given by sampling rate, f_s, can be reduced by increasing the sampling rate. This limitation will be reduced by a factor which is proportional to the increased sampling rate factor.

In ensemble averaging of repetitive signals there are two main results that are used for clinical information extraction. 1) the ensemble averaged (EA) signal and 2) the ensemble deviation (ED) signal. The EA is the result of the average that is used when the deterministic underlying signal is of interest. The ED signal is of interest when studies about morphological variability are done. In these studies the power of the ensemble deviation signal is used as clinical index. We study in the next subsections how the noise, misalignment and signal variability affect, by themself, the estimation of EA and ED signals. We quantify the influence of these factors in terms of the power of the estimated error signal, P_e, and the power of the ensemble deviation signal, P_d.

A. Misalignment in noise-free signals

To study the effect of alignment (or misalignment) we assume that $s(t)$ is a recurrent, deterministic signal which is defined in a time interval (a, b) of L seconds. Then we will suppose each i:th realization in the ensemble is formed by the signal $s(t)$ with a random variable delay τ_i that becomes in the random process $s(t-\tau_i)$. In this way we have the ensemble clean of noise and signal variability, that are the other two sources of misestimation in signal averaging. We are able then to independently study the misalignment effect. The effect in EA signal estimation will be quantified throught the error signal, $e(t)$, between the original signal $s(t)$ and the EA estimated signal, $\hat{s}(t)$:

$$e(t) = s(t) - \hat{s}(t); \quad \text{with} \quad \hat{s}(t) = \frac{1}{N}\sum_{i=1}^{N} s(t-\tau_i). \qquad (1)$$

The effect on the ED signal will be studied through the ED signal, $d(t)$, defined as:

$$d(t) = \sqrt{\frac{\sum_{i=1}^{N}\left(s(t-\tau_i) - \hat{s}(t)\right)^2}{N}}. \qquad (2)$$

Usually the information is obtained with the power of the estimated signal $\hat{s}(t)$, like RMS40 in VLP analysis [1], or the power of the ED signal $d(t)$ as in morphological variability analysis [4]. So we will study the alignment influences in the power of $e(t)$ and $d(t)$. To study this we can make the assumption that the delay τ_i is small compared to the signal variations of $s(t)$; in other words the delay τ_i is small compared to the period of the frequency components of the ECG. This is usually satisfied in ECG averaging applications where the misalignment is of the order of a few milliseconds, and the spectra of the signal do not extend significantly further than 40 Hz. With this assumption we can approximated $s(t-\tau_i)$ with a Taylor series expansion in a similar way as in [6].

$$s(t-\tau_i) \approx s(t) - s'(t)\tau_i + \frac{1}{2!}s''(t)\tau_i^2 \qquad (3)$$

The error signal $e(t)$, assuming the random delay τ_i zero-meaned $\left(\frac{\sum_{i=1}^{N}\tau_i}{N}=0\right)$ with variance $\sigma_\tau^2 = \frac{\sum_{i=1}^{N}\tau_i^2}{N}$, will approach zero in a first order Taylor approximation and in the second order comes:

$$e(t) = \frac{\sum_{i=1}^{N}\left(s'(t)\tau_i - \frac{1}{2!}s''(t)\tau_i^2\right)}{N} = -s''(t)\frac{1}{2!}\sigma_\tau^2. \quad (4)$$

The ED signal $d(t)$, taking only the first order Taylor approximation results in:

$$d(t) = \sqrt{\frac{\sum_{i=1}^{N}\left(-s'(t)\tau_i + \frac{\sum_{i=1}^{N}s'(t)\tau_i}{N}\right)^2}{N}} = s'(t)\sigma_\tau. \quad (5)$$

From (4) and (5) we already note that the effect of a misalignment ensemble average can be more important in the ED indexes that in the EA indexes. To quantify this we can obtain the power of $e(t)$ signal, P_e^τ, and that of the $d(t)$ signal, P_d^τ. The upperindex, τ, refer to power due to misalignment. Operating we obtain:

$$P_e^\tau = \frac{1}{4L}\int_a^b s''^2(t)\sigma_\tau^4 dt = \frac{\sigma_\tau^4}{4M^2T_s^4}\sum_{k=-M/2}^{M/2-1}\left(\frac{2\pi k}{M}\right)^4|S(k)|^2 \quad (6)$$

$$P_d^\tau = \frac{1}{L}\int_a^b s'^2(t)\sigma_\tau^2 dt = \frac{\sigma_\tau^2}{M^2T_s^2}\sum_{k=-M/2}^{M/2-1}\left(\frac{2\pi k}{M}\right)^2|S(k)|^2 \quad (7)$$

Where $S(k)$ is the Discrete Fourier Transform (DFT) of s(t) sampled with a sampling period T_s, M is the number of samples in each recurrence ($MT_s = L$). From this expression we see that the misalignment, σ_τ, will affect the P_e^τ and the P_d^τ differently. If the misalignment σ_τ/T_s expresed in samples, is smaller than one sample the factor $(\sigma_\tau/T_s)^4$ will have much lower effect in the P_e^τ than the factor $(\sigma_\tau/T_s)^2$ in the P_d^τ. Also the frequency distribution energy will influence the expressions in (6) and (7).

B. Noisy signals with no misalignment

Although we have in the previous section studied the influence in alignment with the P_e^τ and P_d^τ, it will be important to compare those values with the contribution of noise in the same parameters. To study that we can easily come with the expression of P_e^n and P_d^n (upperindex, n, referring to noise) when noise is present in some SNR, perfect alignment is achieved, and no morphological variability is present a the signal. Each record in the ensemble will be $s(t) + n_i(t)$.

It is well known [7] that the P_e^n (power of the remaining noise after averaging N realizations) takes the value $P_e^n = \frac{\sigma_n^2}{N}$, where σ_n is the deviation of the noise. Performing a similar calculation for the ED signal $d(t)$, and assuming noise stationary and no correlation between realizations we have:

$$d(t) = \sqrt{\frac{\sum_{i=1}^{N}\left(\left(s(t)+n_i(t)\right) - \frac{\sum_{i=1}^{N}\left(s(t)+n_i(t)\right)}{N}\right)^2}{N}} = \sigma_n(t) = \sigma_n. \quad (8)$$

Then the P_d^n takes the value $P_d^n = \sigma_n^2$. Again we have much more influence of the noise in the ED signal than in the EA estimation. The variability indexes are much more affected by noise than the signal estimate indexes.

Assuming that noise $n(t)$ is uncorrelated with the misalignment effect the combined effect of having both noise and misalignment result in an additive power of the estimation error ($P_e = P_e^n + P_e^\tau$) and similarly with the power of ED signal ($P_e = P_e^n + P_e^\tau$). We can now deal with the adequate sampling rate to have a misalignment effect neglegible with respect to the noise effect. To allow that we can impose that the power given by the misalignment being of lower magnitude order than that of the noise. This means we force $P_e^\tau = P_e^n/10$ and/or $P_d^\tau = P_d^n/10$. Forcing that we get

$$P_e^\tau = \frac{\sigma_\tau^4}{4M^2T_s^4}\sum_{k=-M/2}^{M/2-1}\left(\frac{2\pi k}{M}\right)^4|S(k)|^2 = \frac{\sigma_n^2}{10N} = \frac{P_0}{10\cdot N\cdot SNR_l} \quad (9)$$

$$P_d^\tau = \frac{\sigma_\tau^2}{M^2T_s^2}\sum_{k=-M/2}^{M/2-1}\left(\frac{2\pi k}{M}\right)^2|S(k)|^2 = \frac{\sigma_n^2}{10} = \frac{P_0}{10\cdot SNR_l} \quad (10)$$

where SNR_l represent the signal-to-noise relation measure as linear relation rather than logarithmical, and $P_0 = E_0/L$ is the $s(t)$ signal power. This expressions represent two linear relation between SNR and σ_τ that will give the recommended sampling rate ($f_s = \frac{1}{2\sqrt{3}\sigma_\tau}$ [7]) to have the effect of the misalignment negligible with respect to the noise effect, and assuming alignment only limited by sampling interval.

C. Morphological variability

The final factor affecting signal averaging is the morphological variability present at the signal. To study this effect we will again consider the ideal case where neither noise nor misalignment is present at the signal. Each record in the ensemble can be expressed as $s(t) + v_i(t)$, where $v_i(t)$ represent the variable signal at each record. We can assume that the $v_i(t)$ is not stationary during the record but is along records. In this way we model the fact that the variability can be higher at the central part of QRS than at the terminal parts, and this behavior is repeated across records. We denote P_v the power of the variability signal. This analysis can be made in a parallel way to that made for the noise, assuming that $v_i(t)$ is uncorrelated from record to record in the ensemble. It appears a contribution to the power of the error signal (P_e^v) due to the morphological variability and other for the power of the ED signal (P_d^v):

$$P_e^v = \frac{1}{L}\int_a^b \frac{\sigma_v^2(t)}{N}dt = \frac{P_v}{N} \;;\; P_d^v = \frac{1}{L}\int_a^b \sigma_v^2(t)dt = P_v \quad (11)$$

$$\sigma_v^2(t) = \frac{\sum_{i=1}^{N}v_i^2(t)}{N} \;\; and \; P_v = \frac{1}{L}\int_a^b \sigma_v^2(t)dt \quad (12)$$

Similar reasoning to that made for the case of the noise can be reproduced here, but with a different clinical meaning. Recommended sampling frequency should now be looked as the frequency we need to sample the signal to reliably detect morphological variability as low as having a $SNR_l = P_0/P_v$. Of course this also implies that the noise is at lower levels than the variability.

D. Composite variability effect

The total power of the error or variability signals that we will find in real cases will be the sum of the three contributions, $P_e = P_e^\tau + P_e^n + P_e^v$ and $P_d = P_d^\tau + P_d^n + P_d^v$, assuming that none of the three components are correlated. The objective is to keep the alignment and noise contribution as low as possible, variability is intrinsic to the signal an can have some clinical meaning.

If we know some $t = t_0$ where we expect $\sigma_v^2(t_0) = 0$ and $d^{\tau 2}(t) = 0$, we can make a estimate of the noise contribution $\sigma_n^2 \simeq d^2(t_0)$ and then construct a new ED signal as $\hat{d}^2(t) = d^2(t) - d^2(t_0)$ and measure the P_d on this new signal. If the signal around t_0 is constant the contribution of the misalignment is null (5) and the estimation of the noise power will be reliable. This is a situation that appears at QRS signals since at the PR segment where constant waveform and negligible signal variability is present.

III. Results

To get an idea of the magnitude of these P_e^τ and P_d^τ values in high-resolution ECG analysis, we use a QRS complex recorder at 1000 Hz (Fig. 1). Computing the P_e^τ and P_d^τ for this QRS

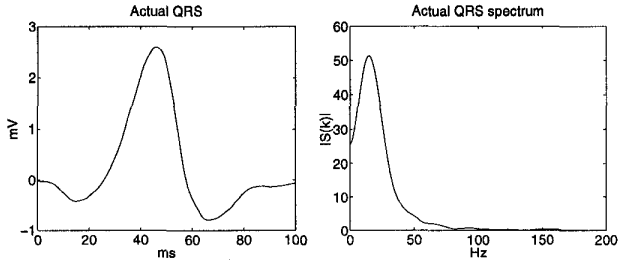

Fig. 1. A QRS and its spectrum, used in the simulations to estimate P_e^τ and P_d^τ.

waveform with a $\sigma_\tau = 1ms = 10^{-3}s$, the usual sampling period we obtain $P_e = 1.580 \cdot 10^{-4} P_0$ and $P_d = 1.2590 \cdot 10^{-2} P_0$. These values, and other waveforms tested, reveal that the effect of a misalignment is between one and three orders of magnitude smaller in the error signal than in the ED signals, for the ECG case. So, the reduction of the alignment error σ_τ will decrease both indexes, but the effect will be much more noticeable in the ED signal than in the error signal. We should keep in mind that the P_e^τ and P_d^τ indexes will have also contributions from noise and signal variability. This makes us predict a much higher effect of alignment improvement in variability analysis than in VLP that are measures of RMS of the EA signal.

A. Sampling rates with the presence of noise

To compare in a realistic situation which contribution is most influential (that from noise or from misalignment) and in which indexes, we will estimate $P_{e,d}^{\tau,n}$ for several SNR and several misalignment σ_τ. We consider the QRS signal of energy E_0, power $P_0 = E_0/L$, and averaging 100 beats (N=100). In Table I we have this comparison. Now we can address the question of which one is the required sampling rate according to the criteria exposed in II-B. For that, we can solve (9) for EA estimation and (10) for ED estimation. Averaging N=100 beats we have the following recommended sampling frequencies depending on the SNR to be the misalignment effect lower

TABLE I

$P_e^{\tau,n}$, AND $P_d^{\tau,n}$ FOR SEVERAL σ_τ AND SNRs. N=100 BEATS

σ_τ (ms)	P_e^τ	P_d^τ
4	$4 \cdot 10^{-2} P_0$	$2 \cdot 10^{-1} P_0$
1	$1 \cdot 10^{-4} P_0$	$1 \cdot 10^{-2} P_0$
0.25	$6 \cdot 10^{-7} P_0$	$0.7 \cdot 10^{-3} P_0$

SNR (dB)	P_e^n	P_d^n
0	$10^{-2} P_0$	P_0
10	$10^{-3} P_0$	$10^{-1} P_0$
20	$10^{-4} P_0$	$10^{-2} P_0$
30	$10^{-5} P_0$	$10^{-3} P_0$

than a 10% of the noise effect. Results are presented in table II. Since the noise also affects the misalignment estimation,

TABLE II

f_s IN Hz

0 dB		10 dB		20 dB		30 dB	
EA	ED	EA	ED	EA	ED	EA	ED
181	102	323	324	576	1024	1023	3239

this values are a lower limit since in real data the σ_τ will be higher than that from the marked sampling rates, making the effect of misalignment larger than 10% that of the noise. Then showing that sampling rates at usual SNR in high-resolution ECG should be increased up to 4 kHz when variability indexes are to be measured.

B. Composite variability simulation

To corroborate the previous derived results we address the following simulation. We take the actual QRS presented in Fig. 1, now upsampled to 4 kHz, and we generate four different ensembles with N=100 realizations. The power of the QRS is normalized to one ($P_0 = 1$). The first one, $s(t) + n_i(t)$ is constructed by adding white stationary Gaussian noise at a SNR=30 dB similar to good quality high-resolution ECG. The overprinted ensemble can be show in Fig. 2. The second ensemble is generated by delaying each realization $s(t - \tau_i)$ a normally distributed random delay τ_i with $\sigma_\tau = 0.288$ ms. This delay emulates the best alignment we can have with a $f_s = 1000$ Hz (Fig. 2). The third ensemble is generated by emulate morphological variability $v(t)$ according to the following rule $s(t) + v_i(t) = s(t) + a_i s(t)$, where a_i is a uniform random variable in (-0.1, 0.1), see Fig. 2. This tries to emulate amplitude variations as occurs with the respiration in normal patients. Finally we construct a fourth ensemble that includes the three effects together $s(t - \tau_i) + n_i(t) + v_i(t)$ (Fig. 2). With these ensembles we estimate in the four cases the estimated signal $\hat{s}(t)$, the error signal $e(t)$, and the ED signal $d(t)$. Then we obtain the powers of each ED signals. In Fig. 3, $d^2(t)$ signals are superimposed together with their total power. The $P_d^n = 0.0010$ agrees with the predictions in Table I, $P_d^\tau = 0.00128$ also agrees with the predicted result. If we are interested in estimating the P_d^v as a clinical index we have a true value of $P_d^v = 0.00341$. When we estimate this value from the ensemble containing all effects (real case) we can take as P_d^v the value of P_d, in this case $P_d = 0.00572$ that is an erroneous estimate. If, as suggested in the method section, we subtract P_d^n estimated at some area where $d^v(t)$ and $d^\tau(t)$ are negligible (last part of the QRS complex in this case) we can obtain a estimate for P_d^v that approximates the value $P_d - P_d^n = 0.0047$ that is also a erroneous estimate of the real value $P_d^v = 0.0034$. This is because the P_d^τ is not negligible with respect to the variability power we are interested in measuring. If we go to Table II, we realize that 30 dB in signal variability requires un

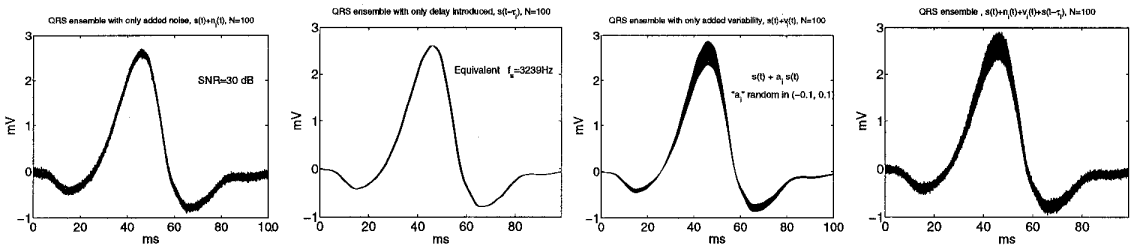

Fig. 2. *From left to right ensembles of 100 realizations when: only adding noise at SNR=30 dB, only adding normally random delay τ_i with $\sigma_\tau = 0.288$ ms (equivalent to best alignment with $f_s = 1000$ Hz), only adding morphological variability as $s(t) + a_i s(t)$ with a_i uniformly random in (-0.1, 0.1), and finally ensemble with the three effect together.*

increase to 3239 Hz in f_s to be detected properly, making the delay negligible with respect to the variability. We performed

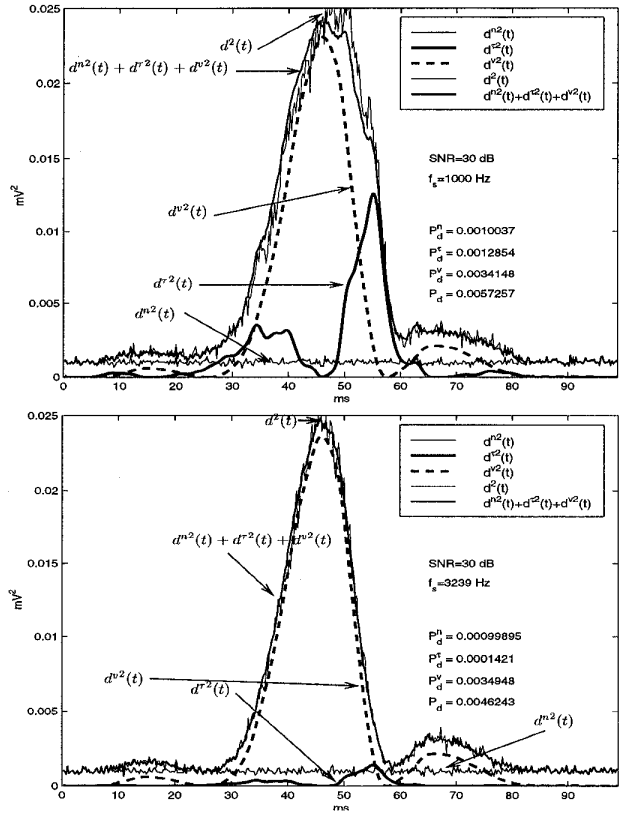

Fig. 3. *Squared ED $d^2(t)$, signals when averaging the different ensembles in Fig. 2. Also the sum of the independently estimated ED is overprinted $d^{n\,2}(t) + d^{\tau\,2}(t) + d^{v\,2}(t)$. Note the basic coincidence between the actual total $d^2(t)$ signal and the sum of the independently estimated. Upper pannel is for f_s= 1000 Hz, lower panel is for f_s=3239 Hz. For more details see text.*

again the previous experiment but now changing the random delay to have a $\sigma_\tau = 0.0891$ ms (equivalent to best alignment with $f_s = 3239$ Hz according to Table II). Proceeding to the shapes and values of the obtained deviation signals (Fig. 3 bottom) we appreciate the same order of values for P_d^n and P_d^v and a decrease of one order of magnitude for P_d^τ, as predicted. Now the same calculation than before will lead us to an estimation of P_d^v of 0.0036 that compared with the real value of 0.0035 becomes in an acceptable estimation. Note that since experiments reflected in Figs. 3 top and bottom correspond to different ensembles the value of P_d^v is not exactly equal in both experiment.

Also note in Figs. 3 that the total $d^2(t)$ signal and that obtained by summing the three individuals $d^{n\,2}(t) + d^{\tau\,2}(t) + d^{v\,2}(t)$ have a good fit then confirming that the assumption of lack of correlation between the three components made in methods section is a reasonable approximation and then the obtained results are validated.

IV. Conclusions

In this study we have consider the sampling rate and the alignment effect when obtaining ensemble averaged and ensemble deviation (ED) signal. We demonstrate that for SNR of 30 dB (typical at HRECG) up to 4 kHz of sampling rate should be necessary to neglect alignment effect in ED signals. It was shown that the ED signal power not only depends of the misalignment but also on the spectrum of the signal components. This spectrum dependency should be considered when studying beat-to-beat variability with the ED signal power, since this will depend of the dynamics of the signal, the misalignment and the spectrum of the analyzing signal. noindent

Acknowledgements

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References

[1] G. Breithardt, M. E. Cain, et al., "Standards for analysis of ventricular late potentials using high resolution or signal-averaged electrocardiography," *Eur. Heart J.*, vol. 12, pp. 473–480, 1991.

[2] H. Ros, A. Koeleman, and T. van den Akker, *The signal averaging technique in clinical cardiology: The Technique of Signal Averaging and its Practical Application in Separation of Atrial and His-Purkinje Activity*, pp. 3–15, F.K. Schattauer Verlag., Stuttgart, 1981.

[3] T. Engel, N. Vallone, and J. Windle, "Signal-averaged electrocardiograms in patients with atrial fibrillation or fluter," *Am Heart J.*, vol. 115, no. 3, pp. 592–597, 1988.

[4] B. Wohlfart, L. Sörnmo, J. Berg, and O. Pahlm, "Beat to beat QRS variability of 12-lead ECG in patients with scintigraphic signs of infarction and ischemia," in *Computers in Cardiology*. IEEE Computer Society Press, 1996, pp. 485–488.

[5] L. Sörnmo, O. Pahlm, and B. Wohlfart, "VCG loop alignment and morphologic variability," in *Proc. 18th Ann. Int. Conf. IEEE Eng. Med. Biol. Soc. Amsterdam, Holland, (CD-ROM)*. IEEE-EMBS, 1996.

[6] G. R. Shaw and P. Savard, "On the detection of QRS variations in the ECG," *IEEE Trans. Biomed. Eng.*, vol. 42, pp. 736–741, 1995.

[7] O. Rompelman and H. H. Ros, "Coherent averaging technique: A tutorial review. part 1: Noise reduction and the equivalent filter. part 2: Trigger jitter, overlapping responses and non-periodic stimulation," *J. Biomed. Eng.*, vol. 8, pp. 24–35, 1986.