Signal processing

Adaptive estimation of QRS complex wave features of ECG signal by the Hermite model

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Abstract—*The most characteristic wave set in ECG signals is the QRS complex. Automatic procedures to classify the QRS are very useful in the diagnosis of cardiac dysfunctions. Early detection and classification of QRS changes are important in real-time monitoring. ECG data compression is also important for storage and data transmission. An Adaptive Hermite Model Estimation System (AHMES) is presented for on-line beat-to-beat estimation of the features that describe the QRS complex with the Hermite model. The AHMES is based on the multiple-input adaptive linear combiner, using as inputs the succession of the QRS complexes and the Hermite functions, where a procedure has been incorporated to adaptively estimate a width related parameter b. The system allows an efficient real-time parameter extraction for classification and data compression. The performance of the AHMES is compared with that of direct feature estimation, studying the improvement in signal-to-noise ratio. In addition, the effect of misalignment at the QRS mark is shown to become a neglecting low-pass effect. The results allow the conditions in which the AHMES improves the direct estimate to be established. The application is shown, for subsequent classification, of the AHMES in extracting the QRS features of an ECG signal with the bigeminy phenomena. Another application is highlighted that helps wide ectopic beats detection using the width parameter b.*

Keywords—*Adaptive estimation, Data classification, Data compression, ECG signal*

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1 Introduction

THE ELECTROCARDIOGRAPHIC (ECG) signal represents the electrical activity of the heart recorded on the body surface. The analysis of this signal is the most common way to study and diagnose cardiac dysfunctions. The ECG signal is characterised by its recurrent or periodic behaviour with each beat. Each recurrence is composed of a wave sequence, P, QRS complex and T waves, where the most characteristic wave set is the QRS complex. This complex represents the depolarisation phenomenon of the ventricles and therefore gives useful information about heart behaviour. The beat-to-beat classification of the QRS complexes will permit us to follow the heart evolution and to detect arrythmias like premature ventricular contractions (PVC). The ECG data compression allows efficient storage of a large amount of ECG data and fast data transmission and signal processing for diagnosis. All these properties are of great importance in healthcare units and in those that need data transmission to a central processing unit.

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Usually, the classification and tracking are performed through pattern recognition techniques (TRAHANIAS and SKORDALAKIS, 1990). These techniques use features that represent the data by a set of either heuristic descriptors (amplitude, area, width etc.) or formal features such as the coefficients of an orthonormal vector set (RAPPAPORT *et al.*, 1982). These features represent the QRS complex as an element in a space where the similarity between beats is measured by the distance between their associated elements. The election of these features is a key point in obtaining a low-dimension feature space with retention of maximum signal information (data compression) and enhancement of distance between different classes of QRS (classification). In previous work, Sörnmo *et al.* (SÖRNMO *et al.*, 1981) have proposed and studied a set of features for QRS classification that consists of the coefficients of the QRS complex modelled by the Hermite functions. These functions are orthonormal; thus each feature has independent information and the signal can be represented with a low number of coefficients. It has been reported that, on average, 98.6% of signal energy can be represented with only three Hermite functions, thus making it an efficient method for classifying QRS complex (SÖRNMO *et al.*, 1981). The performance of the Hermite model as a data compression tool

of the complete ECG signal (PQRST complex) has been presented, obtaining a compression ratio of around 10 on ECG records from the MIT-BIH database (JANÉ *et al.*, 1993). For this reason, this model seems appropriate to efficiently classify QRS complexes and compress the ECG signal.

Early detection and classification of QRS changes are of great interest in real-time monitoring such as in cardiac critical care units or operating room environments. In these circumstances, it is important to develop signal-processing techniques that allow on-line feature extraction for subsequent classification of QRS patterns or data transmission to a central unit. Adaptive signal processing is appropriated for the on-line estimation of non-stationary signals that present a recurrent behaviour, attenuating the noise effect at the estimation (WIDROW and STEARNS, 1985). Many biomedical signals, in particular the ECG, present this recurrent behaviour. The adaptive signal-processing techniques have been applied to biomedical signals like ECG (WIDROW and STEARNS, 1985; FERRARA and WIDROW, 1981; THAKOR and YI-SHENG, 1991; LAGUNA *et al.*, 1992*b*) and evoked potentials (LAGUNA *et al.*, 1992*b*; VAZ and THAKOR, 1989).

In this paper, we present a detailed analysis of an adaptive Hermite model estimation system (AHMES) for the on-line determination of the Hermite model features of the QRS complex reported previously (LAGUNA *et al.*, 1989; LAGUNA *et al.*, 1992*a*). A width parameter is incorporated in this model, related to the width of the QRS complex. It can be considered as a feature specially relevant to ectopic beat detection (they are usually of a greater width than normal beats). We present the theoretical performance of the system using the LMS algorithm (WIDROW and STEARNS, 1985) to adjust the weights (coefficients), and we propose an algorithm to adapt the width parameter. We compare the signal-to-noise ratio improvement (ΔSNR) of the estimated coefficients by the AHMES with the direct estimation (inner product of the ECG signal with the Hermite functions). This comparison establishes under what conditions this estimate yields a better signal-to-noise ratio. The effect of misalignment at the QRS mark is also studied and modelled as a function of the jitter dispersion. Applications of this system to extract the QRS complex features from several examples of ECG signals are presented.

2 Hermite model of the QRS complex

Orthonormal function modelling is a general method for approximating functions with a finite number of parameters (AHMED and RAO, 1975). The approximated function results as the finite linear combination of the orthonormal basis elements.

We can consider the $s'(t)$ signal as composed of the noise-free QRS signal (QRS(t)) over its definition interval $(-T_0/2, T_0/2)$; $t = 0$ is the time given by the QRS detector, and extended to $-\infty$ and $+\infty$ with a zero extension.

$$s'(t) = \begin{cases} \text{QRS}(t) & |t| < T_0/2 \\ 0 & |t| > T_0/2 \end{cases} \tag{1}$$

This $s'(t)$ function represents the QRS complex and is an element of the linear vectorial space $L_2(-\infty, \infty)$, given that it satisfies $\int_{-\infty}^{\infty} s'^2(t)dt < \infty$. The Hermite functions $(\Phi_n(t), n = 0, \ldots, \infty)$ form an orthonormal basis in the $L_2(-\infty, \infty)$ space (FRANKS, 1975) and are expressed as

$$\Phi_n(t, b) = \frac{1}{\sqrt{b2^n n! \sqrt{\pi}}} e^{\frac{-t^2}{2b^2}} H_n(t/b) \tag{2}$$

where $H_n(t/b)$ are the Hermite polynomials. We have considered the parameter b in order to have a scale factor related to the width of the QRS complex. These functions remain orthonormal for any width parameter b. Fig. 1 shows the first six Hermite functions for $b = 25$ ms. Note that they look like the regular QRS shape and are in accordance with previous work (SÖRNMO *et al.*, 1981), where it is shown that the first three functions recover, on average, more than 90% of signal energy.

With these basis functions we can express the QRS signal $s'(t)$ as

$$s'(t) = \sum_{n=0}^{\infty} c_n(b)\Phi_n(t, b)$$

where

$$c_n(b) = \int_{-\infty}^{\infty} s'(t)\Phi_n(t, b)dt \tag{3}$$

and we can consider an N-order approximation of $s'(t)$ as

$$s'(t) \simeq \sum_{n=0}^{N-1} c_n(b)\Phi_n(t, b) \tag{4}$$

In this approximation, the QRS complex is characterised by the parameter b and the coefficients $c_n(b)$. These $N + 1$ parameters are the features that allow the data reduction and subsequent classification for a more efficient data transmission, storage and signal processing for diagnosis.

3 Adaptive Hermite model estimation system

In this Section, we present the adaptive Hermite model estimation system (AHMES) to adaptively calculate the c_n and b coefficients. This system is based on the multiple-input adaptive linear combiner (ALC) with desired response (WIDROW and STEARNS, 1985). The primary input to the AHMES is the digitised QRS signal, and the reference inputs are the digitised Hermite functions. Fig. 2 displays the AHMES, where there are two adaptation processes: the weight adaptation and the parameter b adaptation. The parameter b acts as an input to a Hermite function basis generator that produces the elements of the reference input signals in the AHMES.

The noisy primary input signal $d(t)$ of the AHMES is synthesised as follows. We apply a QRS detector (PAN and TOMPKINS, 1985) to the digitised ECG signal that is suitable for real-time processing. Centred on each jth QRS detection mark of the QRS complex sequence, we define a signal

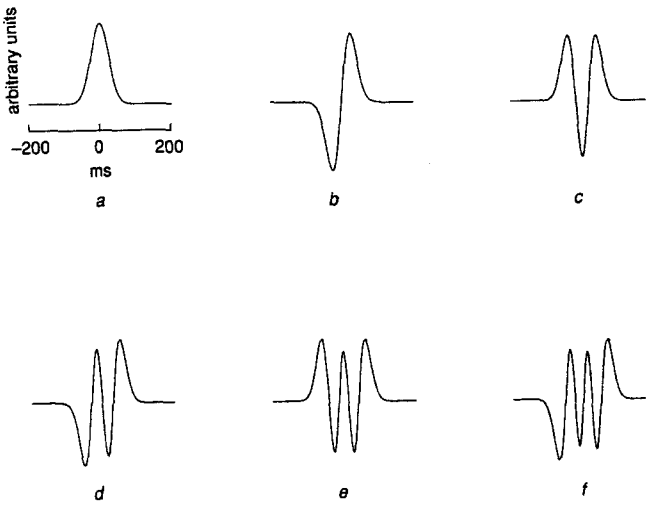

Fig. 1 *First five Hermite functions (Φ_n, $n = 0, \ldots, 5$) in an interval of 400 ms (−200, 200) for $b = 25$: (a) $n = 0$; (b) $n = 1$; (c) $n = 2$; (d) $n = 3$; (e) $n = 4$; (f) $n = 5$*

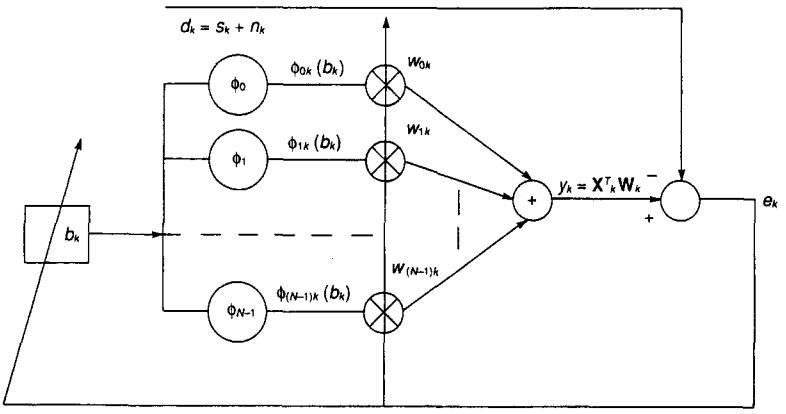

Fig. 2 *Block diagram of AHMES based on multiple-input ALC; d_k = primary input signal composed of QRS signal sequence s_k plus noise n_k not correlated with s_k; AHMES output signal is y_k; w_n = weights; b = width factor that generates Φ_n Hermite basis functions; these functions act as reference inputs to AHMES*

window (200 ms) that is wide enough to contain the QRS complex plus additive noise. This signal window is extended with a zero flat line, on the right and on the left, to form the $d'_j(t) = s'_j(t) + n'_j(t)$ signal, ($n'_j(t)$ is the noise present in the 200 ms of the jth QRS complex with the zero extension), which now extends enough to consider the low-order Hermite functions zero valued outside this time interval. We construct the new signal $d(t)$ as the subsequent linking of all the jth QRS ($j = 1 \ldots nqrs$), $d'_j(t)$, delayed at time l from the $d'_{j-1}(t)$.

$$d(t) = \sum_{j=0}^{nqrs} d'_j(t - jl) = s(t) + n(t) \tag{5}$$

$$s(t) = \sum_{j=0}^{nqrs} s'_j(t - jl) \tag{6}$$

$$n(t) = \sum_{j=0}^{nqrs} n'_j(t - jl) \tag{7}$$

This new signal, by itself, loses the actual timing of beats that contain valuable clinical information. However, it is presented in this way for convenience to analyse the AHMES behaviour, as shown in the following Sections. The primary input has been converted to a pseudo-periodic deterministic (QRSs) component plus added noise not correlated with the former. In real situations, the signal $d(t)$ does not need to be generated; instead we arrange the AHMES to activate when the QRS detector gives a mark and to act only during the 200 ms ECG signal window. After that, the AHMES is inhibited until the next QRS mark appears. In this way, we keep the valuable beat timing information. The two situations are exactly equivalent from the AHMES point of view; the former is much more suitable for AHMES performance analysis.

In Fig. 1 we can note that the exponential part of $\Phi_n(t)$ decreases to zero when time goes on from the origin. This tendency is faster or slower depending on the parameter b and model order N. The time interval $l = LT$ (T is the sampling period) is selected according to b and N values $b < l$ as b closely represents the half-power QRS duration and l is selected to include the complete QRS complex. The sampled signal $d_k = d(kT)$, which acts as primary input to the AHMES, is the deterministic QRS complex signal $s_k = s(kT)$ plus additive noise $n_k = n(kT)$. If QRS signals are free of noise and repetitive, d_k is a periodic signal. The non-repetitive part of the deterministic QRS signal and the noise give the non-periodicity of d_k.

Each reference input is formed by one of the N Hermite functions considered in the model. The construction of these inputs is analogous to the construction of d_k. We concatenate sequences of the Hermite functions defined in an interval of L samples and centred around the synchronisation point (generally the middle point of the interval). The reference inputs Φ_{nk} at the jth QRS recurrence are defined as

$$\Phi_{nk}(b) = \sum_{j=0}^{nqrs} \Phi_n(kT - L(j - \tfrac{1}{2})T, b)\Pi\left(\frac{kT - jLT}{LT}\right) \tag{8}$$

where T is the sampling period and $\Pi(t/a)$ is the rectangular window function of width a centred at $t = 0$, and with unitary amplitude. The parameter b is continuously recalculated and acts as input to the Hermite function generator.

The output signal of the system is y_k which results from the linear combination of the Hermite functions, each one of which is affected by a weight factor w_n. These weight factors characterise the QRS complex in this model. Thus, they are the features that describe the QRS. The error signal e_k, the difference between the original signal and the model output signal, is used to adjust the weights and the parameter b.

We study the theoretical performance of the system according to the standard notation for the ALC (WIDROW and STEARNS, 1985). Then the model output signal is defined as

$$y_k(b) = \sum_{n=0}^{N-1} w_{nk} X_{nk}(b) = \boldsymbol{W}_k^T \boldsymbol{X}_k(b) \tag{9}$$

where $X_{nk}(b) = \Phi_{nk}(b)$, $\boldsymbol{X}_k$ is the reference vector and $\boldsymbol{W}_k$ is the weight vector

$$\begin{aligned} \boldsymbol{X}_k(b) &= [X_{0k}(b), X_{1k}(b), \ldots, X_{(N-1)k}(b)]^T, \\ \boldsymbol{W}_k &= [w_{0k}, w_{1k}, \ldots, w_{(N-1)k}]^T \end{aligned} \tag{10}$$

The error signal e_k can be expressed as

$$e_k = d_k - y_k(b) = d_k - \boldsymbol{W}_k^T \boldsymbol{X}_k(b) \tag{11}$$

The mean-square error (MSE) between the signal under study and the estimated signal can be expressed (WIDROW and STEARNS, 1985) as

$$\begin{aligned} \xi &= E[e_k^2] = E[d_k^2] + \boldsymbol{W}^T \boldsymbol{R} \boldsymbol{W} - 2\boldsymbol{P}(b)^T \boldsymbol{W} \\ \boldsymbol{R} &= E[\boldsymbol{X}_k(b)\boldsymbol{X}_k^T(b)], \quad \boldsymbol{P}(b) = E[d_k \boldsymbol{X}_k(b)] \end{aligned} \tag{12}$$

where $\boldsymbol{R}$ and $\boldsymbol{P}(b)$ are the input correlation matrix and the cross-correlation vector, respectively, and $\boldsymbol{W}$ is the weight

vector W_k, taken as a variable. Given that the Hermite functions remain orthonormal for any parameter b, the R matrix results proportional to the identity matrix and does not depend on parameter b.

$$R = \frac{1}{LT} I \tag{13}$$

In the ECG signal, the noise is not correlated with the beats sequence; thus we can assume the noise n_k is not correlated with the reference inputs X_k. The cross-correlation vector P then results proportional to the coefficients $c_n(b)$ that describe the QRS complex in the Hermite model

$$P = \frac{1}{LT}[c_0(b), c_1(b), \ \ldots, \ c_{(N-1)}(b)]^T \tag{14}$$

If we minimise this MSE ξ with respect to the weight vector (WIDROW and STEARNS, 1985), we obtain the optimum weight vector W^* (Wiener solution), which from eqns. 13 and 14 has the expression

$$W^*(b) = R^{-1}P(b) = [c_0(b), c_1(b), \ \ldots, \ c_{N-1}(b)]^T \tag{15}$$

The optimum weight vector $W^*(b)$, which depends on the b parameter, is the sequence of coefficients that describes the deterministic component s_k of the primary input signal d_k in the Hermite model, defined by the reference inputs $X_{nk} = \Phi_{nk}$.

In the steady state, the AHMES output is

$$y_k = W^{*T}(b)X_k(b) = \sum_{n=0}^{N-1} w_n^*(b)X_{nk}(b) = \sum_{n=0}^{N-1} c_n(b)\Phi_{nk}(b) \tag{16}$$

that means y_k is the N order approximation of the $s'(t)$ signal (QRS complex) in the Hermite model (eqn. 4), for a given parameter b.

If now we minimise the MSE ξ with respect to parameter b, and consider that the R matrix does not depend on b, we find that the optimum parameter $b(b^*)$, which minimises ξ for a fixed W, satisfies

$$E\left[d_k \frac{\partial y_k(b)}{\partial b}\right]_{b=b^*} = 0 \tag{17}$$

When the weight vector and parameter b have both converged to the optimum solution, the MSE becomes minimum ($\xi_{\min}$) and from eqns. 3, 5, 12 and 15 results:

$$\xi_{\min} = E[d_k^2] - P(b)^T W^* = E[n_k^2] + \frac{1}{LT}\sum_{n=N}^{\infty} c_n^2(b^*) \tag{18}$$

The $\xi_{\min}$, and then e_k, is due to the noise present in the primary signal and to the deterministic signal component that cannot be represented by the N order approximation.

Given that the weight vector oscillates around the optimal value $w^*(b)$, y_k will be an unbiased estimation of s_k. The remaining noise due to the misadjustment (M) will depend on the adaptive algorithm used to adjust the weights. It represents the effect of the vector oscillations around the optimum position (WIDROW and STEARNS, 1985). The weight values, picked up when the QRS signal has just passed the AHMES (end of each QRS complex adaptation), will be the estimation of the c_n coefficients of each QRS complex. The quality of the y_k estimation will be directly related to the quality of the c_n estimation.

To measure the excess of the mean squared error, we calculate the misadjustment (WIDROW and STEARNS, 1985)

$$M = \frac{\text{excess MSE}}{\xi_{\min}} \tag{19}$$

which results in a mean squared error ξ

$$\xi = \xi_{\min}(1+M) = (1+M)\left(E[n_k^2] + \frac{1}{LT}\sum_{n=N}^{\infty} c_n^2(b^*)\right) \tag{20}$$

3.1 *SNR improvement of* AHMES *estimation*

Given that at the steady state the estimated signal y_k is orthogonal with the error e_k (WIDROW and STEARNS, 1985), the excess MSE is the excess of error power introduced at the estimation y_k, and the SNR of this estimation SNR_y will be

$$\text{SNR}_y = \frac{\sum_{n=0}^{N-1} c_n^2(b)}{(M)\left(\sum_{n=N}^{\infty} c_n^2(b) + E[n_k^2]\right)} \tag{21}$$

If we consider that the QRS energy is basically concentrated at the N first coefficients, we can neglect the term $\sum_{n=N}^{\infty} c_n^2(b)$, obtaining

$$\text{SNR}_y \simeq \frac{\sum_{n=0}^{\infty} c_n^2(b)}{(M)E[n_k^2]} = \text{SNR}_d \frac{1}{M} \tag{22}$$

where SNR_d is the SNR at the original signal. Comparison of this SNR_y with that obtained from the direct estimation of the c_n will give the SNR improvement (ΔSNR) achieved by the adaptive system. The SNR obtained with the direct c_n estimation ($\text{SNR}_y^{\text{direct}}$) can be estimated, assuming that the noise is white and then the power spectral density is uniformly distributed at the c_n domain. Then

$$\text{SNR}_y^{\text{direct}} = \frac{\sum_{n=0}^{N-1} c_n^2(b)}{E[n_k^2]\frac{N}{L}} \simeq \text{SNR}_d \frac{L}{N} \tag{23}$$

We find that the adaptive c_n estimate AHMES has a SNR improvement ΔSNR of

$$\Delta\text{SNR} = \frac{\text{SNR}_y}{\text{SNR}_y^{\text{direct}}} = \frac{N}{ML} \tag{24}$$

This relation is the SNR improvement at the estimated signals with the c_n coefficients, which equals the SNR improvement at the c_n estimation because it is a linear relation of the c_n and the fixed $\Phi_n(t, b)$ basis. To have a better estimation with the AHMES than with the direct estimation ($\Delta\text{SNR} > 1$), we then need the misadjustment M to satisfy

$$M < \frac{N}{L} \tag{25}$$

This condition will depend on the adaptive algorithm and its parameters. When satisfied, together with noise conditions, the adaptive estimate improves the c_n estimation performed with this AHMES instead of with a direct inner product.

3.2 *Effects of misalignment at* QRS *detection mark*

In this Section, we analyses the effect of a misalignment at the QRS mark location with respect to the period of d_k. AHMES application needs to estimate the occurrence time of the repetitive QRS signal (JANÉ *et al.*, 1991*b*), and in these cases errors can appear. When an error of ($\pm\delta$) appears in these estimations, the reference inputs remain periodic with period L, but the signal s_k changes its period to $s_k = s_{k+L\pm\delta}$. The value of δ is then of a random nature, and so varies from period to period. We study the effect of this misalignment, focusing on a previous study on this topic when then inputs to the ALC are orthogonal functions (JANÉ *et al.*, 1991*a*).

The effect of these errors on the estimated signal are reflected through the effect on the $\boldsymbol{P}$ vector. The optimum weight vector $\boldsymbol{W}^* = \boldsymbol{R}^{-1}\boldsymbol{P}$ is affected through $\boldsymbol{P}$ vector modifications ($\boldsymbol{R}$ does not change). We then analyse the $\boldsymbol{P}$ vector in this case

$$\boldsymbol{P} = E[d_k \boldsymbol{X}_k] = E[s_k \boldsymbol{X}_k] + E[n_k \boldsymbol{X}_k] \tag{26}$$

As noise n_k is supposed to be not correlated with the QRS mark (and thus the reference inputs), in eqn. 26 the second term of $\boldsymbol{P}$ is null and $\boldsymbol{P}$ is reduced to $\boldsymbol{P} = E[s_k \boldsymbol{X}_k]$.

If we assume that the errors of the occurrence time determination δ are expressed in sample values and have a probability distribution $p[\delta]$, the $\boldsymbol{P}$ vector can be expressed as

$$\boldsymbol{P} = E[s_k \boldsymbol{X}_k] = \sum_{\delta=-\infty}^{\infty} \left(\frac{1}{L} \sum_{k=1}^{L} s_{k+\delta} \boldsymbol{X}_k \right) p[\delta] \tag{27}$$

$$\boldsymbol{P} = \frac{1}{L} \sum_{k=1}^{L} \boldsymbol{X}_k \sum_{\delta=-\infty}^{\infty} s_{k+\delta} p[\delta] \tag{28}$$

From this result, we observe that the $\boldsymbol{P}$ vector elements are (s'_i/L), where s'_i are the components of a signal s'_k to which the filter converges, and take the value

$$s'_k = \sum_{\delta=-\infty}^{\infty} s_{k+\delta} p[\delta] \tag{29}$$

Calculating the Fourier transform of this s'_k signal ($S'(\Omega)$), we have

$$S'(\Omega) = S(\Omega) \sum_{\delta=-\infty}^{\infty} e^{j\Omega\delta} p[\delta] \tag{30}$$

where $S(\Omega)$ is the Fourier transform of s_k. The effect of the error in the occurrence time estimation then makes a filtering effect on the signal s_k at the estimated y_k. The transfer function $C(\Omega)$ of this filter ($S'(\Omega) = C(\Omega)S(\Omega)$) is the characteristic function of the δ distribution (BENDAT and PIERSOL, 1986)

$$C(\Omega) = \sum_{\delta=-\infty}^{\infty} e^{j\Omega\delta} p[\delta] \tag{31}$$

In the case that $p[\delta]$ is a Gaussian distribution with standard deviation σ, the characteristic function is (ROMPELMAN and ROS, 1986)

$$C(\Omega) = \sum_{n=-\infty}^{\infty} e^{-(\Omega - 2\pi n)^2 \sigma^2/2} \tag{32}$$

which consists of a low-pass filter with a cut-off frequency f_c at -3 dB of $f_c = 132{\cdot}5/\sigma$, where f_c is expressed in Hz and σ in ms. The estimation of $\boldsymbol{W}^*$ is then the coefficients s'_k of a low-pass filtered deterministic signal component, whose cut-off frequency depends on the error distribution.

This effect should be taken into consideration when estimating high-frequency components (JANÉ *et al.*, 1991*b*). The first-order Hermite functions are already low frequency, and so this effect is negligible in the AHMES estimation. In ECG signals the error in QRS estimation (JANÉ *et al.*, 1991*b*) can be lower than 1 ms, giving a low-pass filter with $f_c = 132{\cdot}5$ Hz that will not affect the AHMES estimation as proposed in this work, because the ECG bandwidth is lower than this value (THAKOR *et al.*, 1984).

4 Adaptive algorithms

The AHMES includes one adaptation process to obtain the estimation of the Hermite model coefficients (weight vector) and another adaptation process to estimate the optimum width parameter b. We use the least-mean square (LMS) algorithm (WIDROW and STEARNS, 1985) to adjust the weight vector, which is implemented by the recursive expression

$$\boldsymbol{W}_{k+1} = \boldsymbol{W}_k + 2\mu_1 e_k \boldsymbol{X}_k(b) \tag{33}$$

To adjust the parameter b we consider a gradient search method ($b_{k+1} = b_k - \mu_2 \nabla_k$) with the same approximation (WIDROW and STEARNS, 1985) used to obtain the LMS algorithm

$$\nabla E[e_k^2] \simeq \nabla e_k^2 = 2 e_k \nabla e_k \tag{34}$$

and then we can write

$$\frac{\partial E[e_k^2(b)]}{\partial b} \simeq 2 e_k(b) \frac{\partial e_k(b)}{\partial b} \tag{35}$$

From eqn. 11 we obtain

$$\frac{\partial e_k(b)}{\partial b} = -\frac{\partial y_k(b)}{\partial b} \tag{36}$$

which leads to the following recursive expression to adapt b

$$b_{k+1} = b_k + 2\mu_2 e_k(b_k) \frac{\partial y_k(b_k)}{\partial b_k} \tag{37}$$

We have considered different μ factors, μ_1 and μ_2, for the weight vector and b parameter, respectively. In order to implement eqn. 37, the value of $\partial y_k(b_k)/\partial b_k$ in each time instant k needs to be known. This value can be proved to be a linear combination of w_n and $\Phi_{nk}(b_k)$. Thus, it can be calculated on-line in each iteration. Eqn. 37 can be reformulated as

$$b_{k+1} = b_k + 2\mu_2 e_k(b_k) \sum_{n=0}^{N-1} w_n \frac{\partial \Phi_{nk}(b_k)}{\partial b_k} \tag{38}$$

and it has been proved that (LAGUNA, 1990)

$$\frac{\partial \Phi_n(t, b)}{\partial b} = \frac{1}{2b} \times \left[-\sqrt{n(n-1)}\Phi_{n-2}(t, b) + \sqrt{(n+2)(n+1)}\Phi_{n+2}(t, b) \right] \tag{39}$$

From eqns. 38 and 39 it is evident that b_k can be recalculated from $\boldsymbol{W}_k$ and $\boldsymbol{X}_k(b)$ directly, with no additional calculations. On the other hand, this implies that Hermite functions and their derivatives, with respect to parameter b, are orthogonal

$$E\left[\Phi_n(t, b) \frac{\partial \Phi_n(t, b)}{\partial b} \right] = 0 \tag{40}$$

These results are used when the convergence of the b_k adaptation algorithm is studied.

In the steady-state, the MSE does not reach the minimum error $\xi_{\min}$ because there are oscillations of the weight vector around the optimum solution $\boldsymbol{W}^*$ and oscillations of parameter b around the optimum b^*. This effect is measured by means of the excess MSE ($\xi = \xi_{\min} + \text{excess MSE}$), which is usually measured in terms of the misadjustment M (WIDROW and STEARNS, 1985), $\xi = \xi_{\min}(1 + M)$. The value of M depends on the adaptation algorithm used.

To study the convergence of the weight vector and the parameter b, it is necessary to study the interaction between both adaptation processes. In a first approximation, we study the convergence of the weight vector while parameter b remains fixed and the convergence of parameter b while the weight vector $\boldsymbol{W}$ remains fixed. This is not the real situation, but the convergence predictions with this approximation are in accordance with experimental results when both adaptations act simultaneously.

4.1 *Convergence of the weight vector*

The convergence of the weight adaptation process depends on the algorithm used. The LMS algorithm converges (WIDROW and STEARNS, 1985) when the μ_1 parameter satisfies the condition

$$0 < \mu_1 < \frac{1}{\text{tr}[\boldsymbol{R}]} = \frac{LT}{N} \tag{41}$$

and the convergence time associated with each weight $w_n(\tau_{w_n})$ is given (WIDROW and STEARNS, 1985) by

$$\tau_{w_n} = \frac{1}{2\mu_1\lambda_n} = \frac{LT}{2\mu_1} \tag{42}$$

where λ_n is the nth eigenvalue of the $\boldsymbol{R}$ matrix. In this case, all the eigenvalues have the same value ($\lambda_n = \lambda = 1/LT$). The convergence time associated with the MSE (τ_{mse}) is (WIDROW and STEARNS, 1985)

$$\tau_{\text{mse}} = \frac{1}{4\mu_1\lambda} = \frac{LT}{4\mu_1} \tag{43}$$

where both convergence times are expressed in number of samples.

Therefore, the gain constant μ_1 controls the stability and the speed of convergence. Thus, the convergence of weights can be obtained in the first record ($\tau_{\text{mse}} < L$) if an appropriate value of μ_1 is selected. This possibility will be very useful for tracking variations in the time-varying QRS signals.

The misadjustment M can be approximated, following a previous method (WIDROW and STEARNS, 1985), as

$$M \simeq \mu_1 \text{ tr}[\boldsymbol{R}] = \mu_1 \frac{N}{LT} \tag{44}$$

The value of the gain constant μ_1 is a compromise between the rate of adaptation and the excess MSE due to the steady-state weight vector oscillations (excess MSE $= \xi_{\min}M$).

The MSE in the steady-state becomes

$$\xi = \xi_{\min}(1+M) = \left(E[n_k^2] + \frac{1}{LT}\sum_{n=N}^{\infty} c_n^2(b^*)\right) \times \left(1 + \mu_1 \frac{N}{LT}\right) \tag{45}$$

4.2 SNR *improvement with* LMS

Considering now the value of the misadjustment M and the expression for the ΔSNR (eqn. 24) of the AHMES, we have for the LMS algorithm:

$$\Delta\text{SNR} = \frac{N}{ML} = \frac{T}{\mu_1} \tag{46}$$

This agrees with our initial expectation that the adaptive estimate, for adequate μ_1 values, improves the c_n estimation performed with this adaptive system instead of with a direct inner product. There appears a more restrictive condition at the μ_1 value ($\mu_1 < T$) to obtain $\Delta\text{SNR} > 1$, than that reached by the LMS stability restriction. The improvement is higher as the μ_1 parameter becomes smaller. However, this gives a longer convergence time and then the typical compromise appears for the adaptive systems. On the other hand, this ΔSNR is inaccurate when sudden changes appear on the ECG signal. This is especially relevant in ectopic beat presence. In those cases, the first beats after the change present a lower ΔSNR controlled by the convergence rate.

4.3 *Convergence of parameter b*

The AHMES presents the adaptation of parameter b with the previously proposed algorithm. To study the convergence (condition and rate) as a function of μ_2, we consider expected values in the recurrent expression to adapt b. From eqn. 37 and using the value of $e_k(b_k)$, we can write

$$E[b_{k+1}] = E[b_k] + 2\mu_2\left(E\left[d_k\frac{\partial y_k(b_k)}{\partial b_k}\right] - E\left[y_k(b_k)\frac{\partial y_k(b_k)}{\partial b_k}\right]\right) \tag{47}$$

Given that the weight vector is supposed to be constant, differentiating the expression $E[y_k^2(b)] = \eta$ we obtain

$$2E\left[y_k(b)\frac{\partial y_k(b)}{\partial b}\right] = 0 \tag{48}$$

and eqn. 47 becomes

$$E[b_{k+1}] = E[b_k] + 2\mu_2 E\left[d_k\frac{\partial y_k(b_k)}{\partial b_k}\right] \tag{49}$$

To analyse the convergence of this expression we calculate the value of $E[d_k(\partial y_k(b_k)/\partial b)]$ as a function of parameter b. Using eqn. 39 and assuming b_k is not correlated with $\Phi_{nk}(b_k)$, we have

$$E\left[d_k\frac{\partial y_k(b_k)}{\partial b}\right] = \frac{1}{2E[b_k]}\sum_{n=0}^{N-1} w_n^*(b_0) \times E\left[d_k\left(-\sqrt{n(n-1)}\Phi_{(n-2)k}(b_k) + \sqrt{(n+2)(n+1)}\Phi_{(n+2)k}(b_k)\right)\right] \tag{50}$$

$$E\left[d_k\frac{\partial y_k(b_k)}{\partial b}\right] = \frac{1}{2E[b_k]LT}\sum_{n=0}^{N-1} w_n^*(b_0) \times \left(-\sqrt{n(n-1)}w_{n-2}^*(E[b_k]) + \sqrt{(n+2)(n+1)}w_{n+2}^*(E[b_k])\right) \tag{51}$$

where the weight vector is supposed to be optimum for the initial value of b, $w_n^*(b_0)$. The values $w_n^*(E[b_k])$ are the coefficients $c_n(E[b_k])$ when parameter b takes the value $E[b_k]$. We also slight the variations of $w_n^*(E[b_k])$ with b_k with respect to the term $1/E[b_k]$. We then consider w_n^* constant for each parameter b, resulting

$$E\left[d_k\frac{\partial y_k(b_k)}{\partial b}\right] = \frac{1}{2E[b_k]LT} \times \left(\sqrt{N(N-1)}w_{N-2}^* w_N^* + \sqrt{(N+1)N}w_{N-1}^* w_{N+1}^*\right) \tag{52}$$

Taking again eqn. 49 and considering the first-order Taylor development, we have

$$E[b_{k+1}] = E[b_k] - \frac{\mu_2}{b^{*2}LT} \times \left(\sqrt{N(N-1)}w_{N-2}^* w_N^* + \sqrt{(N+1)N}w_{N-1}^* w_{N+1}^*\right) \times (E[b_k] - b^*) \tag{53}$$

and calling

$$a = \frac{\mu_2}{b^{*2}LT} \times \left(\sqrt{N(N-1)}w_{N-2}^* w_N^* + \sqrt{(N+1)N}w_{N-1}^* w_{N+1}^*\right) \tag{54}$$

we obtain

$$E[b_{k+1}] = E[b_k](1-a) + ab^* \tag{55}$$

Considering the b initial value at $k = 0$, b_0, we can rewrite

$$E[b_k] = b^* + (1-a)^k(b_0 - b^*) \tag{56}$$

This recursive expression converges when a satisfies $0 < a < 2$ and, recovering the a value, we have the convergence condition for the b adaptation algorithm

$$\mu_2 < \frac{2LTb^{*2}}{\left(\sqrt{N(N-1)}w^*_{N-2}w^*_N + \sqrt{(N+1)N}w^*_{N-1}w^*_{N+1}\right)} \tag{57}$$

To estimate the right-hand side eqn. 57 we approximate $\sqrt{N(N-1)} \simeq \sqrt{(N+1)N} \simeq N$ and then

$$\mu_2 < \frac{2LTb^{*2}}{N(w^*_{N-2}w^*_N + w^*_{N-1}w^*_{n+1})} \tag{58}$$

On the other hand, we can state that for the highest order weight w^*_{N-2}, w^*_{N-1}, $w^*_N y w^*_{N+1}$, the signal energy is lower than the average energy in each weight. If we call $SE = E[s_k^2]LT$, we can suppose that

$$N(w^*_{N-2}w^*_N) < SE \quad \text{and} \quad N(w^*_{N-1}w^*_{N+1}) < SE \tag{59}$$

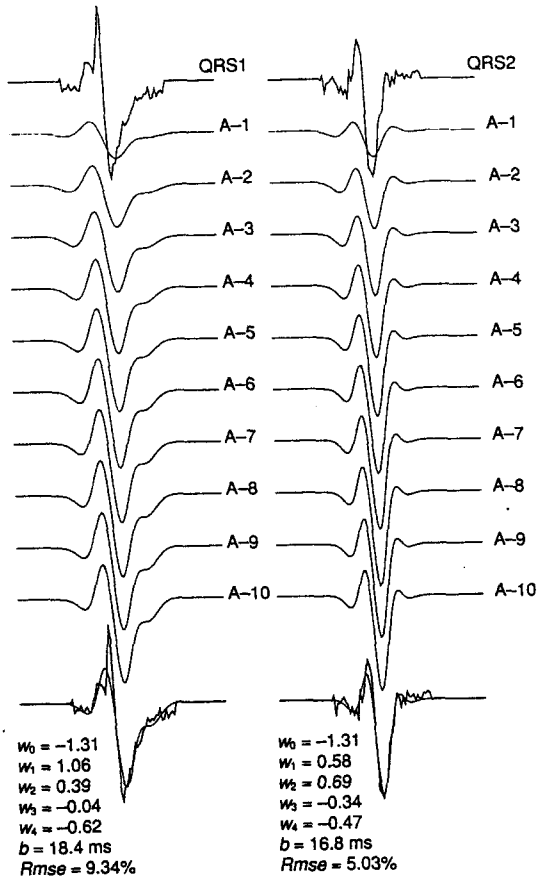

Fig. 3 *AHMES output for two QRS pattern (QRS1 and QRS2) sequences when the same QRS repeats itself; uppermost signals are original QRS complexes, below are reconstructed signals using estimated weights w_n and b after finishing each nth of the first ten adaptations (A-n), using a model order of $N = 5$; lowest signal is that overprinted of the original QRS signal and reconstructed after ten adaptations; plotted parameters w_n, b and Rmse are the values after ten adaptations*

Then we can take a more restrictive convergence limit

$$\mu_2 < \frac{LTb^{*2}}{SE} < \frac{2LTb^{*2}}{N(w^*_{N-2}w^*_N + w^*_{N-1}w^*_{N+1})} \tag{60}$$

and we can consider

$$\mu_2 < \frac{LTb^{*2}}{SE} \tag{61}$$

as the convergence condition. This condition depends on the signal energy SE, and it is in accordance with the adaptation expression eqn. 37, where μ_2 is multiplied by a factor that depends on the primary input signal through y_k. The μ_2 factor then has to consider the signal that will be studied. If this signal energy changes with time, it could be necessary to readjust μ_2.

The convergence time τ_b of the parameter b can be estimated from eqn. 56 as

$$\tau_b \simeq \frac{1}{a} < \frac{LTb^{*2}}{2\mu_2 SE} \tag{62}$$

The expressions obtained with the previous derivation result from some approximations that require their validity to be corroborated through experimentation. In the following we include examples of the AHMES performance and show the validity of previous derivations.

5 Performance analysis of AHMES in simulation

To validate the AHMES, we first consider a real QRS complex repeating itself to form the d_k signal. It can be considered as a stationary deterministic component without noise ($d_k = s_k$). In this way, we ensure that the deterministic component remains constant in the sequence and the AHMES reaches the steady state. We consider a real QRS complex repeated a number A of recurrences. In this situation, we can calculate exactly the signal energy SE and then verify the expressions derived in the previous Section.

The selected QRS is a signal of 200 ms centred in the position given by a QRS detector. In order to avoid base-line variations being modelled as QRS, the window signals are extracted from a high-pass filtered ECG signal. High-pass filtering can introduce some errors, especially in monopolar (high amplitude and large duration) QRS complexes, that require specific filtering (CHRISTOV *et al.*, 1992) or a cubic spline interpolation (MEYER and KEISER, 1977) to suppress base-line variations. The window signals are extended to 400 ms, by adding 100 ms of zero value on the right and on the left. The signal is sampled at 250 Hz ($T = 4$ ms); thus $L = 100$ samples. Fig. 3 shows the estimation results for two different QRS complex patterns (QRS1, QRS2) with $N = 5$. The weights are initialised to zero ($w_{n0} = 0$) and the parameter b is initialised to 25 ms ($b_0 = 25$ ms). The convergence limit for μ_1, considering that the implementation takes ms as the temporal units of Hermite functions, is

$$\mu_1 < \frac{LT}{N} = 80 \tag{63}$$

and the limit for ΔSNR > 1 is

$$\mu_1 < T = 4 \tag{64}$$

We select $\mu_1 = 0{\cdot}75$ that satisfies this condition and gives a $\Delta SNR = 5{\cdot}33 = 7{\cdot}27$ dB. The convergence limit for μ_2 is calculated using the SE of QRS1 ($SE = 1{\cdot}25{\cdot}10^{10}$), and assuming $b^* = 20$ ms

$$\mu_2 < \frac{LTb^{*2}}{SE} = 1{\cdot}28 \cdot 10^{-5} \quad (65)$$

Considering a safety factor, we select $\mu_2 = 10^{-8}$.

In Fig. 3 the w_n and b values are shown after ten recurrence adaptations. To measure the percentage of signal energy that has not been modelled, the relative mean-square error (*Rmse*) is defined as

$$Rmse = \frac{\xi LT}{SE} \cdot 100 \quad (66)$$

The value of *Rmse* after ten adaptations is also shown in Fig. 3. We verify the result reported previously (SÖRNMO *et al.*, 1981), given that with $N = 5$ we recover more than 90% of the signal energy.

Note that QRS1 presents higher b (18·4 ms) than QRS2 (16·8 ms). This is in accordance with the relative QRS width in QRS1 and QRS2 (Fig. 3). Note also that the more significant components w_n are those of lower order in accordance with previous work (SÖRNMO *et al.*, 1981).

5.1 *Weight convergence*

According to eqn. 42 and 43, the predicted convergence time for the weights is

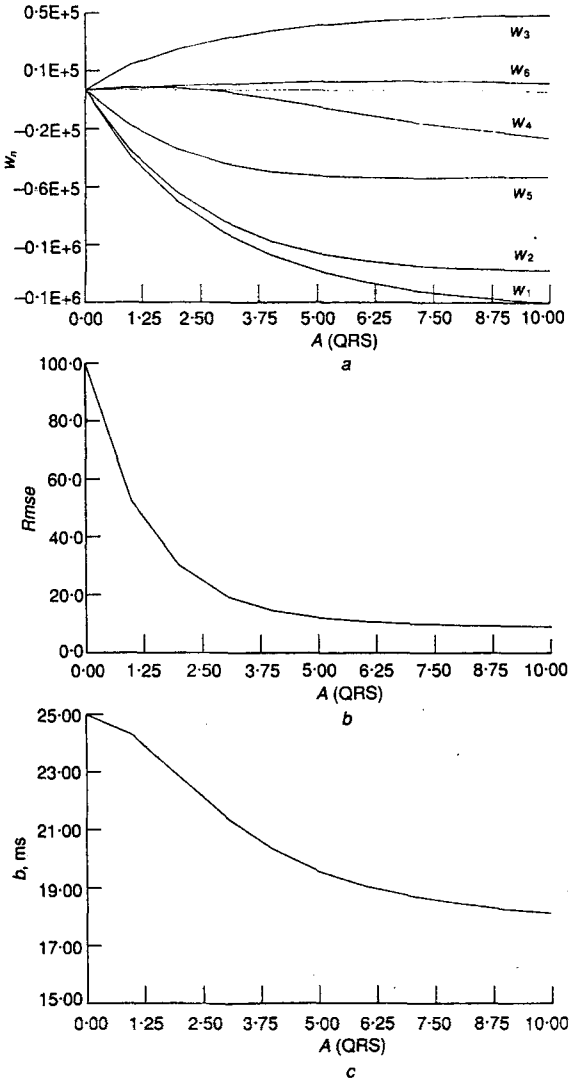

Fig. 4 *Convergence of AHMES in the case of QRS1 pattern: (a) weight convergence; it plots the weights values after each A recurrence adaptation of QRS1; (b) convergence of Rmse; (c) convergence of b*

$$\tau_{w_n} = \frac{LT}{2\mu_1} = 266.7 \text{ samples} \simeq 2{\cdot}7 \text{ QRS recurrences} \quad (67)$$

and for MSE it is $\tau_{mse} = \tau_{w_n}/2 \simeq 1.3$ QRS recurrences. Fig. 4*a* shows the weight evolution after each recurrence adaptation in the case of QRS1. We can corroborate that, after the third recurrence adaptation, all the weights have converged towards more than 60% of the final value. Fig. 4*b* shows the same evolution for the *Rmse* where the convergence time is shown to be half the weight convergence time.

5.2 *Convergence of parameter b*

According to eqn. 62, the predicted convergence time τ_b for the parameter b, in the case of QRS1, can be estimated as $\tau_b < 518{\cdot}4$ samples, which supposes five QRS recurrences. Fig. 4*c* shows the b value after each QRS recurrence adaptation and we can corroborate that, after four recurrences, the b value has already converged to more than 90% of its final value. This result agrees with the convergence time for the parameter b obtained from eqn. 62. Thus, the approximations performed in the derivation of this expression are corroborated to be appropriate.

5.3 *Influence of model order on estimation*

To check the effect of model order N, we have considered the QRS1 signal, and we have applied the AHMES in ten different cases, from $N = 1$ to $N = 10$. Fig. 5 shows the reconstructed signal after ten recurrence adaptations and the *Rmse* for each case. The μ_1, μ_2 parameter and initial conditions

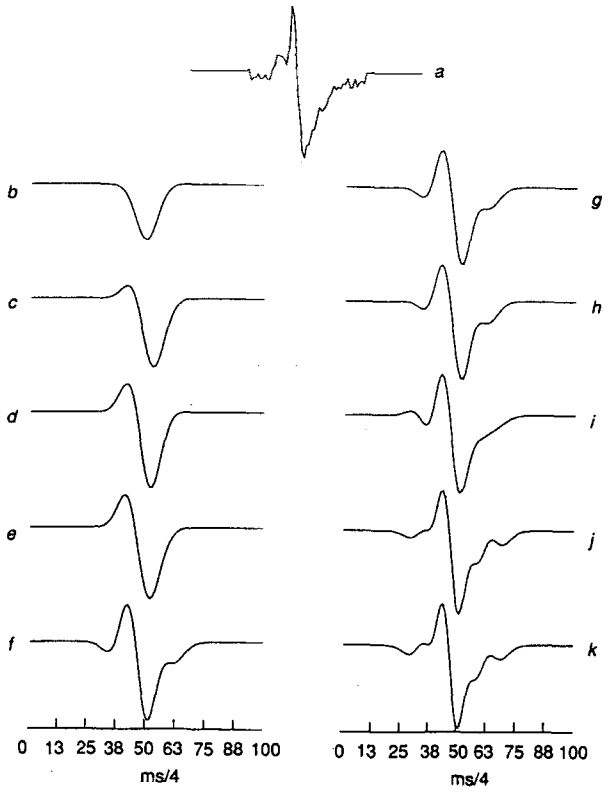

Fig. 5 *QRS1 pattern estimation after ten adaptations for different model order (N = 1, ..., 10); the following are Rmse value and model order: (a) QRS1; (b) N = 1, Rmse = 56·2; (c) N = 2, Rmse = 25·7; (d) N = 3, Rmse = 18·8; (e) N = 4, Rmse = 13·6; (f) N = 5, Rmse = 9·3; (g) N = 6, Rmse = 9·2; (h) N = 7, Rmse = 9·2; (i) N = 8, Rmse = 7·3; (j) N = 9, Rmse = 5·8; (k) N = 10, Rmse = 5·3*

for w_n and b are the same as in the previous case. We can see that low-order estimation results in a low-pass effect of the estimated signal with respect to the original, and when the model order increases, higher frequency components of the original signal are recovered and modelled. However, the high-frequency components of the morphology (such as late potentials, intra-QRS potentials, absolute peak values etc.), which in some cases are clinically relevant, either require a larger model order or are not characterised by the AHMES. The model order (and indirectly the compression ratio) then should be selected with this restriction on the features to be maintained.

6 Applications of AHMES on real QRS complexes

We now consider the coefficient estimation through AHMES of some real sequences of QRS: one that includes ectopic beats wider than normal and another that includes complexes from a patient affected by the bigeminy phenomenon (sequence normal, ectopic etc.) with similar widths.

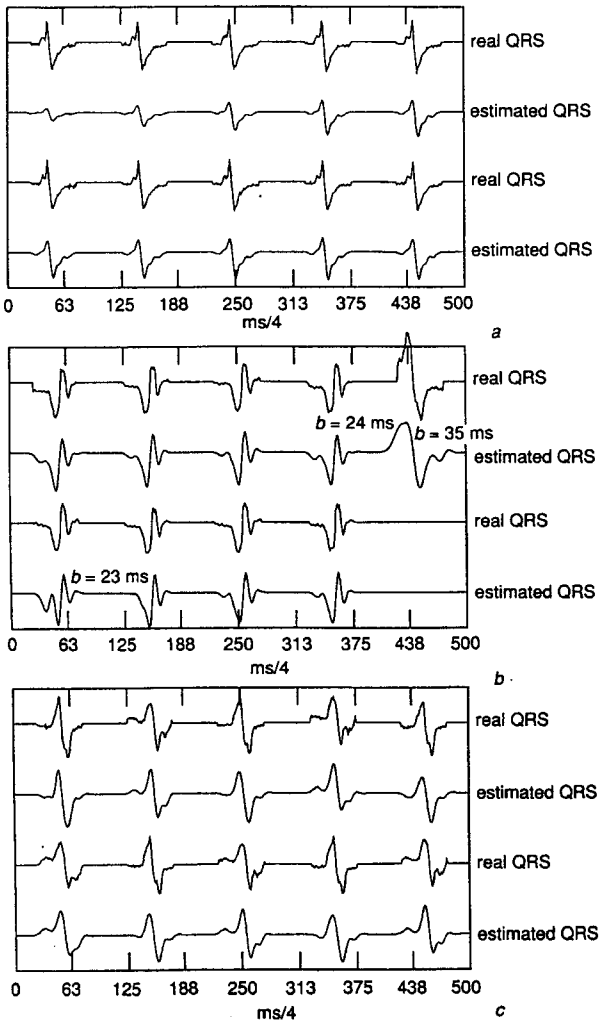

Fig. 6 *Estimation in real QRS sequences with model order $N = 10$; first and third rows are original QRS sequences; second and fourth rows show corresponding estimated QRS sequence after each recurrence adaptation: (a) regular QRS sequence with no important QRS variations ($\mu_1 = 0{\cdot}75$, $\mu_2 = 10^{-8}$; (b) QRS sequence with a PVC in the fifth beat ($\mu_1 = 3{\cdot}4$, $\mu_2 = 10^{-8}$; (c) QRS sequence from a patient affected by the bigeminy phenomenon, two different QRS patterns, ($\mu_1 = 3{\cdot}4$, $\mu_2 = 10^{-8}$*

In real situations, we are interested in studying a regular QRS sequence from a patient. Fig. 6*a* we have the sequence of ten consecutive QRS (first and third rows). Below the original QRS signal is the estimated signal by the AHMES after each QRS adaptation using a model order $N = 10$. Sampling frequency, window size and μ values are maintained as in the previous simulation study.

6.1 *Estimation with ectopic beat presence*

In this Section we consider the case where the ECG signal has abnormal beats like PVC. In this case, it is important to evaluate how the AHMES performs and eventually to detect and discriminate these abnormal beats. Usually, PVC beats have a QRS complex characterised by a greater width than in normal cases. Given that parameter b is related to the width of the Hermite functions considered in the model, we analyse and study the capability of the AHMES to estimate those sudden PVCs of greater width, and eventually detect them through the parameter b.

We take a QRS sequence of nine beats that includes a PVC at the fifth beat (Fig. 6*b*). We applied the AHMES over this beat sequence (model order $N = 10$) with $\mu_1 = 3{\cdot}4$, in order to have a weight convergence time lower than one recurrence (from eqn. 42 we reach $\tau_{w_n} < 1$ QRS recurrence) at the expense of no significant ΔSNR. Thus, it allows us to track beat-to-beat variations, as in the case of isolated PVC QRS complexes. μ_2 remains with the same value ($\mu_2 = 10^{-8}$), given that all QRS sequences have been normalised to satisfy the same convergence condition. Fig. 6*b* and Table 1 show the parameter b value after adaptation of the QRS previous to the PVC beat (24 ms), the PVC (35 ms), and the QRS posterior to the PVC (23 ms). This result shows that the parameter b value after each QRS adaptation can be significantly used to classify PVC beats, even if they appear once in the QRS sequence. The estimated QRS shape after the PVC bears less resemblance to the original than in previous cases, as a result of the great distance (in the feature space) from the previous PVC (the convergence time is that required to achieve 60% of the distance between initial (PVC) and final (regular QRS) value).

The election of the synchronisation point affects the estimation results, and small delays with respect to the window centre can improve the estimation in some morphologies. A posterior study on this topic and how to determine the best synchronisation point would be interesting. Synchronisation with the point that reaches half of the signal energy of the first QRS recurrence can be attempted.

There are also possible variations of the synchronisation point with respect to the deterministic signal given by the variations or jitters of the QRS complex detector marks. This effect has been shown to result in a low-pass effect whose cut-off frequency is proportional to the standard deviation of the jitters.

Fig. 6*b* shows that the reconstructed signal from the AHMES parameters allows beat-to-beat tracking of the QRS shape. This leads us to consider the estimated weights as characteristics to classify the QRS complex. Considering the w_n weight values after each nth QRS adaptation (Table 1), we can see their evolution in the beat sequence and in particular in the sequence normal-ectopic-normal QRS (4th-5th-6th).

6.2 *Estimation with bigeminy phenomenon*

To study the capability of the AHMES for continuous QRS changes, we have selected a QRS sequence from a patient affected by the bigeminy phenomenon (Fig. 6*c*). The QRS signal presents beat-to-beat periodic variations, which basically consist of two different QRS patterns with no differences in

Table 1 b and w_n ($n = 0, \ldots, 4$) feature values after each nth QRS adaptations incase of Fig. 6b (wide ectopic beats).

beat	initially	1th	2th	3th	4th	5th*	6th	7th	8th	9th
b, ms	25·0	25·5	24·0	23·3	24·6	35·1	23·5	21·9	22·4	22·0
w_0	0	−130	−91	−82	−94	−186	−57	−102	−72	−71
w_1	0	138	134	138	126	235	110	140	140	134
w_2	0	−85	−69	−46	−73	102	−56	−63	−76	−61
w_3	0	−64	−72	−83	−71	−218	−55	−69	−64	−75
w_4	0	0·7	14	8	7	135	−14	−14	14	7

* the 5th beat belongs to a PVC

Table 2 b and w_n ($n = 0, \ldots, 4$) feature values after each nth QRS adaptations incase of Fig. 6c (bigeminy phenomenon).

beat	initially	1th	2th	3th	4th	5th*	6th	7th	8th	9th	10th
b, ms	25·0	23·9	24·9	24·8	25·9	26·4	27·8	26·5	26·8	27·1	27·4
w_0	0	−97	−2	−65	52	−89	11	−56	24	−64	34
w_1	0	−186	−168	−172	−166	−152	−189	−162	−166	−160	−169
w_2	0	23	33	37	−90	26	−11	47	−23	45	−18
w_3	0	58	36	48	39	51	14	44	42	42	44
w_4	0	−46	−17	−41	−65	−48	−19	−48	−68	−48	−10

width. We have applied the AHMES to this sequence (model order $N = 10$) with the same μ values as in the previous case ($\mu_1 = 3{\cdot}4$, $\mu_2 = 10^{-8}$) to obtain convergence in one recurrence. The parameter b in this case remains stable (Table 2), and it is not significant to classify the QRS complex sequence. On the other hand, the w_n parameters are able to track the QRS shape variations, as can be corroborated by following the shape estimation recovery in Fig. 6c. Those ectopic beats with similar width can then be detected by the weight vector. We enumerate the $w_n (n = 0, \ldots, 4)$ values from the QRS sequence in Table 2.

It is clear that w_n parameters are adequate to classify the QRS complexes in the two basic shapes (Fig. 6c), following classical pattern recognition techniques as described previously (RAPPAPORT *et al.*, 1982; SÖRNMO *et al.*, 1981). The beat-to-beat convergence $\mu_1 = 3{\cdot}4$ implies a lower $\Delta\text{SNR} = 1{\cdot}18$ with stationary white noise, but still higher than in the case of direct estimation.

When the incoming signal is unknown, we have no idea if the ectopics will be wider than normals; thus a combined b and w_n parameters criterion should be used in general practice for ectopic detection.

7 Conclusions

An adaptive system based on the Hermite functions has been proposed to adaptively estimate and track the QRS complex features in the ECG signal. This system presents better signal-to-noise ratio at the estimation than in the direct estimation. It allows us to model most of the QRS signal energy in a few parameters, due to the similarity of the first Hermite functions with the general shape of the QRS. The orthogonality of Hermite functions leads to non-redundant parameters. The system permits the on-line estimation of the QRS model parameters, with the presence of an on-line QRS detector and with Hermite function generator specially designed for this purpose. The ΔSNR obtained with the system with respect to direct estimation is a function of the μ_1 (for LMS algorithm) adaptation parameter and allows a better estimation of the coefficients than with a direct estimation.

We have studied the theoretical analysis of the AHMES. This system, based on the multiple-input ALC, presents an additional block that adapts the width parameter b. We have proposed an adaptive algorithm for the parameter b value, and the weights are adapted according to the LMS algorithm. The isolated study of both adaptation processes has led us to obtain analytical expressions for the prediction of convergence condition and rate. It has been verified by simulation that these expressions remain valid when both processes act simultaneously. The adaptation of parameter b depends on the primary input signal energy. Thus, it is necessary to estimate this signal energy or to normalise the primary input signal before the adaptation.

It is possible to select the AHMES constants μ_1 and μ_2 in such a way that convergence is achieved in less than one recurrence. Owing to this, it is possible to adapt beat-to-beat changes in the deterministic QRS signal. The extraction of the model parameters after each QRS adaptation provides a description of the QRS signal evolution. In particular, parameter b can be useful for on-line detection of PVC beats. The Hermite model parameters (weights of the AHMES) have been used previously (SÖRNMO *et al.*, 1981) as features for QRS classification and for data compression of the ECG (JANÉ *et al.*, 1993). The AHMES allows the on-line beat-to-beat estimation of these features with a better SNR than the direct estimation.

We have presented an application example of the AHMES in the case of wide ectopic beat presence and bigeminy phenomena with no width differences, in the QRS complex sequence. In these cases, a higher μ_1 value is required to achieve beat-to-beat convergence at the expense of lower ΔSNR. We obtain features relevant for beat classification.

The same study can be performed for the P waves and ST-T complexes, with proper parameter modification, representing in this way the complete ECG signal and allowing the ECG data compression to be presented as previously (JANÉ *et al.*, 1993).

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References

AHMED, N., and RAO, K. R. (1875): 'Orthogonal transforms for digital signal processing' (Springer-Verlag, New York)

BENDAT, J. S., and PIERSOL, A. G. (1986): 'Random data. analysis and measurements procedures' (John Wiley & Sons, New York)

CHRISTOV, I. I., DOTSINSKY, I. A., and DASKALOV, I. K. (1992): 'High-pass filtering of ECG signals using QRS elimination', *Med. Biol. Eng. Comput.*, **302**, (2), pp. 253–256.

FERRARA, E. R., and WIDROW, B. (1981): 'The time-sequenced adaptive filter', *IEEE Trans.*, **CAS-28**, pp. 519–523
FRANKS, L. E. (1975): 'Signal theory' (Prentice-Hall, Englewood Cliffs, New Jersey)
JANÉ, R., LAGUNA, P., and CAMINAL, P. (1991*a*): 'Adaptive estimation of event-related bioelectric signals: effect of misalignment errors'. Proc. 13th Int. Conf. of the IEEE Engineering in Medicine and Biology Society, Orlando, pp. 365–366
JANÉ, R., OLMOS, S., LAGUNA, P., and CAMINAL, P. (1993): 'Adaptive Hermite models for ECG data compression: performance and evaluation with automatic wave detection'. Computers in Cardiology (IEEE Computer Society Press) pp. 389–392
JANÉ, R., RIX, H., CAMINAL, P., and LAGUNA, P. (1991*b*): 'Alignment methods for signal averaging of high resolution cardiac signals: a comparative study of performance', *IEEE Trans.*, **BME-38**, (6), pp. 571–579
LAGUNA, P. (1990): 'New electrocardiographic signal processing techniques: application to long-term records'. PhD Thesis, Science Faculty, Zaragoza, Spain (*in Spanish*)
LAGUNA, P., CAMINAL, P., THAKOR, N. V., and JANÉ, R. (1989): 'Adaptive QRS shape estimation using Hermite model'. Proc. 11th IEEE Ann. Conf. of Engineering in Medicine and Biology Society, pp. 683–684
LAGUNA, P., JANÉ, R., and CAMINAL, P. (1992*a*): 'Adaptive feature extraction for QRS classification and ectopic beat detection'. Computers in Cardiology (IEEE Computer Society Press) pp. 613–616
LAGUNA, P., JANÉ, R., MESTE, O., POON, P. W., CAMINAL, P., RIX, H., and THAKOR, N. V. (1992*b*): 'Adaptive filter for event-related bioelectric signals using an impulse correlated reference input: comparison with signal averaging techniques', *IEEE Trans.*, **BME-39**, (10), pp. 1032–1044
MEYER, C. R., and KEISER, H. N. (1977): 'Electrocardiogram baseline noise estimation and removal using cubic splines and state-space computation techniques', *Comput. Biomed. Res.*, **10**, pp. 459–470
PAN, J., and TOMPKINS, W. J. (1985): 'A real-time QRS detection algorithm', *IEEE Trans.*, **BME-32**, (3), pp. 230–236
RAPPAPORT, S. H., GILLICK, L., MOODY, G. B., and MARK, R. G. (1982): 'QRS morphology classification: Quantitative evaluation of different strategies'. Computers in Cardiology (IEEE Computer Society Press) pp. 33–38
ROMPELMAN, P., and ROS, H. H. (1986): 'Coherent averaging technique: a tutorial review. part 1: noise reduction and the equivalent filter. part 2: trigger jitter, overlapping responses and non-periodic stimulation', *J. Biomed. Eng.*, **8**, pp. 24–35
SÖRNMO, L., BÖRJESSON, P. O., NYGARDS, M. E. and PAHLM, O. (1981): 'A method for evaluation of QRS shape features using a mathematical model for the ECG', *IEEE Trans.*, **BME-28**, (19), pp. 713–717
THAKOR, N. V., WEBSTER, J. G., and TOMPKINS, W. J. (1984): 'Estimation of QRS complex power spectrum for design of a QRS filter', *IEEE Trans.*, **BME-31**, (11), pp. 702–706
THAKOR, N. V., and YI-SHENG, Z. (1991): 'Applications of adaptive filtering to ECG analysis: noise cancelation and arrhythmia detection', *IEEE Trans.*, **BME-38**, pp. 785–794
TRAHANIAS, P., and SKORDALAKIS, E. (1990): 'Syntactic pattern recognition of the ECG', *IEEE Trans.*, **BME-12**, pp. 648–657
VAZ, C. A., and THAKOR, N. V. (1989): 'Adaptive Fourier estimation of time-varying evoked potentials', *IEEE Trans.*, **BME-36**, (4), pp. 448–455
WIDROW, B., and STEARNS, S. D. (1985): 'Adaptive signal proccessing' (Prentice-Hall, Englewood Cliffs, New Jersey)

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