Improved Interpolation of Unevenly Sampled Heart Rate Signals

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Abstract

In the Heart Rate Variability (HRV) analysis, based on Power Spectral Density (PSD), there are two major problems: The accessibility of information (beat timing) and the uneven sampling of the information carrying signal. The first problem has been addressed through the Heart Rate (HR), the Heart Period (HP), and recently we have proposed the Heart Timing (HT) signal. Interpolation has been done with linear or cubic spline methods. In this work, we propose an alternative, that is a combination of the adaptive weights preconditioning method and the method of conjugate gradient for the solution of positive definite linear systems. We show how this reconstruction algorithm allows the recovery of a sequence of regular spaced samples and its spectrum from the proposed Heart Timing signal, with higher precision (lower low-pass effect) than other methods.

1. Introduction

The power spectral density (PSD) estimate of heart rate variability (HRV) is commonly used as a test of the neural cardiovascular system, since it is related to the sympathetic and parasympathetic regulation of the sino atrial node [1]. Many authors have assumed the integral pulse frequency modulation (IPFM) model to explain the mechanisms used by the autonomic system to control the heart rate. This model assumes a modulating signal $m(t)$ that, when acting through the IPFM, generates the beat occurrence times [2]. PSD methods try to infer the spectrum of $m(t)$ signal, $M(\omega)$, from the beat occurrence times, usually from Heart Rate (HR) or Heart Period (HP) signals.

In a recent paper [7], we commented on two major problems in HRV analysis based on PSD: The first was the election of a discrete signal that represents the spectral activity of the neural system. This signal must be built through the only known information: the beat occurrence times. We presented the Heart Timing (HT) signal as a better alternative to the HR or the HP signals because its spectrum is directly related to the one of the modulating signal in the IPFM model. The second problem is related to the inherent uneven sampling of all this discrete signals. There are two main ways to minimise this problem: To use methods for spectral estimation directly from the irregular sampled signal as the Lomb method [3], or use methods that interpolate the irregular sampled signal at regularly spaced time points. The cubic spline interpolation technique has better performance than linear but it still has a low-pass filtering effect. Therefore, we have applied a more elaborated and powerful method of reconstruction of unevenly sampled band-limited signals [5]. The so called, ACT method has been developed by the NUHAG group at the University of Vienna and, it is in this work, that we have applied this method to the Heart Timing and the Heart Rate signals.

The spectrum of an actual signal of the modulating heart activity is unknown. Then, to experimentally compare the performance of this method with the others, a controlled experiment with known modulating signals $m(t)$ has been developed. These signals come from reported AR models representing real data, and other computer generated signals. These $m(t)$ signals, used as inputs to the IPFM model, generate beat sequences analysed with different HRV methods to recover the information present in $m(t)$. We use reported PSD methods to estimate the HRV spectra and the new ones based on the HT signal and the ACT interpolation. The obtained spectra in all these cases are compared with the original one, $M(\omega)$. The results show a comparative analysis between the different methods.

2. The Heart Timing signal

We have summarised here the use of the HT signal more extensively referenced in [7]. The IPFM model is based on the hypothesis that the all influences on the sino-atrial node can be represented by a single function $m(t)$. The time series of beats by means of IPFM model can be generated by

$$k = \int_0^{t_k} \frac{1+m(t)}{T}\, dt \qquad (1)$$

where k is an integer and represents the order number of the kth beat, t_k is the time occurrence of this kth beat [2]. We can see $(1+m(t))/T$ as the instantaneous heart rate. T is the mean of the RR interval and $m(t)/T$ represents the dynamic part and its mean is zero. It is considered that the first beat occurs at $t_0 = 0$ and that $m(t) = 0$ if $t < 0$.

We defined the continuos Heart Timing signal as

$$HT(t) = \int_{-\infty}^{t} m(\tau)d\tau = \int_{0}^{t} m(\tau)d\tau \qquad (2)$$

with its spectrum

$$HT(\omega) = \frac{M(\omega)}{j\omega} + \pi\, M(0)\delta(0) = \frac{M(\omega)}{j\omega} \qquad (3)$$

The spectrum of *HT(t)* is the same as the *m(t)* except for $1/j\omega$ because of the integration. However, we only know the discrete beat occurrence times at t_k.

Using the new defined signal, the equation (1) can be rewritten as

$$HT(t_k) = kT - t_k = \int_{0}^{t_k} m(\tau)d\tau \qquad (4)$$

where $HT(t_k)$ defines the unevenly sampled version of the Heart Timing signal, but its spectrum

$$\mathrm{F}\{HT(t_k)\} = \frac{M(\omega)}{j\omega} * \sum_{k=0}^{\infty} e^{-j\omega t_k} \qquad (5)$$

can be rather different to the one of the modulating signal due to the convolution term. All irregularly spaced HRV signals, HP, HR and HT and its directly related PSD estimation methods experience this problem.

3. Interpolation methods

The interpolation can be seen as a time-variant filter that acts with different frequency response as function of the space between interpolated samples. The frequency response of the linear and cubic spline interpolation methods has been obtained as the frequency transforms of a unit impulse interpolated with each method. In figure 1 we can observe both responses. The linear interpolation has a cut-off frequency that goes from $0.36/T_s$ Hz when the interpolation factor is 2, to $0.32/T_s$ Hz when the interpolation factor is 16. The cubic spline method has a cut-off frequency of $0.44/T_s$ Hz and it has a negligible dependence with the interpolation factor. T_s is the original sampling period. The linear method has a lower cut-off frequency than the cubic spline method as expected by its inferior performance.

When the heart rate is low, the filtering effect is more significant. Thus, with cubic splines, a HR of 40 bpm (T_s=1.5 s.), gives a cut-off frequency of 0.29 Hz that will still distort some clinically useful information.

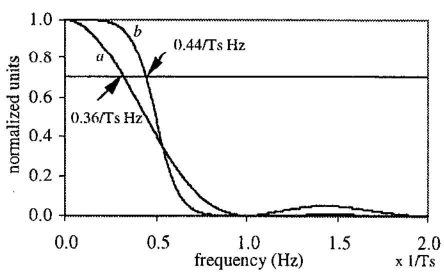


Figure 1. Interpolators frequency response with cut-off frequencies. a) Linear method b) Cubic spline method.

To reduce the low pass filtering effect produced by the cubic spline method, a more powerful reconstruction method of unevenly sampled band limited signals is proposed. This, so called ACT method, is a combination of the adaptive weights method, conjugate gradient acceleration and the use of Toeplitz matrices [5]. This method has been developed by H.G. Feichtinger and the NUHAG group at the University of Vienna and it has been applied to the HRV signals in this work. Due to its small storage requirements, it is particularly well suited for large data sets. It also works well near the critical sampling density (low heart rates) and with a sampling set with large gaps (ectopics presence). It is used as an iterative algorithm to recover the regularly sampled signal and its spectrum from its irregular samples simultaneously. In [5] a detailed description of this method is given.

4. Comparison between the spectra

In the study of HRV, different spectral analysis methods have been used [2], [7]. A comparison of these methods and the new based on the ACT method is presented in this paper. These methods will be compared:

- Spectrum of counts (SPC)
- Lomb method of the HP and HR signals (LHP, LHR)
- DFT of the cubic spline interpolated HP, HR and HT signals (FHPI, FHRI, FHTI)
- ACT method of the HP, HR and HT signals (ACTHP, ACTHP, ACTHT)

Others methods was compared as the direct DFT of the sequences and the "DFT of the low-pass filtered sequences filled with zeros". They are not included because the poor performance of the first ones and the equivalent behaviour to the Lomb method of the second ones.

We have made two kinds of experiments to compare the different PSD estimates proposed. Firstly, we have considered as modulating signal *m(t)* as a Sinc-like and as a Gaussian signal which we exactly known its spectrum.

The second simulation consists in using a realistic AR model to simulate the *m(t)* signal [4]. Then, the series of beats are generated as output of the IPFM model and the PSD estimation is achieved by the different methods.

4.1. Analytical modulating *m(t)* signals

We have used controlled analytical wide band signals to remark the filtering effect of the different methods. In the first case (Figure 2a) *m(t)* is simulated with

$$m(t) = 0.2\frac{\sin(2\pi \cdot 0.4 \cdot (t-512))}{2\pi \cdot 0.4 \cdot (t-512)} \qquad (6)$$

whose spectrum is flat from 0 to 0.4 Hz. The mean HP is *T*=1s (mean HR=60 bpm). In the second case, *m(t)* is simulated by Gaussian signals

$$m(t) = \frac{1}{2}e^{-\frac{(t-512)^2}{200}}\left(\frac{1}{2} + \cos(2\pi\, 0.19\, t) + \cos(2\pi\, 0.31\, t)\right) \qquad (7)$$

whose spectrum is formed by three Gaussian functions centred at 0, 0.19 and 0.31 Hz, and with the same amplitude. The mean HP is T=1s. (Figure 2b). A third case is simulated with the same $m(t)$ but with T=1.2s (mean HR=50 bpm) (Figure 2c). Moreover, to show the robustness of each method with the presence of ectopics, the three cases above have been reproduced with five random beats removed in each case (Figures 2d, 2e, 2f).

In figure 2a can be observed how only SPC and ACTHT methods keep the rectangular shape but SPC introduce high frequency power over 0.3 Hz. The FHTI has a good behaviour but near to 0.35 Hz, the low-pass effect becomes remarkable due to the interpolator. The other methods deform the spectrum appreciably. The HR-based methods overestimate the low frequency components and the HP-based methods tend to underestimate these frequencies. Also, note how for any method the ACT interpolation is the one that gives better high-frequency estimations. In figure 2b can be observed how only SPC, FHTI and ACTHT keep the amplitudes of the Gaussians, but again, the SPC introduces artefacts at high frequencies (over 0.3 Hz). The third case demonstrates how at low heart rates, the low-pass filtering effect of FHTI is more notable and it is here where the improvement of the ACTHT is more remarkable.

Figures 2d, 2e and 2f show the deformation of the obtained spectra due to the presence of ectopics. Only the ACTHT method recovers the information with precision and again, without any filtering effect.

The *Normalised Power Error* has been computed as

$$NPE = \frac{\int \left| |M(\omega)|^2 - |\hat{M}(\omega)|^2 \right| d\omega}{\int |M(\omega)|^2 d\omega} \qquad (8)$$

where $M(\omega)$ is the original spectrum and $\hat{M}(\omega)$ is the estimation obtained with each method. Table 1 shows the *NPE* obtained in the six cases. The first three rows correspond to the cases without ectopics and the next three rows show the results obtained with ectopics.

This simulation demonstrate that the ACTHT method is the one that recovers the spectrum with precision in the presence of ectopics while the other methods deteriorate its behaviour notably. Especially, the SPC method becomes unusable in these conditions (no guessing of beats has been made in the SPC).

Table 1. Normalised Power Error ($\times 10^{-2}$)

	SPC	LHP	LHR	FHPI	FHRI	FHTI	ActHP	ActHR	ActHT
Case 1	1.19	16.3	20.4	33.4	15.9	2.14	33.7	14.1	0.11
Case 2	9.61	27.9	113	55.5	21.5	8.84	54.7	10.0	8.73
Case 3	7.84	33.7	92.5	59.8	34.0	12.7	55.7	10.8	7.28
Ectp 1	-	16.1	22.8	33.2	24.4	11.1	51.4	48.8	0.11
Ectp 2	129	39.0	114	62.2	26.8	17.3	58.6	44.9	8.73
Ectp 3	136	45.9	80.9	69.9	57.9	44.7	61.1	11.8	7.28

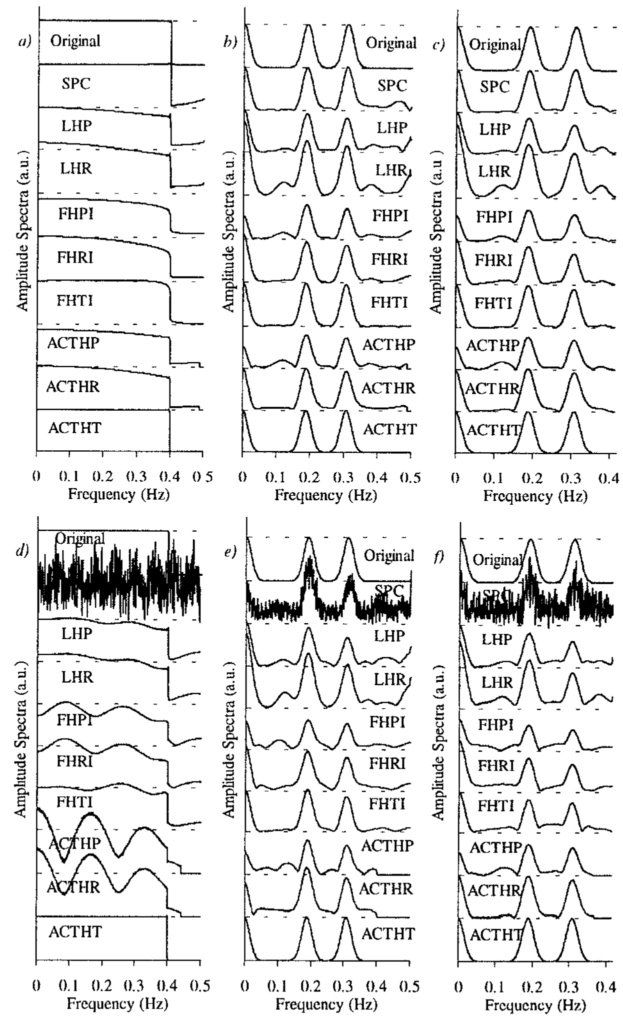
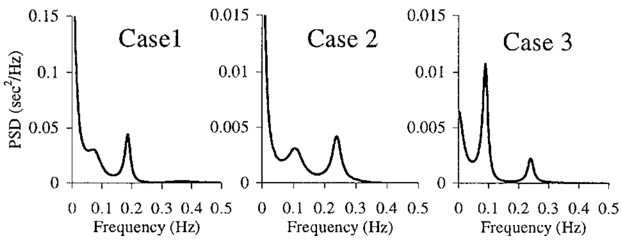


Figure 2. Spectra obtained for analytical $m(t)$ signals. a) Sinc signal with T=1s. b) Gaussian modulated signal with T=1s. c) Gaussian signal with T=1.2s. d) Sinc signal with ectopics. e) Gaussian signal with ectopics and T=1s. f) Gaussian signal with ectopics and T=1.2s.

4.2. AR simulation

To compare the behaviour of the spectra estimates with a more realistic signal we generate the beat series from an $m(t)$ signal following typical spectra from a real subject. Three AR models have been used for generating twenty sequences of the $m(t)$ signal. The first model is the one proposed in [4] with T = 1.2s. and $\sigma_{RR} \approx$ 42ms. A second model was made based on the typical rest tachogram suggested in [6]. The proposed T = 842.5ms. and $\sigma_{RR} \approx$ 42ms. have been used. Finally, a third model was made based on the tilt tachogram, which was also presented in [6]. Again, the proposed T = 564.7 ms. and $\sigma_{RR} \approx$ 27 ms. have been used. Figure 3 shows the characteristics and the coefficients of these AR models.



$$a_k = \begin{vmatrix} 1.0701 \\ -0.3360 \\ -0.0117 \\ -0.0758 \\ 0.4281 \\ -0.2354 \\ -0.1165 \\ 0.0119 \\ 0.1435 \end{vmatrix} \quad a_k = \begin{vmatrix} 1.6990 \\ -2.0322 \\ 2.0713 \\ -1.5485 \\ 0.9674 \\ -0.5488 \\ 0.2872 \end{vmatrix} \quad a_k = \begin{vmatrix} 1.9282 \\ -2.3335 \\ 2.4816 \\ -2.0508 \\ 1.2438 \\ -0.8116 \\ 0.4130 \end{vmatrix}$$

Figure 3. Spectra of the AR models and its coefficients

The PSD is frequently divided in three bands of frequency: VLF (0.003 - 0.04 Hz), LF (0.04 -0.15 Hz) and HF (0.15 - 0.4 Hz) to get the clinical indexes [6]. The relative power at each band, VLF/AF, LF/AF and HF/AF, where AF=VLF+LF+HF, has been calculated to compare each method with the original spectrum in each case. Then, the error in each band for each realisation has been computed. Finally, the mean of the error in each band and its standard deviation has been obtained considering the twenty realisations. Figure 4 shows the obtained results.

We can observe that the SPC, FHTI and ACTHT methods have a mean error much smaller than the others do, which have a strong low pass response. Moreover, its standard deviation is also smaller. The FHTI has a higher low pass response in the case 1 due to the low heart rate.

5. Conclusion

The ACT method in conjunction with the Heart Timing signal has been demonstrated to be a good alternative to estimate the PSD of HRV signals. This method has not any low pass filtering effect and recovers precisely the spectrum of the modulating signal. The improvement respect to FHTI is more visible in signals with power in high frequencies or with lower mean heart rate. It is especially remarkable the ACTHT behaviour with signals with ectopics or missed beats. This method is the only one that recovers the spectrum with good fit to the original. Then, the ACTHT method is the more suitable for HRV analysis studies, especially in cases of low HR, high frequency components at the HRV signal or presence of PVC or false negatives QRS detection.

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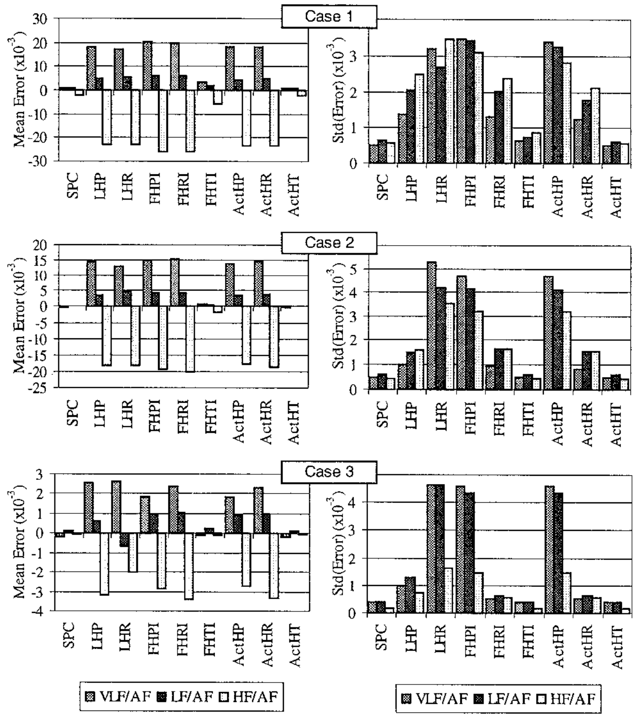

Figure 4. Mean error and standard deviation of error of relative clinical indexes in each band.

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