Noise effect on orthogonal transform compression of ECG signals *

Salvador Olmos, Raimon Jané (&), José García and Pablo Laguna

Centro Politécnico Superior, Univ. de Zaragoza, María de Luna 3, 50015 Zaragoza, Spain
(&)Institut de Cibernètica (UPC), Barcelona, Spain.

***Abstract*— Modeling of signals using orthogonal transforms is a very appropriate technique for ECG data compression. An increase of coefficient number (n) increases both signal and noise power reconstruction. A method for selecting the optimum number of coefficients (n^*) in noisy records is presented. Adaptive algorithms achieve better coefficient estimation when the input signal is corrupted with uncorrelated noise. In this work we analyze LMS algorithm that improves the estimation of classical inner product. A selection criteria of the μ parameter is obtained in order to outperform inner product performance.**

I. INTRODUCTION

The overall goal of data compression is to represent a signal with the minimum number of bits, thereby speeding transmission and minimizing storage requirements, with the best possible fidelity. The literature reports a great variety of methods for ECG compression [1]. Their effectiveness is generally evaluated on very different databases by means of dishomogeneous conditions (sampling frequency, precision, noise level). Evaluating the quality of the compressed signal is a very difficult task. Utility of the signal depends critically on the quality, but quality is itself an attribute with many possible definitions, depending on the signal use. However quadratic error indexes such as MSE or PRD are easy to calculate and they have become standard.

The intent of this paper is to evaluate the effect of noise in evaluating the mean squared error (MSE) of the signal in ECG data compression with orthogonal transforms.

II. ECG DATA COMPRESSION

The operation of an orthogonal transform data-compression system is illustrated in figure 1. The digital ECG signal $\mathbf{X} = [x_0, x_1, ..., x_{N-1}]^T$ is operated on by the orthogonal transform $\mathbf{T}$, to produce the sequence $\mathbf{K} = [k_0, k_1, ..., k_{N-1}]^T$. The elements of $\mathbf{K}$ are the magnitude of the projections of $\mathbf{X}$ vector onto the $\mathbf{T}$ basis. The data can be compressed by selecting less than the entire set of coefficients $n < N$.

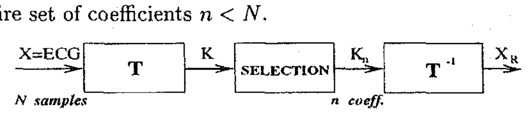

Figure 1: *Block diagram of data compressor.*

The mean squared error associated exclusively with the approximation of $\mathbf{X}$ with the rank n projection $\mathbf{X_R}$ is

$$MSE_n = \frac{1}{N}\sum_{i=n}^{N-1} k_i^2 \qquad (1)$$

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In orthogonal data compression systems there is a general tradeoff between the quality of reconstructed signal (it requires a high number of coefficients n) and the compression ratio. Several orthogonal transforms have been applied to ECG data compression (DCT, LT, HT, KLT). The Karhunen-Loeve transform (KLT) is the optimal in the sense that it needs the minimum number of coefficients for a given MSE. This is the domain where signal and noise are most separated.

If noise $\mathbf{N} = [n_0, n_1, ..., n_{N-1}]^T$ is added to the ECG signal $\mathbf{X}$, the MSE between $\mathbf{X}$ and the compressed signal will be now

$$MSE_n^{direct} = \frac{1}{N}\sum_{i=n}^{N-1} k_i^2 + \frac{1}{N}\sum_{i=0}^{n-1} \alpha_i^2 \qquad (2)$$

where α_i are the coefficients of the noise signal in the transformed domain. The values of MSE_n (assuming white noise and using KLT calculated from a test set over 110.000 beats from MIT and ST-T databases) are shown in figure 2 for several values of signal to noise ratio SNR. So, there is an optimum value of n (n^*) that minimizes the MSE of noisy signals and is directly related to the SNR of the input signal, and the transformed representations of signal and noise. Expressions have been simplified to the particular case of white noise ($\alpha_i = \sigma$).

The SNR of the reconstructed signal will be (3) where σ^2 is the variance of the white noise.

$$SNR_n^{direct} = \frac{\sum_{i=0}^{n-1} k_i^2}{\sum_{i=0}^{n-1} \alpha_i^2} = \frac{\sum_{i=0}^{n-1} k_i^2}{n\sigma^2} \qquad (3)$$

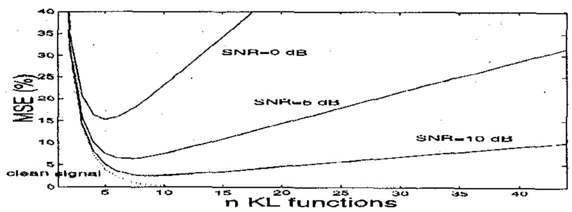

Figure 2: *MSE for KLT inner product of noisy signals.*

The selection of the optimum number of coefficients n^* is a tradeoff between compression ratio and reconstruction error (MSE).

III. ADAPTIVE ESTIMATION

Adaptive estimation of the coefficients achieves reduction of noise uncorrelated with the signal. In this case an adaptive linear combiner [2] is used with the orthogonal KL basis functions as multiple reference input. The correlation matrix is $\mathbf{R} = \frac{1}{N}\mathbf{I}$ and the cross-correlation vector is $\mathbf{P} = \frac{1}{N}[k_0, k_1, ..., k_{n-1}]^T$. The

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optimal solution for the coefficients is $\mathbf{R^{-1}P} = [k_0, k_1, ..., k_{n-1}]^T$, the projection of the clean signal.

When the LMS adaptive algorithm is used to estimate the coefficients it can be proved that the SNR of the estimated signal is

$$SNR_n^{LMS} = \frac{\frac{1}{N}\sum_{i=0}^{n-1} k_i^2}{(\mu\frac{n}{N})(\sigma^2 + \frac{1}{N}\sum_{i=n}^{N-1} k_i^2)} \quad (4)$$

The reconstructed SNR improvement of LMS algorithm *vs* inner product will be

$$\Delta SNR_n^{LMS\ vs\ direct} = \frac{SNR_n^{LMS}}{SNR_n^{direct}} = \frac{\sigma^2}{\mu(\sigma^2 + \frac{1}{N}\sum_{i=n}^{N-1} k_i^2)} \quad (5)$$

The value of μ that achieves equal SNR will be

$$\mu_{lim} = \frac{\sigma^2}{\sigma^2 + \frac{1}{N}\sum_{i=n}^{N-1} k_i^2} \quad (6)$$

If a value of $\mu < \mu_{lim}$ is selected the adaptive estimation of the coefficients gets cleaner reconstructed signals than inner product. In figure 3(a) it can be seen the value of μ_{lim} for the KLT of ECG test set with various levels of noise.

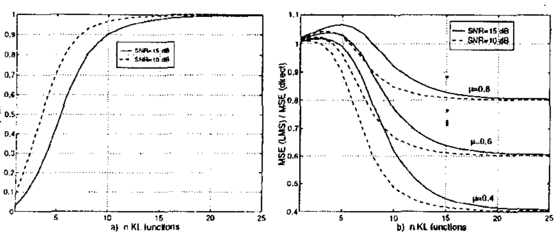

Figure 3: *a) Value of μ_{lim} in LMS for equal SNR to inner product. b) MSE ratio of LMS vs inner product.*

The MSE for LMS algorithms will then be

$$MSE_n^{LMS} = (1 + \frac{\mu n}{N})\frac{1}{N}\sum_{i=n}^{N-1} k_i^2 + \frac{\mu n}{N}\sigma^2 \quad (7)$$

In figure 3(b) it is presented the ratio between MSE_n^{LMS} and MSE_n^{direct} for some values of SNR=10, 15 dB and μ=0.6, 0.4. It can be seen that when the whole basis is used (n=N) the decreasing factor in MSE of LMS *vs* direct is μ. Improvement is achieved with $\mu < \mu_{lim}$ (fig. 3(a) and (b)).

This study is done assuming stationary ECG signals. The choice of a very low value of μ could have problems with the dynamic ECG changes and will increase the value of MSE. Then a study with real signals is required.

IV. SIMULATION

In order to study the effect of noise in estimating the coefficients of orthogonal transforms it is proposed the following study. White noise has been added to 23 records from the MIT-BIH Arrythmia Database. The level of noise added is much higher than unavoidable noise present in original records.

The mean value of MSE (T: theoretical, E: experimental) and standard deviation obtained for the 23 records with KLT and n=15 coefficients and a SNR=15 dB is shown in table 1.

It can be seen that LMS for values of $\mu < \mu_{lim}$ gets lower values of MSE than inner product, but they are not so good

		$\overline{MSE}(\%)$	σ_{MSE}	$(\frac{MSE^{LMS}}{MSE^{direct}}) \pm \sigma$
Inner Product		T: 1.15	—	—
		E: 1.55	0.57	—
LMS	$\mu_{lim} = 0.98$	T: 1.15	—	1
		E: 1.14	0.37	0.71 ± 0.12
	$\mu = 0.8$	T: 0.96	—	0.83
		E: 1.07	0.37	0.70 ± 0.16
	$\mu = 0.6$	T: 0.73	—	0.64
		E: 1.11	0.43	0.75 ± 0.25
	$\mu = 0.4$	T: 0.51	—	0.45
		E: 1.28	0.59	0.87 ± 0.40

Table 1: *Values of $\overline{MSE}$ and σ_{MSE} for real ECG set.*

as the predicted from theoretical study because of the dynamic changes of the ECG signal. This points are marked with (*) in figure 3(b).

In figure 4 it is presented an example of the original and reconstructed signals with inner product and LMS algorithm with original SNR=15 dB in QRS complex and $\mu = 0.6$.

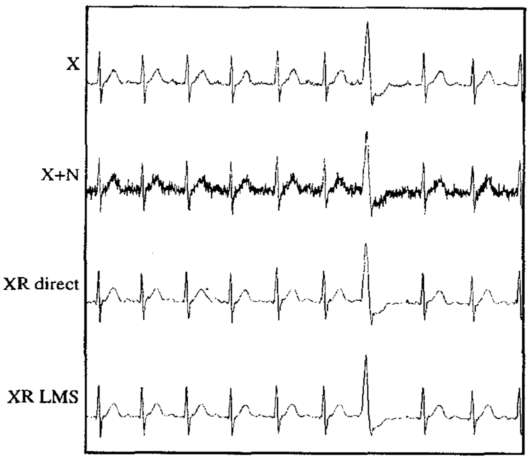

Figure 4: *Clean, noisy and reconstructed ECG signals.*

V. CONCLUSIONS

The effect of noise in orthogonal transform data compression of ECG signals has been studied. Two alternatives have been analyzed: inner product and adaptive estimation with LMS algorithm. Both have been compared in MSE and ΔSNR. For appropriate values of $\mu < \mu_{lim}$ LMS gets higher SNR than inner product and lower MSE.

A simulation study has been made with white noise added to ECG records from the MIT database showing that a μ=0.8 gives the best performance for MSE in ECG data compression decreasing MSE in a factor about 30% respect to inner product.

References

[1] S. M. S. Jalaleddine, C. G. Hutchens, R. D. Strattan, and W. A. Coberly, "ECG data compresssion techniques: A unified approach", *IEEE Trans. Biomed. Eng.*, vol. BME-37, no. 4, pp. 329-341, Apr. 1990.

[2] B. Widrow and S. D. Stearns, *Adaptive Signal Proccessing*, Prentice-Hall, Englewood Cliffs, New Jersey, 1985.