Power Spectral Density of Unevenly Sampled Heart Rate Data *

Pablo Laguna, George B. Moody, and Roger G. Mark

Centro Politécnico Superior, Univ. de Zaragoza, Maria de Luna 3, 50015 Zaragoza, Spain

Abstract - **This work studies the frequency behavior of the Lomb method to estimate the power spectral density (PSD) of heart rate (HR) unevenly sampled (UNS) signals. When the UNS can be modeled as uniform plus a random deviation (approximation well satisfied in HR signals), this spectra results in a periodic repetition of the original continuous time spectrum at the mean Nyquist frequency, with a low pass effect affecting the upper bands that depends on the sampling dispersion. In this case the estimation at the base band is unbiased with practically no dispersion. The performance has been tested with real HR signals corroborating the theoretical results. We have found that the Lomb method avoids the classical methods problem of low pass effect due to resampling. We conclude that the Lomb method is more suitable than classical methods for PSD estimation of UNS signals.**

I. INTRODUCTION

Heart rate variability (HRV) has become an useful tool for analyzing cardiac dysfunction [1]. The HRV is analyzed from the heart rate (HR) series that are not evenly sampled. The power spectral density (PSD) of the HR series seems to be the index that best recover the information present in the HR series [1]. Estimation of the PSD of HR series by classical methods can not be done directly from the time series signal, instead it requires resampling. This resampling introduces artifacts in the estimated spectrum. In addition, when ectopic or noisy beat detection occurs, a broadband noise contamination appears at the spectrum. Elimination of these ectopic or noisy beats and subsequent resampling introduce further alteration of HR spectrum. Autoregressive PSD also needs resampling. This problem has been recently overcome [2] by using a PSD estimation method that deals with unevenly sampled (UNS) data. This method was proposed in [3] (from now Lomb method) and it does not need to interpolate noisy or ectopic beat detection, avoiding spectrum distortion. In this paper we present a detailed analysis of the frequency behavior of the Lomb PSD estimation method. We particularize to the case of actual HR series and we derive a frequency limit estimation up to which the Lomb PSD estimate is free of aliasing.

II. THE LOMB PSD ESTIMATION METHOD

The Lomb method for PSD estimation is based on the minimization of the squared differences between the basis function of the transform and the signal under study [3]. Let's suppose $x(t)$ is the continuous signal under study and $b_i(t)$ is an orthogonal basis set that defines the transform. The coefficients $c(i)$ that represent $x(t)$ in the transform domain are those that minimize the squared error $e(c)$ defined as: $e(c) = \int_{-\infty}^{\infty}(x(t) - c(i)b_i(t))^2\, dt$. When the signal $x(t)$ is accessible only at unevenly spaced samples at t_n instants, Lomb [3] proposes to estimate the Fourier spectra of an UNS signal adjusting the model $x(t_n) + \varepsilon_n = c\ \ b_i(t_n)$, in such a way that the variance of ε_n is minimized resulting in a value for c

$$c(i) = \frac{1}{k}\sum_{n=1}^{N} x(t_n)b_i(t_n). \qquad k = \sum_{n=1}^{N} b_i^2(t_n) \tag{1}$$

This result can be call as a generalized Lomb method to estimate transforms of UNS data. The signal power at index i of the transformation ($P_x(i)$) will be [3]

$$P_x(i) = k\cdot c^2(i) = \hat{c}^2(i) \qquad \hat{c}(i) = c(i)\ \sqrt{k} \tag{2}$$

If the transform is the Fourier Transform (FT) then $k = N$ and $P_x(i) = P_x(f) = \hat{c}^2(f)$. Later, in [4] a fast algorithm was presented to estimate the Lomb spectrum. The remaining question of this PSD estimation method is: how is it related the original spectrum of $x(t)$ signal ($|x(f)|^2$) with the estimated spectrum from the Lomb estimate? From equation (1), we see that it can be rewritten in a different way in terms of the Dirac function $\delta(t)$.

$$\hat{c}(i) = \frac{1}{\sqrt{k}}\int_{-\infty}^{\infty}\sum_{n=1}^{N} x(t)\ \delta(t-t_n)b_i(t)\ dt. \tag{3}$$

We see that the coefficients obtained from the generalized Lomb method are those that become from the projection of the UNS signal $x_s(t) = \frac{1}{k}\sum_{n=1}^{N} x(t)\ \delta(t-t_n)$ onto the base signal $b_i(t)$. It means that the Lomb spectrum is the spectrum of the continuous time UNS signal $x_s(t)$, so in fact the Lomb spectra is the $|x_s(f)|^2$ spectra. Note that we have substituted k by N since the sum of $\sum_{n=1}^{N} |e^{i2\pi f t_n}|^2 = N$. The relationship between $x_s(f)$ and $x(f)$ will give the relationship between the estimated spectrum and the real one from the original signal.

IV. PSD WITH RANDOMLY DISTRIBUTED SAMPLING

Knowledge of $x_s(f)$ requires knowledge of the t_n distribution. However, in signals like HR, the distribution of t_n series can be well modeled (over stationary periods), as a uniform sampling with a random deviation. This fact leaded us to consider the particular case where the t_n distribution satisfies $E[t_n - t_{n-1}] = T$ and $E[t_n - nT] = 0\ \forall n$. Then, t_n can be expressed as $t_n = n\cdot T + \alpha_n$ where α_n is a random variable with zero mean and probability distribution function $P_{\alpha_n}(\alpha_n)$. Moreover $x(t)$ is assumed to be a deterministic signal to be estimated, the random variable α_n makes $x_s(t)$ to be a random process, and then also its FT $x_s(f)$, is a random process. In this case we will consider the mean $E[x_s(f)]$ and the variance $\sigma_{|x_s(f)|^2}$ to have information of how the estimated Lomb spectra $|x_s(f)|^2$ is related to the real signal spectrum $|x(f)|^2$. Calculating the $E[x_s(f)]$ we have:

$$\bar{x}_s(f) = \frac{1}{\sqrt{N}}x(f) * \left[P_\alpha(f)\ \left(f_s\sum_{n=-\infty}^{\infty}\delta(f-nf_s) * W(f)\right)\right], \tag{4}$$

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where $f_s = 1/T$ is the mean sampling frequency, $W(f)$ is the window FT applied to the signal and $P_{\alpha_n}(f)$ is the characteristic function of the probability density function $P_{\alpha_n}(\alpha_n)$. This expression now allows to interpret $E[x_s(f)]$ in terms of the known uniform sampling FT. $E[x_s(f)]$ is the convolution of the original FT signal $x(f)$ with the FT of a uniform sampling function widowed by the finite time observation interval $W(f)$ and now weighted by the characteristic function of the randomly distributed sampling $P_\alpha(f)$. Again appears the sampling theorem to avoid aliasing, where now the Nyquist frequency comes from the mean sampling interval T. So, to have a correct estimate, the mean sampling frequency should be higher than twice the higher frequency of the signal $x(t)$. The other ith frequency bands will present (in mean) the repetition of the base band spectrum weighted by the value of the characteristic function $P_\alpha(i \cdot f_s)$ evaluated at multiples of the mean sampling function f_s. In case that $P_\alpha(\alpha)$ is a Gaussian distribution with standard deviation σ, the characteristic function is also Gaussian, $P_\alpha(f) = e^{-(2\pi^2 f^2 \sigma^2)}$, and consists in a low-pass filter with a cutoff frequency (f_c) at -3 dB of $f_c = 132.5/\sigma$. Then the estimated mean base-band spectrum with Lomb method is unbiased and each ith mean upper band will be affected by a factor that depends on the sampling distribution t_n throughout the σ value, and on band order i.

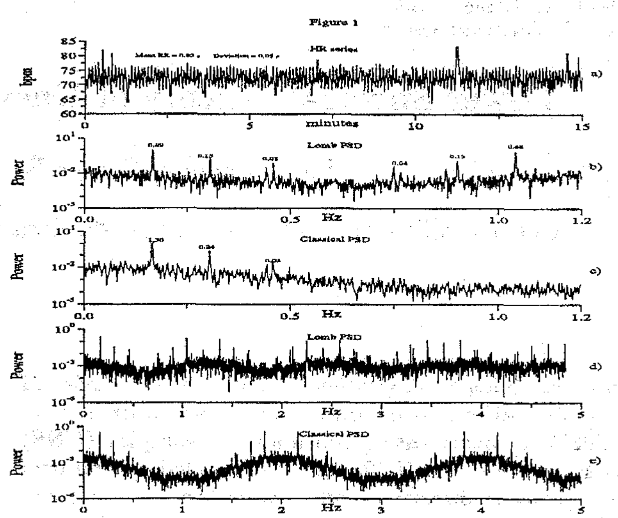
From this result we see that, at these kind of UNS, The mean base-band Lomb spectrum is the $x(f)$. However, it remains to study the variance of the Lomb estimate to see how much a single trial estimate will differs from the mean value. Calculating this variance we get

$$\sigma_{|x_s(f)|^2} \leq 2\sqrt{E[x_s(f)]E[x_s(f)]^* + 2(\pi\sigma n f_s)^2}\sqrt{2(\pi\sigma n f_s)^2}. \quad (5)$$

where n is the band order and thus, at the base-band $(n = 0)$ becomes zero and certificates the single trial Lomb estimate as correct estimate of the PSD of $x(t)$ signal. Note that the deviation increases also with the value of the spectrum at given frequencies. If the signal is evenly sampled $(\sigma = 0)$ we recover the spectrum of the classic uniformly sampled signals. As the band order increases $n > 0$ either the estimate is a biased estimate with a variance that increases with the band-order, the sampling frequency and the dispersion of the sampling.

V. ANALYSIS OF HEART RATE SPECTRUM

In figure 1a we have the HR series of a paced patient from record 102 of the MIT-BIH ECG database. In this record it was introduced an artifactual peak at the frequency domain (0.167 Hz) due to a non symmetric capstan used in the playback system. This artifact and its harmonics will serve in this study as the test for the Lomb PSD method. In this case the assumption of stationary UNS is well satisfied. Figure 1b displays the Lomb spectra of this data. Also it is displayed, 1c, the PSD estimated through resampling the data to a sampling frequency of 2 Hz and estimating the PSD with classical FFT algorithms. The mean sampling frequency at the original HR series is 1.21 Hz and the deviation is 31 ms. Analyzing the Lomb spectrum we corroborate that appears a periodicity which period is the mean sampling frequency, 1.21 Hz, as predicted by the study. There appears at the spectrum three harmonically related peaks at 0.16, 0.30 and 0.45 Hz that correspond to the artifact introduced by the capstan. We can note, fig. 1b, how these peaks have lower amplitude at the second band (0.6 - 1.8 Hz) than at the primary (-0.6 - 0.6 Hz) as a result of the low pass effect introduced by the deviation at the UNS. In fig. 1d we have the Lomb spectrum for several cycles of the mean sampling frequency 1.21 Hz. We can corroborate the



periodic behavior of this spectrum and the low-pass effect given by the sampling dispersion at the upper bands. Note that the peaks and mean shape of the signal at higher bands have lower amplitude, and the signal becomes more embedded by the noise.

Considering now the spectrum obtained from resampling and classical spectrum estimation, fig. 1e, we note that the spectrum has the expected 2 Hz (sampling frequency) periodicity, and also note, fig. 1c, that the spectrum at high frequencies (HF) is attenuated as a result of the resampling. This is particularly evident at the third peak of the spectrum that is much less marked that at the Lomb spectrum. The effect of HF attenuation introduced by the resampling is particularly important in HRV analysis where the ratio between the energy at different bands is used as a clinical marker of cardiac dysfunction [1]. Then, we corroborate the theoretical behavior study in this paper and that the Lomb PSD estimate is a better estimate than any resampled estimation.

VI. CONCLUSIONS

In this work we have presented a detailed analysis of the Lomb method for PSD estimation of UNS signals. We have noted that when the UNS can be modeled as uniform with random variations, the Lomb spectrum repeats with the mean Nyquist frequency, being an unbiased estimate of the PSD at the base-band and with a low-pass effect at upper bands that depends of the sampling distribution. This study has corroborate the theoretical predictions about the Lomb estimation. Comparison with classical PSD estimation method applied after resampling, has shown the limitations of the resampling. Concerning with the HR spectrum we have evidentiated that the Lomb estimate is a better estimate than the classical estimates, since the power ratio between low and HF components is relevant in clinical diagnosis.

References

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