New Heart Rate Variability Time-Domain Signal Construction from the Beat Occurrence Time and the IPFM Model

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Abstract

Heart rate variability (HRV) is an extended tool to analyse the mechanisms controlling the cardiovascular system. It is assuming the integral pulse frequency modulation model (IPFM) to explain the beat occurrence times generation. Spectral estimate methods try to infer the modulating signal characteristics from the available beat timing on the ECG signal. These methods usually estimate the spectrum through the heart period (HP) or the heart rate (HR) signals. In this work, we introduce a new HRV signal, the Heart Timing Signal, (HT) that allows to recover an unbiased estimate of the modulating signal spectra avoiding the spurious components produced when analysing the modulating signal through HR or HP.

1. Introduction

Power spectral density (PSD) estimate of heart rate variability (HRV) is commonly used as a test of the neural cardiovascular system, since it is related with sympathetic and parasympathetic regulation of the sino atrial node [1]. Many authors have assumed the integral pulse frequency modulation (IPFM) model to explain the mechanisms used by the autonomic system to control the heart rate. This model supposes a modulating signal *m(t)* that when acting through the IPFM generates the beat occurrence times [2]- [4]. PSD methods try to infer the spectrum of *m(t)* signal, *M(ω)*, from the beat occurrence times, usually from Heart Rate (HR) or Heart Period (HP) signals.

In this work, we introduce a new HRV time domain signal, the Heart Timing Signal (*HT(t)*), used to deduce, with the PSD estimation, the characteristics of the heart control modulating signal *m(t)*. This signal is constructed as $HT(t_k) = kT - t_k$, where t_k is the time associated to the *k*th beat and *T* the mean heart period in the time of analysis. We demonstrate that the amplitude spectrum *HT(ω)* of this signal can be used to recover that of the input signal to the IPFM model, *M(ω)*, with no spurious contribution, as happen with HR or HP signals.

To experimentally show the validity of this method we have developed a controlled experiment with known modulating signals *m(t)*. These signals come from reported AR models representing real data, and other computer generated signals. These *m(t)* signals, used as inputs to the IPFM model, generate beat sequences analysed with different HRV methods to recover the information present in *m(t)*. We use reported PSD methods to estimate the HRV spectra using HR or HP signals. The obtained spectra in all these cases are compared with the original *M(ω)* one and that obtained from the proposed Heart Timing Signal. The results show how this newly proposed method is the one that better recovers the original spectrum of the modulating signal and then the more appropriated for HRV analysis.

2. The Heart Timing signal

The IPFM model is based on the hypothesis that the all sympathetic and parasympathetic influences on the sino-atrial node can be represented by a single function *m(t)*, and the beat is generated when the integral of this function reaches a threshold. Then, the integral process begins newly [2].

The time series of beats by means of IPFM model can be generated by

$$k = \int_0^{t_k} \frac{1+m(t)}{T}\, dt \qquad (1)$$

where *k* is an integer and represent the order number of the *k*th beat, t_k is the time occurrence of this *k*th beat [4]. We can see *(1+m(t))/T* as the instantaneous heart rate. *T* is the mean of the RR interval and *m(t)/T* represents the dynamic part and its mean is zero. The dynamic part is normally small compared to the mean of the heart rate. It is considered that the first beat occurs at $t_0 = 0$ and that *m(t)* is causal, hence $m(t) = 0$ if $t < 0$.

The equation (1) can be rewritten as

$$kT - t_k = \int_0^{t_k} m(t)\, dt \qquad (2)$$

If we write the continuous form of this equation sampled at t_k

$$HT(t) = (kT - t)\sum_{k=0}^{\infty} \delta(t - t_k) = \int_0^{t} m(\tau)\, d\tau \sum_{k=-\infty}^{\infty} \delta(t - t_k) \qquad (3)$$

The summation in the second member of (3) can be extended to $k = -\infty$ because $m(t)$ is causal.

The Fourier transform of both members is

$$\mathcal{F}\{HT(t)\} = \mathcal{F}\left\{(kT - t)\sum_{k=0}^{\infty}\delta(t - t_k)\right\} = \left(\frac{1}{j\omega}M(\omega) + \pi\, M(0)\,\delta(\omega)\right) * \sum_{k=-\infty}^{\infty} e^{j\omega t_k} \quad (4)$$

where $M(\omega)$ is the Fourier transform of $m(t)$. Because the mean of $m(t)$ is zero then $M(0)=0$, hence

$$HT(\omega) = \mathcal{F}\{HT(t)\} = \frac{1}{j\omega}M(\omega) * \sum_{k=-\infty}^{\infty} e^{j\omega t_k} \quad (5)$$

The signal $HT(t)$ is the new Heart Timing proposed in this paper and his spectrum $HT(\omega)$ coincide with the spectrum of $m(t)$, $M(\omega)$, except for $1/j\omega$ and for the convolution with the spectrum of the deltas train at t_k. We can multiply $HT(\omega)$ by $j\omega$ for compensating the first difference, obtaining an estimation $\hat{M}(\omega) = j\omega\, HT(\omega)$ of $M(\omega)$. Now, the main problem is the inherent unevenly sampling of this sequence. If t_k was regularly spaced, we can recover the spectrum of $m(t)$ if the Nyquist criterion was satisfied. The irregular sampling forces to the convolution with $\sum_{k=-\infty}^{\infty} e^{j\omega t_k}$ to cause aliasing. This problem is always present in Heart Rate signals but if $m(t)$ is of low enough frequencies compared with the heart rate the estimation will be close to the real one.

The classical utilisation of the usual HR or HP signals to estimate $M(\omega)$ implies furthermore the additional introduction of spurious spectral components that are not in $M(\omega)$.

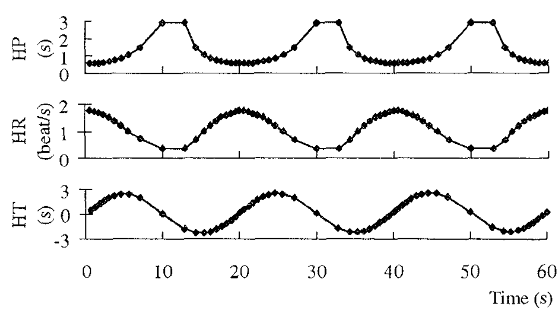

Figure 1. Signals $HP(t)$, $HR(t)$ and $HT(t)$ with $m(t)=0.75\cos(2\pi\, 0.05\, t)$ and $T = 1$s.

The Figure 1 shows graphically this effect when $m(t)=0.75\cos(2\pi\, 0.05\, t)$. In this figure we can anticipate that the spurious components are most important in HP than in HR signals since HR is closer to a pure sinusoid than HP. The proposed HT signal is exactly a pure sinusoid irregular sampled at t_k as

$$HT(t) = kT - t_k = \frac{0.75}{2\pi \cdot 0.05}\sin(2\pi \cdot 0.05 t_k) \quad (6)$$

3. Spectral analysis methods

In the study of HRV, different spectral analysis methods have been used [3], [4], [7]. We show a comparison of these methods and the new one based on the Heart Timing signal presented in this paper. We give a little description of these methods:

- **Spectrum of counts (SPC)**

The spectrum of counts is the power spectrum of a sequence of delta functions placed on the true time occurrence of the R wave [3]. In [4] an analytical PSD is given. We will calculate the amplitude spectrum as the square root of PSD.

- **DFT of the Heart Period (FHP)**

The amplitude spectrum is directly computed by taking the module of the FFT of the $\{t_k - t_{k-1}\}$ sequence. This method is known as Spectrum of Intervals [3], [4].

- **DFT of the Heart Rate (FHR)**

The amplitude spectrum is computed by means of the FFT of the $\{1/(t_k - t_{k-1})\}$ sequence. This method is known as Spectrum of Inverse Intervals [4].

- **DFT of the interpolated HP signal (FHPI)**

The signal $HP(t_k) = t_k - t_{k-1}$ is previously interpolated at regularly spaced time intervals. Then, the amplitude spectrum is computed as in preceding cases. We have used cubic spline interpolation at a sampling frequency of 1 Hz.

- **DFT of the interpolated HR signal (FHRI)**

The signal $HR(t_k) = 1/(t_k - t_{k-1})$ is interpolated as the precedent case. Then, the amplitude spectrum is computed by means of FFT.

- **Lomb method of the HP signal (LHP)**

The Lomb-Scargle periodogram is a means of obtaining power spectra density estimates directly on unevenly sampled data [5]. The intrinsic irregular sampling of HR, HP or HT signal makes suitably this method for estimating the spectrum of this kind of signals. The fast algorithm proposed in [6] is used because the approximation is good and the analysis is much faster. We apply this method to the $HP(t_k) = t_k - t_{k-1}$ signal.

- **Lomb method of the HR signal (LHR)**

The Lomb method is applied to the $HR(t_k)=1/(t_k - t_{k-1})$ signal.

- **AR method of the interpolated HP signal (ARHP)**

A nine-order AR method is used for estimating the spectrum of the HP signal previously interpolated. The AR method will be applied in a set of experiences where the series of beats is built by means of an AR model also of order nine [7].

- **AR method of the interpolated HR signal (ARHR)**

The same AR method is applied but for the HR sequence previously interpolated. We use the same interpolation than in FHPI and FHRI.

- **DFT of the interpolated HT signal (FHTI)**

The new signal $HT(t_k) = kT - t_k$ presented in this paper is used. T is easily computed as t_N/N where N is the number of beats observed. This signal is interpolated by means of cubic splines as in the previous cases. The amplitude spectrum is computed by means of FFT. Finally, each frequency component at ω_j is multiplied by ω_j to correct the effect of integration as described in the equation (5).

4. Comparison between the spectra

We have made two kinds of experience to compare the different PSD estimates proposed. At first, we have made a simple assumption with $m(t)$. We supposed that is formed by one or two pure tones. We pretend to show easily the spurious contribution to the spectra when HR or HP is used instead of HT. The second kind consists in generating the series of beats by means of a realistic AR model described in [7] for modelling the $m(t)$ signal. Then, the series of beats is generated as output of the IPFM model and the PSD estimation achieved by the described methods.

4.1. Simulation by tones

We did the simulation by the same tones as used in [3], [4]. Figure 2 shows the results with the eight methods described (The two AR methods are not applicable here).

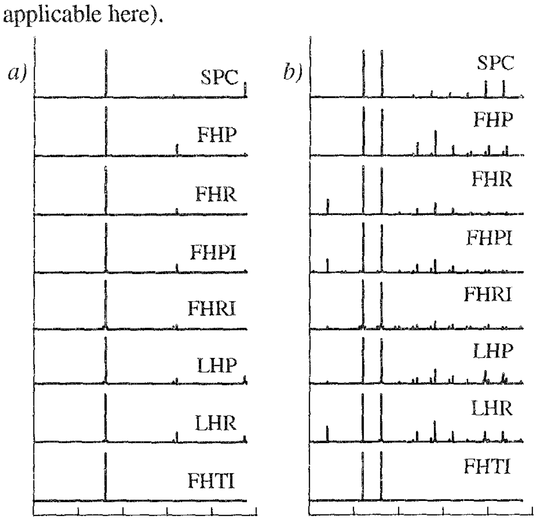

Figure 2. Amplitude Spectra with the eight proposed methods. *a)* $m(t)=0.3\cos(2\pi f1\ t)$, where $f1=0.16$ Hz and $T=1.05$ *s.*; *b)* $m(t)=0.3\cos(2\pi f_1 t) + 0.3\cos(2\pi f_2 t)$, where $f_1=0.12$ Hz, $f_2=0.16$ Hz and $T=1.05$ *s.*

All spectra were computed with a sequence of 1024 beats. In this figure 2 we observe how the HR or HP based methods introduce spurious, more intense when a multitone $m(t)$ signal is used. This phenomenon is more pronounced on HP than on HR signals, as was predicted from figure 1. The HT signal avoids these spurious at the PSD estimation.

4.2. AR simulation

To compare the behaviour of the spectra estimates with a more realistic signal we generate the beat series from an $m(t)$ signal following a typical spectrum from a real subject. We used the nine-order AR model proposed in [7] for generating eight sequences of $m(n)$, signal, as

$$m(n) = \sum_{k=1}^{P} a_k m(n-k) + n(n) \qquad (7)$$

where a_k are the AR parameters showed in the Table 1, P the order and $n(n)$ is white zero-mean noise with $\sigma = 0.072$. With this model, the standard deviation of $m(t)$ is approximately $\sigma \approx 0.145$.

Table 1. Coefficients a_k of the nine-order AR model

a1	*a2*	*a3*	*a4*	*a5*	*a6*	*a7*	*a8*	*a9*
-1.0701	0.3360	0.0117	0.0758	-0.4281	0.2354	0.1165	-0.0119	-0.1435

The $m(n)$ signal, after interpolated to 16 Hz, is the input to an IPFM model for obtaining the sequence of the beat occurrence times. Then, we apply the ten methods described to the eight realisations. We used 1024 beats and $T=1s$ in the IPFM model. Figure 3 presents the spectra obtained in a case, the mean of the spectra of the eight cases, and the $HR(t)$ and $HT(t)$ signals.

The PSD is frequently divided in three bands of frequency: LF (0.01 - 0.08 Hz), MF (0.08 -0.15 Hz) and HF (0.15 - 0.5 Hz) to get the clinical indexes. We have calculated the relative power LF/AF, MF/AF and HF/AF where AF=LF+MF+HF to compare each method with the original spectra in each case. Then, we have calculated the error in each band as the difference of the relative power obtained with each method and at obtained from the realisation of $m(n)$. Finally, we present in the Table 2 and in the Figure 4, the mean of the error (ME) and the standard deviation (σ) in the eight realisations.

In the Figure 4, we can observe that the SPC estimate foments the high frequency band. The interpolated FHPI and FHRI estimates have a strong low pass response. The AR methods are also seen as low pass filters. Lomb methods and the based in the proposed Heart Timing signal, are the ones with better response. We must advertise that the apparently good behaviour of FHP is because the integral in only three wide bands is maintained but we can observe in the Figure 3 that the form of the spectrum is lost. The FHTI method is the one with lower standard deviation from those with lower mean error (FHP, LHP, LHR).

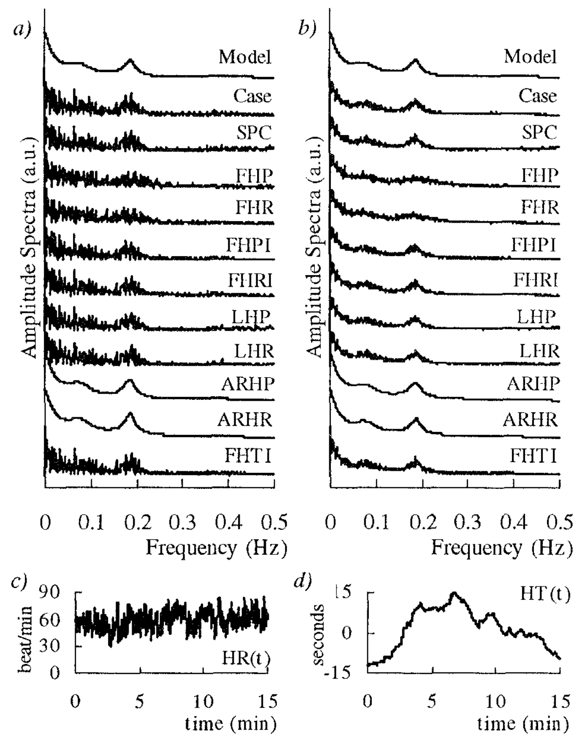

Figure 3. *a)* Amplitude Spectra of a realisation of the AR simulation. b*)* Mean Amplitude Spectra of the eight realisations. c*)* Heart rate signal. *d)* Heart timing signal. The "Model" spectrum is the AR model with a_k coefficients and the "Case" spectrum is the obtained from each *m(n)* realisation.

Table 2. Mean error (ME) and standard deviation (σ) in each band (×10^{-3})

	SPC	FHP	FHR	FHPI	FHRI	LHP	LHR	ARHP	ARHR	FHTI
ME_{LF}	-15.94	2.97	28.73	28.95	30.95	6.91	9.01	23.85	24.95	6.60
σ_{LF}	4.45	12.58	10.07	15.11	2.11	4.34	14.53	25.83	18.33	1.85
ME_{MF}	-5.78	5.89	6.89	2.15	1.68	-5.80	3.36	2.18	1.79	0.83
σ_{MF}	1.82	17.94	17.46	9.48	1.42	2.06	11.03	15.90	10.24	1.78
ME_{HF}	21.72	-8.87	-35.62	-31.10	-32.64	-1.10	-12.36	-26.03	-26.74	-7.43
σ_{HF}	5.33	19.27	11.17	15.91	1.94	5.39	11.63	20.31	12.58	0.96

5. Conclusion

The new Heart Timing signal has been demonstrate to be a good alternative to estimate the PSD of HRV signals. The method used, has not significant deviation between HF and LF, as the others based on HP or HR have. Also, it recovers the form of the original spectra better than the other estimates because the Heart Timing signal does not generate spurious spectral components. Then, the information obtained from this estimate more accurately reflect the real information present at the modulating signal as is the objective of any PSD HRV estimation method.

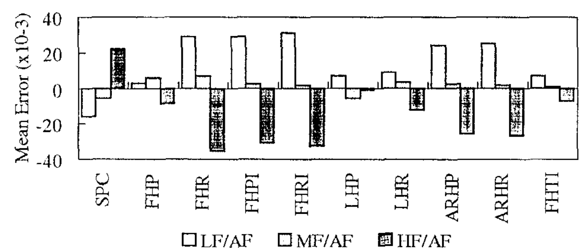

Figure 4. Mean of the error in each band

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