Performance of RLS and LMS Algorithms in KL Estimation of Ischemic ECG Records *

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Abstract— **The Karhunen-Loève (KL) transform is a tool to analyze the repolarization period in the ECG. Adaptive algorithms improve the KL series estimation. The Recursive Least Squares (RLS) and Least Mean Squares (LMS) algorithms are studied when applied to estimate the KL coefficients of the ST-T complex in the ECG signal. The performance of RLS and LMS algorithms are compared both in improvement of signal-to-noise ratio (SNR) and in convergence rate. It is presented a specific initialization for the LMS algorithm that obtains same performance than RLS with lower calculations and without the numerical instability problem, making it the most suitable for the KL estimation.**

I. INTRODUCTION

Ischemia is a lack of oxygen in cardiac cells that if it is maintained can result in myocardial infarct. The ST-T complex of the ECG reflects the repolarization phase of the cardiac electrical cycle and ischemia is ussually reflected as changes in ST-T shape. Thus it would be very interesting to study the dinamic behavior of the ECG, tracking these changes to detect and/or monitorize ischemic episodes. The Karhunen-Loève transform (KLT) has been previously applied to study the ST-T complex [1]. It has been shown that adaptive estimation increases the quality of KL estimation reducing noise effects uncorrelated with the signal (50/60 Hz, muscular, baseline wander, artifacts, ...). A detailed description of the performance of two adaptive algorithms (RLS and LMS) in estimating the KL series of ischemic ECG records is presented.

II. THE KARHUNEN-LOEVE TRANSFORM ESTIMATION

The KLT is a signal-dependent linear transform which is optimal: it concentrates the maximum signal information in the minimum number of parameters, and it defines the domain where the signal and noise are most separated. The KL basis functions for the ST-T complex were derived from a training set of 100.000 beats [1]. The KL basis KL_{ik} are the n reference inputs to a transversal adaptive linear combiner filter whose primary input d_k is the concatenation of ST-T complexes from consecutive beats s_k plus noise n_k (figure 1). The weights after filtering each ST-T complex are the estimated KL coefficients.

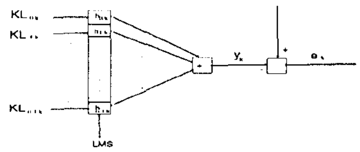

Figure 1: *Adaptive Linear Combiner Filter.*

*This work was supported by grant TIC94-0608-02 from CICYT, and PIT06/93 from CONAI. Spain

III. RECURSIVE LEAST SQUARES

The aim of RLS algorithm is to obtain, at each time k, a multiple linear regression model of the inputs and desired responses of the filter up to this time, in a recursive way. Formally, we want to minimize a cost function $\mathcal{E}(k)$ which exponentially weights the differences between the desired responses $d(i)$ and inputs $\mathbf{X}(i)$. It it expressed as $\mathcal{E}(k) = \sum_{i=1}^{k} W^{k-i}\left[d(i) - \mathbf{h}^T(k)\mathbf{X}(i)\right]^2$, where W is a *forgetting factor*, $\mathbf{X}(i)$ represents the KL basis function vector, and $\mathbf{h}(k)$ is a weight vector adapted to minimize $\mathcal{E}(k)$. The RLS algorithm produces after convergence an unbiased estimation of the desired signal. The exponential nature of the estimators, i.e., the finite window effect leads to an *estimation-noise* which results in a misadjustment $\mathcal{M}$ of the weight vector from its optimal setting [2]:

$$\mathcal{M} = \frac{1-W}{1+W}n \qquad (1)$$

where n is the number of taps (basis functions) in the adaptive filter. In nonstationary environments a $W < 1$ value is selected so that the algorithm presents finite memory and tracking capability. This choice results in a convergence time of [2]: $\tau = \frac{1}{1-W}$. Now the SNR improvement at the estimated KL series achieved by the RLS adaptive estimation in stationary state is derived. The Excess of MSE of the filter is:

$$ExcessMSE = \mathcal{M}\mathcal{E}_{min} = n\frac{1-W}{1+W}\left(\frac{1}{N}\sum_{i=n}^{N-1} kl_i^2 + E[n_k^2]\right) \qquad (2)$$

where n_k is the noise signal corrupting the ECG. The Excess MSE depends on the noise power, the power of the ST-T complex not represented by the first n KL coefficients and the misadjustment of the algorithm; N is the number of samples in each ST-T complex. When the forgetting factor approaches to unity (infinite memory algorithm), the $ExcessMSE$ will be minimum. The dependence on the number of tap weights n leads to a tradeoff between to reduce the misadjustment and to minimize the signal power not represented. The KLT is optimal in the sense that it minimizes (2) with the lower n value possible.The SNR obtained at the output of the RLS adaptive filter neglecting the term $\frac{1}{N}\sum_{i=n}^{N-1} kl_i^2$ is:

$$SNR_y \approx \frac{\frac{1}{N}\sum_{i=0}^{n-1} kl_i^2}{n\frac{1-W}{1+W}E[n_k^2]} = SNR_d\frac{1}{n}\frac{1+W}{1-W} \qquad (3)$$

where SNR_d is the SNR at the original ECG signal. The SNR obtained using the inner product to estimate the KL coefficients and assuming white noise is:

$$SNR_y^{direct} = \frac{\frac{1}{N}\sum_{i=0}^{n-1} kl_i^2}{\frac{n}{N}E[n_k^2]} = SNR_d\frac{N}{n} \qquad (4)$$

Thus the improvement in SNR using RLS algorithm versus direct estimation will be:

$$\Delta SNR_{RLS} = \frac{SNR_y}{SNR_y^{direct}} = \frac{1}{N}\frac{1+W}{1-W} \quad (5)$$

When W approaches to unity the improvement ΔSNR will be better, but the time constant τ will be greater.

IV. COMPARISON BETWEEN RLS AND LMS

Similar analysis shows that ΔSNR obtained with the LMS algorithm is given by $\Delta SNR_{LMS} = \frac{1}{\mu}$. Thus the improvement of the RLS versus the LMS is:

$$\Delta SNR_{RLSvsLMS} = \frac{\mu}{N}\frac{1+W}{1-W} \quad (6)$$

In figure 2 the iso-improvement curves as function of its features parameters (μ (step parameter in LMS) and W (forgetting factor in RLS)) can be seen: the equal ΔSNR curves correspond to sampling rates of 1000, 360 and 250 Hz, i.e. to 600, 216 and 150 samples in the ST-T complex, respectively. The region above each curve corresponds to a better improvement in SNR produced by the RLS algorithm and the region below conversely. It is well known that RLS algorithm is highly useful

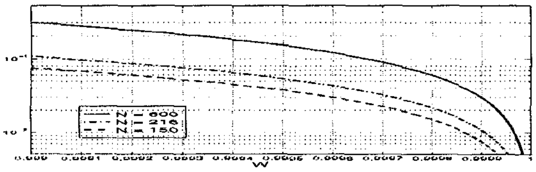

Figure 2: *Equal ΔSNR curves.*

when the eigenvalues of correlation matrix are widely spread, because the time constant of each filter tap does not depend on them. In contrast the LMS algorithm shows a different time constant for ech tap and the convergence rate is limited by the smallest eigenvalues. Nevertheless, this property does not make difference in the case of KL coefficients estimation because the KL eigenvectors are orthonormal and all eigenvalues are equal. In order to obtain the same SNR improvement it is necessary to select μ and W related accordingly to (6) and this leads roughly to the same rate of converegence for both algorithms:

$$M_{LMS} = \mu\frac{n}{N} = N\frac{1-W}{1+W}\frac{n}{N} = M_{RLS} \quad (7)$$

$$\tau_{LMS} = \frac{N}{2\mu} = \frac{1}{2}\frac{1+W}{1-W} \approx \tau_{RLS} \quad (8)$$

So their performances become equal looking at the adaptive trade-off between $\mathcal{M}$ and τ but with higher simplicity for the LMS. However the initial convergence rate is much better in RLS algorithm as it can be seen in the kl_0 estimated on the record e0103 of the *European ST-T Database* (figures 3(b) and 3(c)) for equivalent W and μ values.

V. ISCHEMIC ECG ANALYSIS

KL coefficients were calculated with direct estimation (fig. 3(a)) , RLS (fig. 3(b)) and LMS (fig. 3(c)) algorithms. The adaptive estimation shows a large SNR improvement of the estimated KL series (19dB). The LMS estimation can lead to errors in the initial detection of ischemic episodes as it is shown comparing figures 3(b) and 3(c) where there are two episodes with the first underestimated with the LMS because of the convergence time, whereas the RLS overrides this problem. In the rest of the record the signal tracking is essentially the same as it has been proved in (7) and (8). Because of roundoff errors, when a W less than unity is used, the RLS algorithm becomes unstable and it is necessary to periodically restart the algorithm. This improves the convergence rate but it increases the misadjustment (fig. 4(c) shows two hour kl_0 series of the same record). Once the RLS and LMS algorithms have been compared it is propossed an *ad-hoc* initialization for the LMS weights (instead to zero, to the inner product of the first ST-T complex with each basis function) which gives a better initial convergence rate. The result (fig. 3(d) and 4(b)) is that the same tracking properties than RLS for the KL coefficients estimation is achieved with a lower computational load.

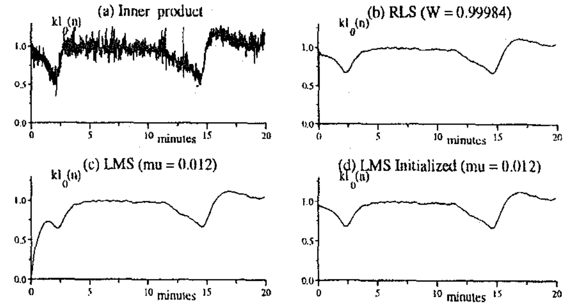

Figure 3: *Estimation of $kl_0(n)$ in record e0103.*

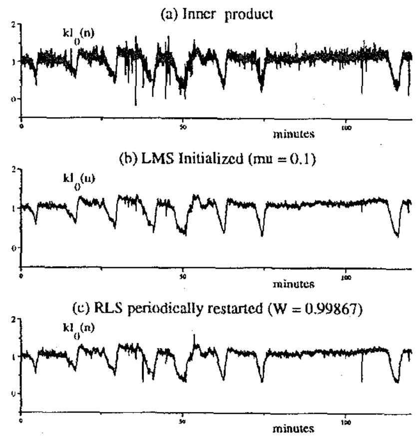

Figure 4: *Estimation of $kl_0(n)$ in record e0103.*

VI. CONCLUSIONS

The performance of RLS and LMS algorithms have been compared in the adaptive estimation of KL series of ST-T complex. It has been shown that, for the same signal-to-noise ratio improvement, equal convergence rate is obtained in both cases, although RLS algorithm has a better initial convergence rate. Nevertheless, RLS algorithm has shown a less robust behavior because of the roundoff errors which results in instabilities for $W < 1$ values and also requires a higher computational complexity. Finally it has been proposed an initialization to the inner product for the LMS algorithm which leads to the same tracking behavior than RLS but with lower computational work making it the most suitable to estimate the KL series.

References

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