ECG Data Compression with the Karhunen-Loève Transform

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Abstract

In this work we analyze a Karhunen-Loève transform technique for ECG data compression. This transform has been applied in two different ways: to the entire beat signal and to independent windows (P wave, QRS complex and ST-T complex). The optimum number of coefficients and bits for coding the signal is analyzed for the MIT-BIH Arrythmia database. The data compression performance of both choices are: for the entire beat a mean compressio ratio of 12.1 with a mean MSE of 0.3% and for shorter windows a mean compressio ratio of 17.12 with a mean value of MSE of 0.44%

1. Introduction

The great amount of data generated from digital recording of ECG signals forces data compression techniques for redundancy removal. In this work we analyze a Karhunen-Loève Transform (KLT) technique for ECG data compression. The KLT is a rotational transform of a n-dimensional Euclidean pattern space that performs optimal signal energy concentration into the minimum number of coefficients. Data compression can be obtained selecting a fraction of the coefficients giving a reduced loss of information.

Data compression can be defined as the process of reducing redundancies in a signal [1]. In the case of ECG signal we can distinguish two kinds of redundancies: the first one due to excess in sampling frequency and/or wordlength (shown in the correlation between samples of the same beat) and the second one due to the quasiperiodic nature of ECG signal (shown in the correlation between samples of adjacent beats).

We propose a KLT system in order to reduce the first kind of redundancies and a differential coding system for the kl coefficients to reduce the second one.

The KLT has been applied in two different ways: firstly, using the entire beat as pattern vector of the KLT, and the second one using independent signal subwindows: P wave, QRS complex and ST-T complex. The latter method (KLw) achieves better performance. A comparison is presented with a suboptimal transform (DCT) applied to the entire beat showing that the performance of KLT is superior to DCT.

2. The Karhunen-Loève transform

The KLT is a signal dependent transform that is optimal in the sense that is the solution to the problem of minimizing the mean squared error between a signal and a reduced linear combination of orthogonal basis functions [2].

The derivation of the KLT basis begins by estimating the covariance matrix $\mathbf{C}$ of the pattern vectors of the training set $\mathbf{C} = E\{\mathbf{x}\mathbf{x}^T\}$ where $\mathbf{x}$ are the ECG signal windows [3]. The covariance matrix reflects the distribution of the pattern vectors in the pattern space. The orthogonal eigenvectors of $\mathbf{C}$ are the basis functions of the KLT, and the eigenvalues, λ_k, represent the average dispersion of the projection of a pattern vector onto the corresponding basis function. After sorting the eigenvectors by decreasing eigenvalues, such that $\lambda_k \geq \lambda_{k+1}$, for $k = 0, 1, ..., P-1$, the corresponding basis functions are arranged in order of representational strength.

3. Training set and KLT basis

To obtain a representative training set of normal and abnormal ECG waveforms we have selected a wide variety of ECG records from different databases (MIT-BIH databases and European ST-T database). First, QRS complexes were detected and labeled using ARISTOTLE [4]. We grouped all the labels in three groups: Normal beats (N), Ventricular beats (V) and Left Bundle Branch Block beats (L).

For normal beats (N) the training set was 114 records in all (36 from the MIT-BIH Arrhythmia Database and 78 from the European ST-T Database) with a total of over 220.000 beats. For ectopic beats (V) the trainig set was 63 records in all (35 from the MIT-BIH Arrhythmia Database, 13 from the European ST-T Database, and 5 from the MIT-BIH long term database) with over 62.000 beats. For (L) beats the trainig set was 4 records from the MIT-BIH Arrhythmia Database with over 8.400 beats.

The alignment of the windows contaning the ECG signals is necessary to compute the covariance matrix. All windows were aligned to the fiducial point of the QRS complex q_i. For (N) beats the entire beat window is taken as $(q_i - 250\,ms, q_{i+1} - 250\,ms)$. If the RR_i interval (between QRS marks) is $q_{i+1} - q_i < 720\,ms$, the end of the window is located at $q_i + \frac{2}{3} \cdot RR_i$. (L) beats windows are similar to (N) but the quantities are 380 ms in contrast 250 ms and $\frac{2}{5}$ in contrast to $\frac{2}{3}$. (V) beats window is defined as $(q_i\text{–}0.2 \cdot RR_{i-1}, q_i\text{+}0.8 \cdot RR_i)$.

For (N) and (L) beats additional segmentation of the pattern vector achieves better performance. Three shorter windows are defined: P wave defined as $(q_i - 250, q_i - 60)$, QRS complex $(q_i - 60, q_i + 85)$, ST-T complex $(q_i + 85, q_{i+1} - 250)$.

Baseline wander removing is applied to the records with cubic splines interpolation previous to estimate the covariance matrix to only use the clinical information.

The pattern vectors were normalized by magnitude (i.e., scaled such that the signal energy was constant); in this way, each pattern vector is given equal importance when deriving the KLT basis functions. Since the durations of the windows vary the estimation of certain elements of the covariance matrix is problematic. We address this issue by estimating each element of the covariance matrix using only those windows for which the corresponding elements are available. This procedure avoids the introduction of varying length artifacts into the covariance matrix estimate; its consequence is that the final portions elements of the basis functions are derived from a smaller sample set than the initial portions.

The eigenvalues of the covariance matrix show the energy concentration performance. Figure 1 shows the mean squared error (MSE)

$$MSE_p = \frac{\sum_{i=p}^{P-1} \lambda_i}{\sum_{i=0}^{P-1} \lambda_i} \cdot 100 \qquad (1)$$

as a function of the number of coefficients p when the entire beat window is selected as pattern vector. The eigenvectors corresponding to the largest eigenvalues reflect the most common ECG waveform (figure 2).

4. Data compression performance

The operation of an orthogonal transform data-compression algorithm is illustrated in figure 3. The ECG window signal $\mathbf{X}$, an P-element long sequence of M-bits integer numbers, is operated on by the orthogonal transform $\mathbf{T}$, to produce the sequence $\mathbf{Y}$. The elements of the $\mathbf{Y}$ sequence are the magnitude of the projection of $\mathbf{X}$ vector onto the $\mathbf{T}$ basis. The data can be compressed in any of these two ways: first, by selecting less than the entire set of coefficients

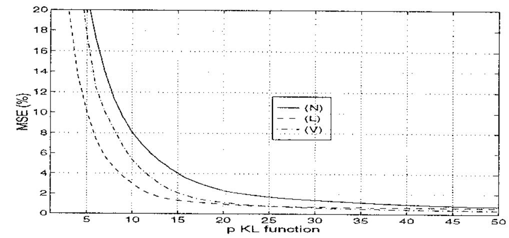

Figure 1: *MSE as function of p for (N), (V) and (L) beats.*

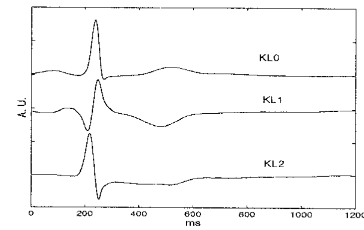

Figure 2: *First KLT basis functions for normal beats.*

$p < P$, or second, by using lower number of bits for the coefficients than those used to represent the original signal $(m < M)$. The compression ratio (CR), expressed as the ratio between the total number of bits of the original signal and the total number of bits of the reconstructed, will be $CR = \frac{P\,M}{p\,m}$.

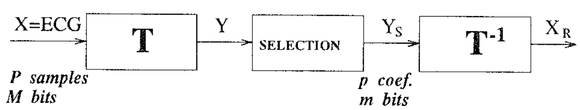

Figure 3: *Block diagram of a transform data compression system.*

The ECG signal has a high correlation between successive beats, so more redundant information can be eliminated with differential coding of the coefficients (DPCM). So, DPCM is employed to achieve higher compression ratio. Three series of coefficients are independently generated for every morphology (N,V,L), avoiding transient generated from changes between types. DPCM of order 1 has been applied to reduce the number of bits for coding the coefficients.

A reconstructed signal $\mathbf{X_R}$ can be obtained with the inverse orthogonal transform $\mathbf{T^{-1}}$. The original and the reconstructed signal will differ in the error sequence $\mathbf{E}$, $\mathbf{X} = \mathbf{X_R} + \mathbf{E}$. There is a clear tradeoff between compression ratio and error signal. A low number of functions p gets higher compression ratios, but the error signal is higher. On the contrary, a higher number of functions achieves very accurate reconstructions with lower compression ratios. The objective of a data compression algorithm is to get the maximum CR accepting a fixed "low error level".

The couple (p, m) sets the functioning conditions for the data compression system, and it is defined as *operating point.* Figure 4 shows the performance of all possible operating points for (N) beats obtained from the first 5 minutes of ECG records of MIT-BIH Arrythmia and European ST-T databases. The MSE of residual $\mathbf{E}$ decreases when both the number of functions p and the number of bits m increase. On the analogy of many disciplines we name this plot as *characteristic curves chart*, since it relates the input parameters (p, m) to the output parameters (MSE).

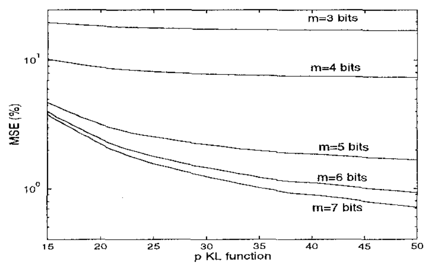

Figure 4: *Characteristic curves chart for (N) beats.*

For example, using $m = 6$ bits and $p = 40$ coefficients we get a MSE of 1% and a compression ratio of $CR = \frac{P\,M}{p\;m} = 13.2$ for beats of 800 ms (P=288 samples with sampling frequency 360 Hz) and M=11 bits/sample.

In order to determine the optimum operating points (p^*, m^*) it is very useful to represent the same information on a (p, m) plane (figure 5). The reconstruction error is represented as solid contour lines. The dotted lines represent the points of iso-compression ratio, that are hyperbola functions ($p\,m = constant$) that implies Bits Per Beat (BPB)=constant. The minimum MSE is obtained at the upper-right corner, but it presents minimum compression ratio.

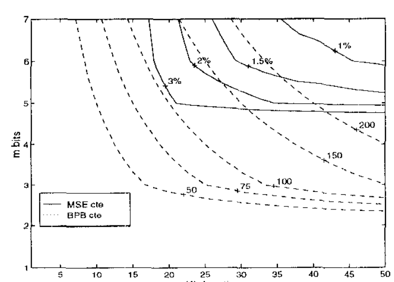

Figure 5: *Average error surface contour for (N) beats.*

This chart is a wise compendium of all the involved variables in orthogonal transform data compression systems. Firstly, it clearly shows the trade-off between compression ratio and reconstruction error: signals with low level error have a low compression ratio and vice versa. Secondly, the optimum operating point are the "knees" of the contour lines, and they are easily determined on the chart.

For a given value of BPB, the optimum operating point is found as the integer values (p_1^*, m_1^*) nearer to the "knee" of the contour line tangent with the given hyperbola. Similarly, for a given MSE the optimum operating point is found as the integer values (p_2^*, m_2^*) nearer to the "knees" of the given contour line. It must be noted that only integer values of p, m can be considered, even though continous lines are plotted for clearer representation.

The presented method gets the optimum tradeoff between data compression and fidelity because it searches the optimum operating point over the entire (p, m) plane. This chart and its optimum operating points are only valid for a specific beat type. The average chart is obtained by averaging charts from representative morphologies of ECG databases.

The optimum operation point for (N) beats are in table 1 for a value of MSE of 1% (equivalent to a value of 10% PRD [1]). The values for the DCT are also presented, and it is corroborated the superior performance of the KLT. Similar results are obtained for other morphologies.

Table 1: Optimal operating points for (N) beats.

	DCT			KL			KLw	
MSE	m	p	CR	m	p	CR	$\sum m_i\, p_i$	CR
1%	6	69	7.65	6	40	13.2	185	17.1

Many other optimal operating points can be selected from the contour map for several values of MSE. The number of Bits Per Beat (BPB) used when an optimal point is selected is constant, and the bit rate (bit/s) for every record will be function of the heart rate. Similar charts have been obtained for subwindows P, QRS and ST-T.

5. Results

This method has been applied to the MIT-BIH Arrythmia. The selected operation point is ($p = 40$, $m = 6$) for (N) beats, ($p = 23$, $m = 6$) for (L) beats and ($p = 24$, $m = 6$) for (V) beats. The values of MSE and CR are shown in figures 6 and 7 respectively. The two methods (entire beat and short windows) are compared. The entire beat method achieves lower values of MSE, but its compression ratio is lower than the windows method. Obtained results are better than those reported in [1].

In figure 8 there is an example of original signal (above) and reconstructed signals (entire beat method

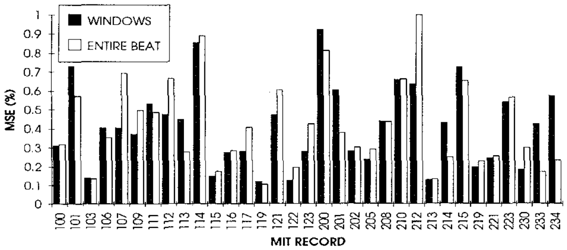

Figure 6: *Mean squared error for the MIT-BIH Arrhythmia database.*

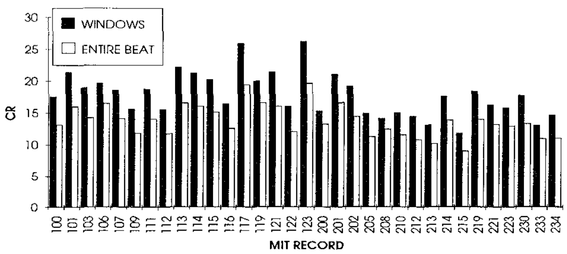

Figure 7: *Compression ratio for the MIT-BIH Arrhythmia database.*

in the middle and windows method at bottom) for record 106.

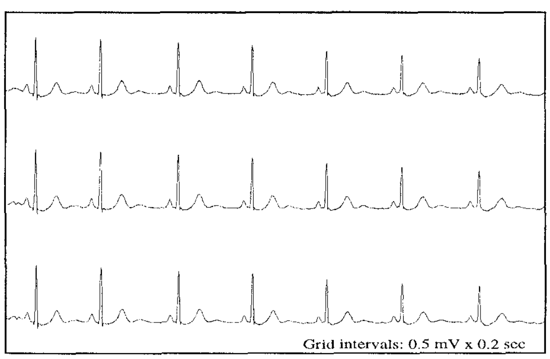

Figure 8: *Original and reconstructed signals for record 106.*

In figure 9 there is an example of (V) beats in record 221.

6. Conclusions

The Karhunen-Loève Transform (KLT) has been applied for ECG data compression. A "universal" basis has been calculated from a very large training set of ECG records from MIT-BIH databases and European ST-T database. Three different basis are calculated for the groups of morphologies Normal beats (N), Ventricular beats (V) and Left Bundle Branch Block beats (L).

An exhaustive analysis of the possible operation

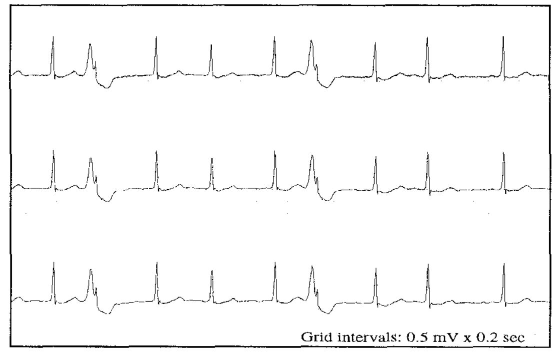

Figure 9: *Original and reconstructed signals for record 221.*

points has been made for selecting the optimum number of coefficients and bits. The performance of the KLT has been compared with the DCT in terms of MSE and CR and it has been showed that KLT is superior to DCT. Moreover, when the KLT is applied to shorter windows (P wave, QRS complex and ST-T complex) the best performance is obtained. Also it requires lower computational load.

These methods have been applied to 39 records from MIT-BIH (nine records were rejected with non well defined P, QRS T waveforms), obtaining a mean compresion ratio of 12.1 and a mean MSE value of 0.44% for the entire beat method, and compression ratio of 17.2 and a mean MSE of 0.3% for the window method.

Acknowledgements

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