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The adaptive linear combiner with a periodic-impulse reference input as a linear comb filter

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Abstract

In this paper we present a particularization of the adaptive linear combiner (ALC) filter structure, that results in a linear time-invariant comb filter suitable for the estimation of periodic signals and repetitive time-locked signals. The ALC is used in its transversal form, and the reference input is a periodic unitary-impulse train signal. When the LMS algorithm is used we show that the structure results in a simple and efficient linear time-invariant comb filter, taking as output the output of the ALC. This comb filter has lobe widths proportional to the μ gain parameter of the LMS algorithm, and the separation between lobes is controlled by the period L of the periodic impulse train. We have also analyzed this filter when applied to estimate repetitive signals time-locked to a stimulus, and we show that the effect of a temporal misalignment in the determination of the stimulus results in a low-pass filtering effect, with cut-off frequency inversely proportional to the dispersion of the impulse estimation. This effect is specially important when the time occurrence of the stimulus is not directly accessible and needs to be estimated from some 'noise affected' procedures, as in Electrocardiographic signals. The filter is also shown to be equivalent to a time-sequenced adaptive filter with one weight. Finally, an application to somatosensory evoked potentials estimation is presented.

Zussammenfassung

In diesem Aufsatz stellen wir eine Spezialisierung der "adaptiven linearen Kombinier"- (ALC-) Filterstruktur vor, die auf ein lineares zeit-invariantes Kammfilter hinausläuft, welches geeignet ist, periodische und repetierende, zeitlich gekoppelte Signale zu schätzen. Der ALC wird in seiner Transversalform benutzt, und der Referenzeingang besteht aus einem periodischen Einheitsimpulszug. Wir zeigen, daß die Struktur bei Verwendung des LMS-Algorithmus' in einem einfachen und wirksamen zeit-invarianten Kammfilter resultiert, wenn man als Ausgang den ALC-Ausgang betrachtet. Dieses Kammfilter besitzt Durchlaßbänder, deren Breiten zum Adaptionsfaktor μ des LMS-Algorithmus' proportional sind, und der Abstand zwischen ben Bändern wird durch die Periode L des periodischen Impulszuges festgelegt. Wir haben dieses Filter auch in der Anwendung zur Schätzung repetierender Signale analysiert, die zeitlich mit einem Stimulus gekoppelt sind, und wir zeigen, daß die Wirkung zeitlicher Fehlanpassung bei der Stimulusbestimmung in einem Tiefpaß-Filtereffekt resultiert mit einer Grenzfrequenz, die zur Dispersion der Impulsschätzung umgekehrt proportional ist. Dieser Effekt ist besonders wichtig, wenn der Zeitpunkt des Stimulus nicht direkt zugänglich ist und

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mittels irgendwelcher, durch Rauschen beeinträchtigter Prozeduren (wei etwa bei EKG-Signalen) zu schätzen ist. Es wird auch gezeigt, daß unser Filter einem adaptiven sequentiellen Filter mit einem Koeffizienten entspricht. Zum Schluß wird eine Anwendung auf die Schätzung somatosensorischer Potentiale vorgestellt.

Résumé

Dans cet article nous présentons une structure particulière du filtre combineur linéaire adaptatif (CLA), qui donne un filtre en peigne linéaire approprié à l'estimation de signaux périodiques et de signaux répétitifs fixés en temps. Le CLA est utilisé dans sa forme transverse, et l'entrée de référence est un train d'impulsions périodiques unitaires. Quand l'algorithme LMS est utilisé, nous montrons que la structure se résume à un filtre en peigne linéaire simple et efficace, prenant comme sortie la sortie du CLA. Ce filtre en peigne a des largeurs de lobes proportionnelles au paramètre de gain μ de l'algorithme LMS, et la séparation entre lobes est controlée par la période L du train d'impulsions périodiques. Nous avons également analysé ce filtre lorqu'il est appliqué à l'estimation de signaux répétitifs fixés en temps à un stimulus, et nous montrons que l'effet d'un mauvais alignement temporel dans la détermination du stimulus résulte en un effet de filtrage passe-bas, avec une fréquence de coupure inversement proportionnelle à la dispersion de l'estimation de l'impulsion. Cet effet est particulièrement important quand l'occurrence temporelle du stimulus n'est pas directement accessible et doit étre estimée à partir de procédures "affectées par le bruit", comme pour les signaux électrocardiographiques. On montre aussi que le filtre est équivalent à un filtre adaptif séquencé en temps avec un facteur de pondération. Finalement, une application à l'estimation des potentiels évoqués somatosenseurs est présentée.

Keywords: Adaptive filters; Periodic signals; Comb filters; Electrocardiogram (ECG); Evoked potentials (EP)

1. Introduction

Adaptive filtering technique is a wide-spread tool for estimating signals embedded in uncorrelated noise [18]. Adaptive filters have a structure that allows the filter to 'learn' the input statistics and to track them when they are time-varying. To achieve this property they make use, in addition to the primary input signal, of one or more auxiliary inputs, called reference inputs, that contain noise-correlated signals [18, 17, 7, 8]. Very often, the performance of adaptive systems depends on the adequate selection of the primary and reference inputs, and the availability of the reference inputs correlated with the noise but not with the signals of interest. These filters are time-varying and self-adjusting, allowing the separation of signals and uncorrelated noise even if they appear in the same frequency band.

One special structure of adaptive filters, the adaptive linear combiner (ALC), using as reference inputs two sinusoids with a phase difference of 90°, has been shown to become a linear notch filter [18] when the LMS algorithm [19] is used in the adaptation process. Particularization of this notch to a high-pass comes from taking the reference input as a unitary constant [18]. The advantage of this filter is its simple implementation together with the control possibility of the notch bandwidth directly from the μ gain constant of the LMS algorithm.

In this paper we present another particularization of the ALC structure that becomes a linear comb filter. The interest of a comb filter lies in its capacity to estimate periodic signals [12]. This feature can be used to estimate signals of an intrinsic periodic nature, or to eliminate noise of periodic characteristics. In the first case many applications have been found until now. In [3] one application is presented to separate solar and lunar components from ionospheric measurements of electron concentration. Other applications have considered the suppression of the clutter from fixed objects in moving-target indicator (MTI) radars [14]. In biomedical signal processing, many signals, even those that are not strictly periodical, are finite-length repetition signals time-locked to a stimulus. The detection of the stimulus allows the signal to be studied as a periodic signal composed of the concatenation of subsequent recurrences. Among these signals we found the electrocardiographic signal (ECG) that is time-locked to an internal stimulus [16] generated at the heart. In this case the detection of the stimulus is performed through a QRS detector [6] that locates the more prominent wave (called QRS complex) in the signal belonging to each cardiac cycle. Other biomedical signals that fulfill this requirement are the evoked potentials (EP) [16]. In this case the stimulus is external (visual, auditory, or electrical) and the signal is the brain's electrical response to these external stimuli. In these cases we have a precise estimation of the time stimulus, given that it is externally controlled by the user. Among the noise rejection applications the most characteristic is the rejection of powerline (50/60 Hz) harmonics through a comb filter structure.

The filter that we analyze in this paper is based on the ALC with two inputs: the signal to be estimated (primary input) and the reference input that consists of a periodic unitary-impulse train with period of that of the signal to be estimated. The length of the transversal filter matches the period of the impulse train. We will see that this structure (Fig. 1) belongs to a linear time-invariant comb filter when the LMS algorithm is used in the adaptation process, with the advantage of its easy implementation and the direct controlling of the width lobes through the μ factor. Also, by simple inspection, we show that this filter has the same

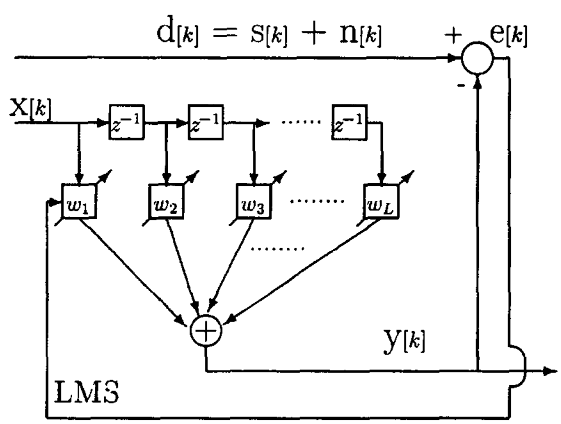

Fig. 1. Filter structure using the transversal adaptive linear combiner. $d[k] = s[k] + n[k]$ is the periodic signal to be estimated (primary signal), composed of a strictly periodic signal $s[k] = s[k+L]$ plus additive noise $n[k]$ not correlated with the signal $x[k]$ is the periodic unitary-impulse train $x[k] = x[k+L]$, and $y[k]$ is the filter output.

behavior as the time-sequenced adaptive filter presented in [4], using one weight in each weight vector.

This filter was previously applied to biomedical signals [11] (ECG and EP), where we showed its tracking capability of time-varying signals. In this paper we present the filter study and we show that the filter is a linear time-invariant comb filter. We also study the effect of a dispersion or misalignment in the synchronization (periodicity) of the impulse train with respect to the periodic signal showing that this results in a low-pass effect controlled by the deviation of the alignment dispersion.

2. The filter structure

The filter structure is based on the transversal adaptive linear combiner (Fig. 1) with L taps, where the primary input ($d[k] = s[k] + n[k]$) is the noise contaminated periodic signal (in the case of periodic signals) or the consecutive linking of recurrences (in the case of event-related signals). Thus, $d[k]$ is considered to be composed of a strictly periodic signal $s[k] = s[k+L]$ to be estimated, plus additive noise $n[k]$ not L-periodic and so not correlated with $s[k]$. If there is noise L-periodic, obviously this filter will not reject it since it will be indistinguible from the signal $s[k]$. The reference signal ($x[k]$) is a periodic unitary-impulse train with L sample period (the period of the signal to be estimated),

$$x[k] = \sum_{m=0}^{\infty} \delta[k - mL - 1], \tag{1}$$

where $\delta[k] = 1$ for $k = 0$ and $\delta[k] = 0$ otherwise. The output of this filter is taken at $y[k]$, and $e[k]$ is the error signal between the estimated signal $y[k]$ and the input signal $d[k]$.

The construction of the impulse train will depend on the application. In those cases where the period of $s[k]$ is well-known a priori (solar and lunar ionosferics components [3], radar applications [14] (maybe with time delay estimates), 50/60 Hz interferences, etc.) the construction of $x[k]$ is immediate. In those cases where the signal $d[k]$ is the consecutive linking of finite time recurrences of a time-locked signal, the impulse generation comes from the time occurrence determination of the event that generated the signal. If the event is external (evoked potential) again the generation of $x[k]$ is immediate from the generator of the event, but if the event is internal (ECG signals) a time occurrence detector [6] or a more sophisticated alignment method [10] will be required. The effect of a misalignment error at the impulse determination will be analyzed in Section 4.

In case of event-related signals the $d[k]$ signal, by itself, loses the actual timing of recurrences that can contain a valuable information. However, it is presented in this way for convenience to analyze the filter behavior, as evidenced in following sections. The primary input $d[k]$ has been converted to a pseudo-periodic deterministic component plus added noise not correlated with the former. In real situations the signal $d(t)$ does not need to be generated, instead we arrange the filter to activate when the recurrence detector gives a mark and to act only during the determined signal window. After that the filter is inhibited until the next recurrence mark appears. In this way we keep the valuable timing information. The two situations are exactly equivalent from the filter point of view, being the former much more suitable for filter performance analysis.

A careful analysis of the time-sequenced adaptive filter [4, 15] verified that the filter proposed in this work presents the same behavior as the time-sequenced one restricted to have only one weight at each LMS filter as described in [4].

2.1. The adaptive filter behavior

Here, we briefly study the behavior of the filter through the well-known adaptive theory. More detailed analysis can be found in [11]. The filter output can be expressed as

$$y[k] = \sum_{i=1}^{L} w_i[k]x[k-i+1] = \boldsymbol{W}_k^{\mathrm{T}}\boldsymbol{X}_k, \tag{2}$$

where $\boldsymbol{W}_k = (w_1[k]\ w_2[k]\ \dots\ w_L[k])^{\mathrm{T}}$ and $\boldsymbol{X}_k = (x[k]\ x[k-1]\ \dots\ x[k-L+1])^{\mathrm{T}}$ are the weight vector and the reference vector, respectively. The mean-square error ξ between the signal under study $d[k]$ and the estimated one $y[k]$, can be expressed by

$$\begin{aligned}\xi &= E\{\varepsilon^2[k]\} = E\{(s[k]-y[k])^2\} + E\{n^2[k]\}\\ &= E\{d^2[k]\} + \boldsymbol{W}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{W} - 2\boldsymbol{P}^{\mathrm{T}}\boldsymbol{W},\\ \boldsymbol{R} &= E\{\boldsymbol{X}_k\boldsymbol{X}_k^{\mathrm{T}}\}, \qquad \boldsymbol{P} = E\{d[k]\boldsymbol{X}_k\},\end{aligned} \tag{3}$$

where $\boldsymbol{R}$ and $\boldsymbol{P}$ are the input correlation matrix and the cross-correlation vector, respectively. Note that even if $x[k]$ is not a stochastic signal the calculations can also be done since the expectations are taken as time averages; see [18]. Considering in this case that the reference signal $x[k]$ is a periodic unitary-impulse train uncorrelated with the noise, we obtain a simple expression for $\boldsymbol{R}$ and $\boldsymbol{P}$:

$$\boldsymbol{R} = \frac{1}{L}\boldsymbol{I}, \qquad \boldsymbol{P} = \frac{1}{L}(s[1]\ s[2]\ \dots s[L])^{\mathrm{T}}. \tag{4}$$

The optimum weight vector [18] that minimizes the ξ from (4) is

$$\boldsymbol{W}^* = \boldsymbol{R}^{-1}\boldsymbol{P} = (s[1]\ s[2]\ \dots s[L])^{\mathrm{T}}. \tag{5}$$

In the steady state the optimum filter output $y^*[k]$ becomes

$$\begin{aligned}y^*[k] &= \boldsymbol{W}^*\boldsymbol{X}_k = \sum_{i=1}^{L} w_i^* x[k-i+1]\\ &= w^*_{\mathrm{mod}\,(k/L)} = s[k],\end{aligned} \tag{6}$$

where

$$\mathrm{mod}(k/L) = \begin{cases}\mathrm{modul}(k/L), & k \neq m'L,\\ L, & k = m'L.\end{cases} \tag{7}$$

So, we see that, when the weight vector has converged to the optimum, the filter behaves like an ideal comb filter, estimating only the periodic signal $s[k]$. However, the misadjustment $\boldsymbol{M}$ of the adaptation algorithm, that represents the excess of ξ, leads the filter out of this behavior. The minimum mean-squared error ($\xi_{\min}$) takes the value $\xi_{\min} = E\{d^2[k]\} - \boldsymbol{P}^{\mathrm{T}}\boldsymbol{W}^* = E\{n^2[k]\}$.

Using the LMS algorithm [19] $\boldsymbol{W}_{k+1} = \boldsymbol{W}_k + 2\mu e[k]\boldsymbol{X}_k$, the convergence condition is [5]

$$0 < \mu < \frac{1}{3\,\mathrm{tr}[\boldsymbol{R}]} = \frac{1}{3}, \tag{8}$$

and the time constant (τ_{mse}) for the convergence of the ξ is $\tau_{\mathrm{mse}} = 1/4\mu\lambda = L/4\mu$, where $\lambda = 1/L$ is the eigenvalue of the matrix $\boldsymbol{R}$ (τ_{mse} is measured in number of sampling periods). The misadjustment $\boldsymbol{M}$ takes the approximated value [18]

$$\boldsymbol{M} = \frac{E\{(s[k]-y[k])^2\}}{E\{n[k]^2\}} \simeq \mu\,\mathrm{tr}[\boldsymbol{R}] = \mu,$$

$$\text{which implies } \xi \simeq E\{n^2[k]\}(1+\mu). \tag{9}$$

Note from this expression that M can be interpreted as a normalized estimate of the residual noise at the filter output. The signal-to-noise ratio improvement $\Delta\mathrm{SNR}_y$ at the filter output after convergence is

$$\Delta\mathrm{SNR}_y = \frac{\mathrm{SNR}_y}{\mathrm{SNR}_d} = \frac{E\{s^2[k]\}/E\{(y[k]-s[k]^2\}}{E\{s^2[k]\}/E\{n^2[k]\}} \simeq \frac{1}{\boldsymbol{M}} = \frac{1}{\mu}. \tag{10}$$

In this study, we have shown the convergence of the filter output $y[k]$ to the periodic signal $s[k]$ that we want to estimate. Here, the well-known trade-off for the selection of μ value appears. Large μ values make the convergence time (τ_{mse}) smaller in exchange for a higher misadjustment and hence a lower signal-to-noise improvement. Eventual period-to-period variations of the $s[k]$ periodic signal will be tracked by this filter if $\tau_{\mathrm{mse}} = 1/4\mu < L$, that gives ($1/3 > \mu > 1/4$). These μ values at the upper band of convergence imply higher $\boldsymbol{M}$ at the output.

This is the analysis that can be made with the adaptive study. We corroborate that the filter is suitable to estimate periodic signals but we do not have information about residual noise ($n'[k]$) characteristics at the output signal ($n'[k] = y[k] - s[k]$), and how this residual noise depends or not on the input noise, given that adaptive formalism considers expected values losing any other useful information of the signals. In the next section we will analyze this filter without considering the LMS adaptive approximation. This will be possible because of the special simplicity of the $x[k]$ reference signal.

2.2. The filter output as an exponentially weighted averager

In this section we show that the filter output $y[k]$ is exactly equivalent to an exponentially weighted average of the preceding periods of the primary input signal, with a forgetting factor multiplying each period. From these results we will show in Section 3 that this linear filter is a comb filter.

To show this equivalence, we make use of the LMS algorithm expression for each weight i of the vectorial equality. Considering the expression of the error $e[k]$, we can write

$$w_i[k+1] = w_i[k] + 2\mu(d[k]-y[k])x[k-i+1], \quad i = 1,\ldots,L. \tag{11}$$

Given that $x[k]$ is a periodic unitary-impulse train the weight $w_i[k]$ will only be actualized once in each period. We will denote w_i^m the value of the weight w_i when the filter is processing the period $m+1$ of the $d[k]$ signal ($d[k] = d[mL+i]$, $i = 1, 2, \ldots, L$). With this notation (11) can be written as

$$w_i^{m+1} = w_i^m + 2\mu(d[mL+i] - y[mL+i]), \quad i = 1,\ldots,L. \tag{12}$$

From (6) we know that $y[mL+i] = w_{\mathrm{mod}((mL+i)/L)}[mL+i] = w_i[mL+i] = w_i^m$. Then (12) results in

$$w_i^{m+1} = w_i^m + 2\mu(d[mL+i] - w_i^m), \quad i = 1,\ldots,L. \tag{13}$$

This equation is a recursive equation for the weights where it is shown that the weight w_i is only affected by the samples ith of each signal period in $d[k]$. If we express the recursive relation (13) as a function of the initial weight w_i^1, which is initialized to be zero ($w_i^1 = 0$), then

$$w_i^{N+1} = \sum_{m=1}^{N} 2\mu(1-2\mu)^{N-m} d[mL+i]. \tag{14}$$

Considering again that $y[mL+i] = w_i^m$ we obtain the expression of the output signal $y[k]$ as

$$y[(N+1)L+i] = \sum_{m=1}^{N} 2\mu(1-2\mu)^{N-m} d[mL+i]. \tag{15}$$

From expression (15) we see that $y[(N+1)L+i]$ (output of the filter after processing N periods) is a weighted average of the ith samples of previous signal periods m. The weight factor $2\mu(1-2\mu)^{N-m}$ decreases when $N-m$ increases as long as $|1-2\mu|<1$ (convergence condition of the LMS). This equivalence with the weighted averager implies that the studied adaptive filter using the LMS algorithm is a linear time-invariant filter (15). However, this fact is not true in general for all adaptive filters. Regarding Eq. (15) the output at time instant i of the $(N+1)$th period depends only on the previous $d[mL+i]$ $(m=1,\dots,N)$ noise samples at the same time instant i. Thus, the requirement of the noise to be filtered is that it must be uncorrelated between the signal recurrences. It does not need to be white noise to satisfy the theoretically predicted filter performance, since LMS theory assumes white noise to study $\boldsymbol{M}$ behavior [18]. This is very important in signals such as evoked potentials in which it is well known that the noise, usually the background electroencephalogram (EEG), is a highly correlated signal in each record [1].

In [11] we show that the exact estimation of the signal-to-noise ratio improvement $\Delta\mathrm{SNR}_y$ when $N\to\infty$ (steady state in adaptive formalism) is

$$\lim_{N\to\infty}\Delta\mathrm{SNR}_y=\frac{1-\mu}{\mu}. \tag{16}$$

This is in disagreement with the previous result with the adaptive approximation. This comes from the approximated $\boldsymbol{M}\simeq\mu\,\mathrm{tr}[\boldsymbol{R}]=\mu$ approximation. However, considering the estimation of $\boldsymbol{M}$ presented in [8] or the estimation for this particular case presented in [11] we see that the misadjustment takes the value $\boldsymbol{M}=\mu/(1-\mu)$, and then the $\Delta\mathrm{SNR}_y$ estimated with the adaptive formalism or with the exact deduction are identical.

3. The filter structure as a comb filter

In this section we study the frequency behavior of the filter. To study the frequency response of the filter we will express the fitter output given in (12) as a function of the $y[k]$ and $d[k]$ signals, considering that the weight w is the signal $y[k]$ at times k, Eq. (6). With this substitution we have

$$y[k+L]=y[k](1-2\mu)+2\mu d[k]. \tag{17}$$

Taking the z transform of this equation we obtain

$$y(z)z^L=y(z)(1-2\mu)+2\mu d(z), \tag{18}$$

and thus the z-transform transfer function of the filter is

$$H(z)=\frac{y(z)}{d(z)}=\frac{2\mu z^{-L}}{1+(2\mu-1)z^{-L}}, \tag{19}$$

and the Fourier transform

$$H(w)=\frac{2\mu\mathrm{e}^{-\mathrm{j}wL}}{1+(2\mu-1)\mathrm{e}^{-\mathrm{j}wL}}, \tag{20}$$

where w is the normalized frequency.

This filter is a comb filter in which lobes repeat at frequencies which are multiples of the fundamental normalized frequency f_0 $(f_0=2\pi/L)$. It includes a time delay factor of L samples, due to the fact that in a given period of the signal the filter ouptut is the weights, actualized last time at the previous period. This is evident at the first period $k=0,\dots,L$ where the output will be zero (assuming weight initialization as zero) independent of the signal input.

The -3 dB cut-off frequency of each lobe is $(\mu/\pi)f_0$ far from the central frequency of the lobe. Analyzing this fact in detail we found that wider lobes (higher μ) implied more non-periodic signal component (noise) at the output. This is in agreement with the result from adaptive study that gives lower $\Delta\mathrm{SNR}_y$ for higher μ values. We can interpret the misadjustment as the residual noise that passes through the lobes spread.

In Fig. 2 we present the transfer function module for a sampling frequency of $f_s=1000$ Hz, $L=1000$ samples $(f_0=1$ Hz$)$ and $\mu=0.05$. This implies a cut-off frequency of each lobe with respect to the central frequency of $f_c=0.016$ Hz.

Recovering the adaptive and linear points of view we have already interpreted the misadjustment as the noise passed through the lobes spread. The convergence time can be interpreted as the time the filter needs to identify a signal as a periodic signal with L period. This is easy to visualize at the beginning (note that we have assumed $d[k]=0$ for $k<0$) of the filter operation. Even if the signal is

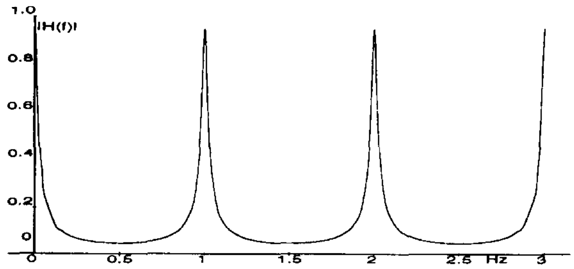

Fig. 2. Transfer function of the filter for a sampling frequency of 1000 Hz, $L = 1000$ samples, and $\mu = 0.05$.

strictly periodic ($d[k] = d[k + L]$ for $k > 0$) at the first period the filter has the information of an infinite null signal $k < 0$ and one period of some values $d[k]$; this is not at all a periodic signal and the filter response is near zero. Going further the signal input $d[k]$ takes more periods (its spectrum concentrates more around the lobes) and the estimation approaches the periodic component. The same reasoning can be used with a sudden change in the periodic signal that will require a time τ_{mse} to recover this change at the output signal.

From another point of view, this fact can be explained by the impulse response $h[k]$ that extends from zero to ∞, but with an exponentially decreasing weighted factor that makes the significant contribution no longer than τ_{mse}:

$$h[k] = 2\mu \mathrm{e}^{-(k-L)/\tau L} u[k - L] \sum_{m=-\infty}^{\infty} \delta[k - mL]. \tag{21}$$

So the filter considers the periodicity of the input signal mostly by 'looking' at the last τL time. Again this reasoning justifies the convergence time from the adaptive point of view.

This filter can also be compared to a classical averaging over the last N periods of the $d[k]$ signal. In [13] the frequency behavior of such a procedure is presented, which also results in a comb filter where the lobe width is inversely proportional to the number N of averaged periods. The advantage of the filter analyzed here is the implementation efficiency, low memory requirements, and the signal tracking capability.

4. Effects of misalignment at the unitary impulse location

In this section we will analyze the effect of a misalignment at the impulse location with respect to the period of $d[k]$. This problem will not appear at signals that are of a periodic nature by itself, since the period L of the impulse train will be fixed by the user at the frequency he wants to filter the incoming signal. If this value is not adequately selected the output will reflect the periodic input signal components at the new fundamental frequency. This procedure can be used to estimate periodic signals of unknown period. The variation of L will allow us to find out what the period of the incoming signal is.

The application of a comb filter to repetitive time-locked signals, mentioned in the introduction, needs to estimate the occurrence time of the repetitive signal [10] to generate the $d[k]$. In real practice the filter works L samples after the trigger activates the filter and waits until a new trigger signal appears. This is equivalent to considering the filter as continuously working with $d[k]$ built as described in the introduction (continuous linking of the different recurrences). In these cases timing errors can appear. When an error of ($\pm\delta$) appears in these estimations the unitary-impulse train remains periodic with period L, but the signal $s[k]$ will change its period to $s[k] = s[k + L \pm \delta]$. The value of δ will be of random nature and so will vary from period to period. We will study the effect of this misalignment at the filter output through the adaptive formalism, particularizing a previous study on this topic when the inputs to the ALC are orthogonal functions [9].

The effect of these errors on the estimated signal will be reflected through the effect on the $\boldsymbol{P}$ vector. The optimum weight vector $\boldsymbol{W}^* = \boldsymbol{R}^{-1}\boldsymbol{P}$ will be affected through $\boldsymbol{P}$ vector modifications ($\boldsymbol{R}$ does not change). Then we will analyze the $\boldsymbol{P}$ vector in this case:

$$\boldsymbol{P} = E\{d[k]\boldsymbol{X}_k\} = E\{s[k]\boldsymbol{X}_k\} + E\{n[k]\boldsymbol{X}_k\}. \tag{22}$$

As noise $n[k]$ is supposed to be not correlated with the stimulus the second term of $\boldsymbol{P}$ in (22) is null and $\boldsymbol{P}$ is reduced to become $\boldsymbol{P} = E\{s[k]\boldsymbol{X}_k\}$.

If we assume that the errors of the occurrence time determination (δ) are expressed in sample values and have a probability distribution $p[\delta]$, the $\boldsymbol{P}$ vector can be expressed as

$$\boldsymbol{P} = E\{s[k]\boldsymbol{X}_k\}$$

$$= \sum_{\delta=-\infty}^{\infty}\left(\frac{1}{L}\sum_{k=1}^{L} s[k+\delta]\boldsymbol{X}_k\right)p[\delta], \tag{23}$$

$$\boldsymbol{P} = \frac{1}{L}\sum_{k=1}^{L}\boldsymbol{X}_k \sum_{d=-\infty}^{\infty} s[k+\delta]p[\delta]. \tag{24}$$

From this result we observe that the $\boldsymbol{P}$ vector elements are $(s'[i]/L)$ where $s'[i]$ are the components of a signal $s'[k]$ to where the filter converges, and takes the value

$$s'[k] = \sum_{\delta=-\infty}^{\infty} s[k+\delta]p[\delta]. \tag{25}$$

Calculating the Fourier transform of this $s'[k]$ signal $(S'(\Omega))$ we have

$$S'(\Omega) = S(\Omega)\sum_{\delta=-\infty}^{\infty} \mathrm{e}^{\mathrm{j}\Omega\delta}p[\delta], \tag{26}$$

where $S(\Omega)$ is the Fourier transform of $s[k]$. So the effect of the error in the occurrence time estimation makes a filtering effect on the signal $s[k]$ at the estimated $y[k]$. The transfer function $C(\Omega)$ of this filter $(S'(\Omega) = C(\Omega)S(\Omega))$ is the characteristic function of the δ distribution [2]

$$C(\Omega) = \sum_{\delta=-\infty}^{\infty} \mathrm{e}^{\mathrm{j}\Omega\delta}p[\delta]. \tag{27}$$

In the case that $p[\delta]$ is a Gaussian distribution with standard deviation σ, the characteristic function is [13]

$$C(\Omega) = \sum_{n=-\infty}^{\infty} \mathrm{e}^{-(\Omega-2\pi n)^2\sigma^2/2} \tag{28}$$

that consists of a low-pass filter with a cutoff frequency (f_c) at -3 dB of $f_c = 132.5/\sigma$, where f_c is expressed in Hz and σ in ms. Thus, the estimation of W^* will be the coefficients $s'[k]$ of a low-pass filtered deterministic signal component, whose cutoff frequency depends on the error distribution.

The effect should be taken into consideration when estimating signals of high-frequency components [10] around the limit reached in this study. This limitation results in the same effect that was observed in [13] for classical signal averaging. So, we can state that the improvement in implementation efficacy and tracking capability of this filter with respect to the classical averaging is continued by maintaining the misalignment effect. These advantages are at the expense of a limit in the SNR improvement given by the lobe width (misadjustment).

4.1. Simulation study

In order to test the previous results we have taken 150 ms of a deterministic signal that belongs to a real QRS complex from an ECG signal (Fig. 3), sampled at 1000 Hz. We have extended this signal

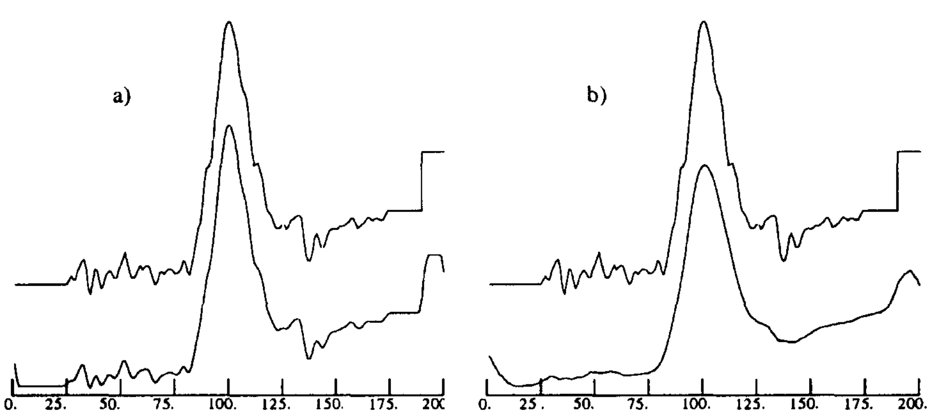

Fig. 3. Output of the filter (lower signals) after processing 500 periods of the upper signal with a Gaussian (σ) distributed delay at the impulse filter input: (a) $\sigma = 1$ ms that implies $f_c = 132.5$ Hz; (b) $\sigma = 5$ ms that gives $f_c = 26.5$ Hz.

to 200 ms with a 25 ms flat line of the left and another 25 ms on the right. The time domain extension includes a step at the 190 ms of the total signal (upper signals of Fig. 3). With this signal we have generated the signal in study d_k as composed of the succession of 500 recurrences of the same signal. In this way we can consider that all the signal is deterministic and there is not noise ($n[k] = 0$). We have estimated the deterministic signal with the filter, considering that $L = 200$. Then all the deterministic signal can be represented. In the estimation we have started the adaptation of each recurrence with a Gaussian distributed delay (σ) with respect to the exact occurrence point of each realization. In Fig. 3 we have the original deterministic signal and below the estimated after processing 500 recurrences for $\sigma = 1$ ms (Fig. 3(a)) and for $\sigma = 5$ ms (Fig. 3(b)). From the cutoff frequency obtained in the theoretical study we know that $f_c = 132.5$ Hz for $\sigma = 1$ ms, and $f_c = 26.5$ Hz for $\sigma = 5$ ms. Figs. 3(a) and 3(b) are in accordance with these results, where we can see that filtering effect (f_c) occurs at lower frequencies for higher σ, according to the expression $f_c = 132.5/\sigma$.

5. Application to the estimation of event-related somatosensory evoked potential signals

In this section we briefly present the filter estimation of a real somatosensory evoked potential (SEP) signal [1] that records the brain response to an electrical stimulus. In this case the electrical stimulus is a current of 20 mA given at the rate of 5.9/s. The response is recorded from 40 ms before and 40 ms after the stimulus with a sampling frequency of 3200 Hz. Thus, in each SEP, we have the first part with only the EEG signal and the second part with the EEG + SEP. After 1420 recurrences, etomidate (0.2 mg/kg) was administrated to the subject and an additional 1420 SEP recurrences were recorded. The SEP response changed as a result of the etomidate administration, and we can see in Fig. 4 how the filter estimates the dynamic changes. In this figure we have in the first row different periods (N) of the SEP that compose the primary input d_k to the filter. In the second row are the results after classical averaging of the previous N SEP that can also be seen as the output of a linear comb filter [13] whose lobe width gets narrower as N increases. These characteristics made the classical averaging unable to follow the dynamic changes of the signal that occurs after the etomidate administration. In the following rows are the comb filter estimates for different values of the μ parameter. We can note how a low μ value gets a fast amplitude detection of the EP change at the cost of a poor SNR, as a result of having a wide lobe spread. As the μ value decreases the SNR improves as a result of the decrease in the lobe spread. However, at this special time-varying signal, smaller μ requires higher time to manifest the

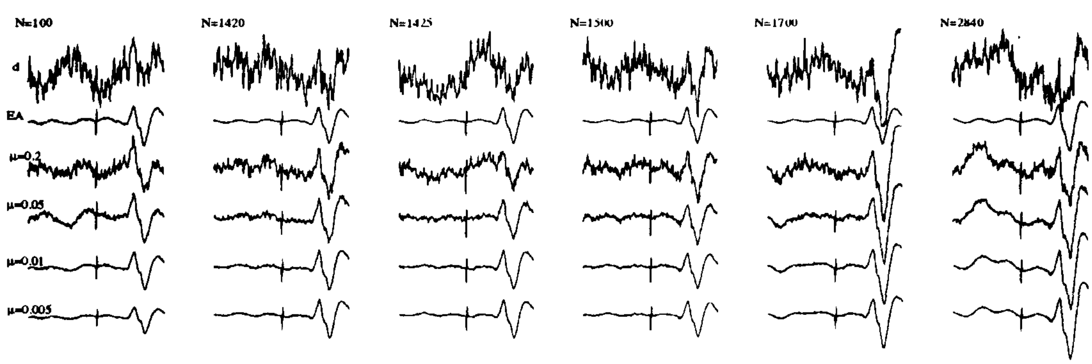

Fig. 4. Application to time-varying somatosensory evoked potentials (2840 recurrences) from a subject with application of etomidate in the 1420 recurrence. The records extend 80 ms, 40 ms previous to the electrical stimulus (20 mA) and 40 ms after the stimulus. The first row displays the SEP recurrences. In the second row are the results after classical EA of N records. The following rows are the filter output after processing N records for different μ values.

EP amplitude changes as a result of the well-known trade-off of the adaptive systems between speed of convergence and misadjustment at the steady state.

6. Conclusions

In this paper we have presented an efficient comb filter implementation from the adaptive linear combiner. This filter results from the particularization of the adaptive linear combiner using the LMS algorithm and a periodic impulse-train reference input. Also it is remarked that it obtains the same behavior than the time-sequenced adaptive filter with only one weight. We have presented the filter from the point of view of adaptive theory and we have shown that its behavior results in a linear time-invariant comb filter suitable for: estimation of periodical signals, selection of signal components (should they exist) of a given periodicity, rejection of periodical noise and estimation of repetitive time-locked signals. The misadjustment and convergence time from the adaptive theory has been interpreted in the linear frequency domain, showing that both are different interpretations of the same fact as expected. The width of the comb lobes has been shown to be proportional to the μ LMS gain allowing a direct and simple control of the lobe width. The period of the signal to be estimated (fundamental frequency of the comb) is directly adjusted through the period of the impulse train that we use at the reference input and the length of the filter: both should match the period of the signal to be estimated.

We have also studied the effect of applying the filter with an error at the synchronization time given by the occurrence of the impulse. This is specially important at event-related signals with no direct access to the event. We have shown that this error results in a low-pass effect at the estimated signal whose cut-off frequency is inversely proportional to the dispersion of the errors. This effect should be taken into consideration when estimating signals of relatively high-frequency components with the filter, and noise estimation of the occurrence event.

In conclusion, we have presented a linear comb effect, with a simple and stable implementation through the adaptive linear combiner structure. Direct lobe width control with the μ parameter of the LMS algorithm is allowed. The filter is suitable for real-time implementation with wide-spread possibilities of applications, some of them briefly enumerated in this work. Results are presented in the case of somatosensory evoked response estimation of the brain response to an electrical stimulus.

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