# Scalar Quantizers for Compression of ECG Signal with Orthogonal Transforms

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## Abstract

*In this work we analyze several scalar quantizers applied to transform coding of ECG signal. In order to reduce the variance of the transform coefficient series predictive coding is used improving scalar quantizers performance. Due to the non-stationary nature of ECG signal, adaptive predictors and quantizers are more suitable than fixed configurations. Best results are obtained with an adaptive-predictive adaptive uniform quantizer for on line operation mode.*

## 1. Introduction

The compression of biomedical signals is often used in clinical applications where high volumes of data are generated [1]. The aim of any data compression system is to minimize the volume of data without loss of significant clinical information. Data compression can be defined as the process of detecting and reducing redundancies in a signal.

In the case of ECG signals we can distinguish several kinds of data redundancies. One reflected as a statistical dependence between adjacent samples of the same beat. Other reflected in the non-uniform probability density function (*pdf*) of the amplitudes. This will lead us to consider optimal quantizers. Finally, there exist correlation between samples of different beats due to the quasi-periodic characteristics of the ECG signals.

While source coding can be performed on the original signal directly, it is usually more efficient to find an appropriate transform. The first useful property of appropriate transforms is their energy packing property, that is, the signal energy is almost completely concentrated in a few number of transform coefficients. Another advantage of transform coding is that the new domain is often more appropriate for quantization. Firstly, some basis functions are more relevant for coding the signal, so optimal bit allocation algorithms improve coding performance. Secondly, the correlation between ECG samples of different beats due to the quasi-periodicity of ECG signals produces high correlation between adjacent transform coefficients. Predictive quantization techniques [2] are suitable in this case.

In this work we analyze several uniform and non-uniform scalar quantizers for coding the coefficients obtained when the Karhunen-Loève (KLT) transform is applied to the ECG signal [3]. Results are obtained for all signals from MIT-BIH Arrythmia database.

## 2. Scalar Quantization of KLT Coefficients

Assume $\mathbf{X}=[x_1, x_2, \cdots, x_N]^T$ a vector of $N$ consecutive samples representing a beat of the ECG signal. Typically, these samples are correlated and independent coding of the samples is inefficient. The idea is to apply a linear transform $\mathbf{T}$ (see figure 1) so that the signal energy is more concentrated in the first transform coefficients $(y_1, y_2, \cdots, y_M)$ $M << N$ . The orthogonal transform that achieves the best energy packing is the KLT [4]. Moreover, in this domain the transform coefficients are decorrelated. While there is no general formal result that guarantees more efficient compression by decorrelation, it turns out in practice (and for certain cases in theory) that scalar quantization of decorrelated transform coefficients is more efficient than scalar quantization of the samples [5]. Details of the application of the KLT to the ECG signal can be found in [3].

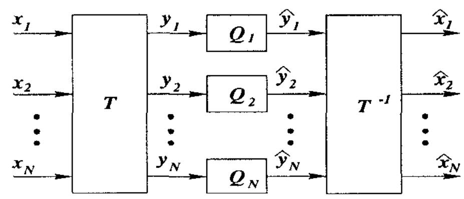

Figure 1: *Transform coding.*

In order to achieve compression in the transform domain we need to map the real value coefficients $(y_1, y_2, \cdots, y_N)$ into a discrete set or codebook. This process of mapping the real line to a countable discrete alphabet or codebook is called *quantization* [5]. In practical situations the discrete alphabet is finite. When the samples are individually quantized we call it scalar quantization. The process of quantization is crucial in compression systems because is where the compression and reconstruction error is generated. The fundamental trade-off in quantization is between rate (number of bits used) and distortion and is known as rate-distortion theory.

Scalar quantizers can be classified as uniform and non-uniform. Uniform quantization, while not optimal for nonuniform pdf's, is very simple and thus often used in practice. The only design parameters are the quantization step $\Delta$, and the number of levels. For selecting the step $\Delta$ there are some different approaches [2]: for Gaussian pdf's the optimum step $\Delta$ is proportional to the standard deviation of the input signal $\sigma$, for non-Gaussian pdf's $\Delta$ is selected to minimize distortion (usually measured as MSE) with an iterative minimization procedure.

When the pdf is not uniform, optimal quantization will not be uniform either. An optimal MSE quantizer is one that minimizes the distortion for a given number of quantization levels. Optimal quantizers can be calculated using the *Lloyd-Max* algorithm [5]. While distortion of non-uniform quantizers is less than for uniform ones, the overhead information needed for coding the non-uniform codebook is much larger. This effect is more important when the quantizer should be periodically actualized in order to follow the non-stationary behavior of the input signal. This will be the case of transform coefficients of ECG signal.

## 3. Predictive Quantization

An important and useful technique is when, instead of quantizing the samples $y[n]$ of the signal to be compressed, one quantizes the difference between a prediction $\widehat{y}[n]$ and $y[n]$, or $e[n] = y[n] - \widehat{y}[n]$. Obviously, if the prediction is accurate, $e[n]$ will be small and for a given number of quantization levels, the quantization error will decrease as compared to straight quantization of $y[n]$ (see figure 2). Prediction is usually linear and based on a finite number of past samples. Closed loop quantization is an efficient and very well known quantization technique (DPCM) [2].

The predictor order was selected with Akaike's criteria. A reasonable value of the model order ex-

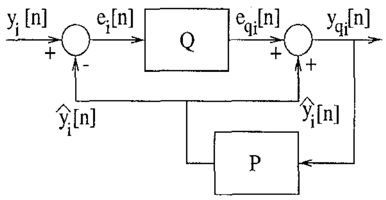


Figure 2: *Predictive quantizer.*

perimentally obtained for the prediction of the KL coefficients of ECG signals was $P$=4.

When the input signal is non-stationary adaptive prediction models (ADPCM) are necessary in order to maintain prediction error variance at low values. There are two main approaches for adaptive prediction [2]: *forward* and *backward*. We selected backward adaptation where predictor coefficients are updated on the basis of quantized and transmitted data $y_q[n]$, so there is no overhead information. Several algorithms for adapting such a predictor have been presented in estimation theory literature. Gradient Adaptive Lattice (GAL) is a steepest descent iteration [6] designed to minimize the sum of forward and backward prediction error of the lattice predictor of figure 3. Reflection coefficients of the lattice structure $K_m^f$ and $K_m^b$ are adaptively estimated with a gradient method as LMS algorithm resulting the updating equation

$$\begin{aligned} K_m^f[n+1] &= K_m^f[n] + \mu_m^f[n] \cdot b_m[n] \cdot f_{m-1}[n] \\ K_m^b[n+1] &= K_m^b[n] + \mu_m^b[n] \cdot f_m[n] \cdot b_{m-1}[n-1] \end{aligned} \quad (1)$$

where $b_m[n]$ and $f_m[n]$ are the backward and forward prediction errors respectively and $\mu_m^f[n]$ and $\mu_m^b[n]$ are the normalized updating steps

$$\begin{aligned} \mu_m^f[n] &= \frac{1-\beta}{\beta\, w_m^f[n-1] + (1-\beta) f_{m-1}^2[n]} \\ \mu_m^b[n] &= \frac{1-\beta}{\beta\, w_m^b[n-1] + (1-\beta) b_{m-1}^2[n-1]} \end{aligned} \quad (2)$$

with $0 < \beta < 1$ and $w_m[n]$ the recursive estimation of prediction error energy.

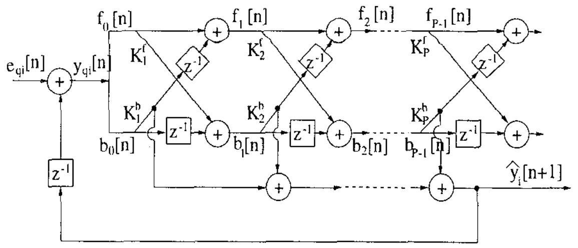


Figure 3: Block diagram of lattice predictor.

## 4. Bit Allocation

The coefficients resulting of transform coding the ECG signal $(y_1, y_2, \cdots, y_N)$ are not of equal significance and there is good reason to allocate bits in an unequal way to quantize these values. The problem of bit allocation can be stated as find the number of bits $b_i$ for every coefficient series to minimize a distortion function (usually mean squared reconstruction error) subject to the constraint of a total number of bits $\sum_i b_i \leq B$. Several solutions have been presented for this problem [5]. A simple (not optimal) algorithm consists of, in each of $B$ iterations, allocating one bit where the overall distortion is minimized at each step given the current partial allocation. In the special case where the high resolution quantizer assumption holds (high number of quantization levels), the distortion is proportional to the variance of the $i$th quantizer times the factor $2^{-2b_i}$. Then, the greedy algorithm simplifies as follows: use the standard deviations as the initial demands for each coefficient. For each iteration, one bit is assigned to the coefficient with higher standard deviation and then it is reduced it by a factor of 2.

## 5. Results

The aim of this work is to compare the performance of several scalar quantizers for coding the KLT coefficients of ECG signals from MIT-BIH Arrythmia database. The following quantizers were selected: DPCM with fixed and uniform quantizer, and with fixed predictor (Q1); DPCM with adaptive and uniform quantizer, with the same fixed predictor (Q2); and ADPCM with adaptive-uniform quantizer with adaptive GAL predictor (Q3). Non-uniform quantizers designed with *Lloyd-Max* algorithm were also tested. For the same bit-rate non-uniform quantizers performance was lower than quantizer Q1 due to the high overhead information needed by the codebook.

The non-stationary behavior of the ECG signal usually leads to a non-stationary series of transform coefficients, where the adaptive quantizers schemes are suitable configurations. This behavior is illustrated for example in figure 4 where the first KL coefficient series $y_1[n]$ of normal beats of record 106 is shown.

If the compression system should work on line, a fixed predictor should be trained during the first beats, giving large prediction error variances where mismatches between input signal and predictor occurred. The prediction error signal obtained with an order $P$=4 linear predictor trained with the first 10 samples is shown in figure 5. Some redundancies

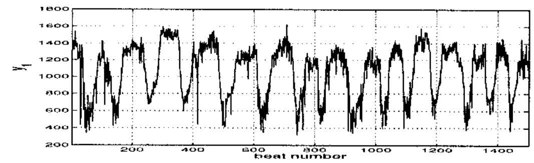

Figure 4: *Suddenly changes in the first KL coefficient series $y_1[n]$ of normal beats of record 106.*

are removed (reduction rms value from 1106 to 157), but the dynamic range of prediction error is high due to the non stationarities. In off line systems, the predictor can be trained with the complete variant signal. The off line predictor gets only a bit larger energy reduction (rms value 137).

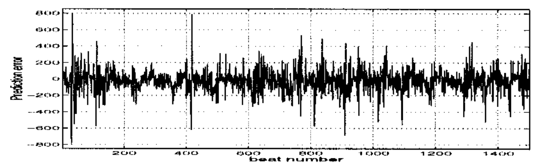

Figure 5: *Prediction error with an order of $N$=4.*

If we apply a fixed uniform quantizer Q1 to prediction error signal we will have some troubles: if our compression system should work on line, quantizer Q1 should be trained and suited during the first samples. In the example of figure 5 the step factor $\Delta$ will be low (1.80 for a quantizer with $N$=7 bits) producing many overload errors where the prediction error variance is high (overload errors represents 99.9% of total error). If the compression system can work off line, the step factor $\Delta$ will be a compromise between granular and overload errors. Step factors are calculated minimizing the mean squared value of quantization error. Results of step factor $\Delta$, quantization error variance $\sigma_q$ and percentage of overload errors $\%OL$ are shown in table 1.

| | *on line* | | | *off line* | | |
|---|---|---|---|---|---|---|
| | $\Delta[n]$ | $\sigma_q$ | $\%OL$ | $\Delta$ | $\sigma_q$ | $\%OL$ |
| **Q1** | 1.80 | 93.9 | 99.9 | 12.35 | 3.73 | 1.34 |
| **Q2** | 12.1±6.1 | 4.54 | 27.7 | — | — | — |
| **Q3** | 10.1±6.9 | 3.57 | 85.24 | — | — | — |

Table 1: *Quantization results for selected quantizers Q1 and Q2 with 7 bits of 1^st^ KL series of rec. 106.*

High values of overload quantization error in on line systems are highly undesirable because these errors are unbounded, and the clinical information of the ECG signal may be hardly distortioned. For non stationary signals adaptive uniform quantizers (Q2) may improve performance of on line systems updating the step factor $\Delta[n]$ according to the variance of the input signal. Backward adaptation should be made with quantized and transmitted prediction errors of previous samples in order to avoid overhead information. Results for the adaptive quantizer Q2 applied to the input signal of figure 4 are much better than results achieved with an on line fixed quantizer Q1 and similar to off line Q1 (see table 1).

Quantizers Q1 and Q2 need some little overhead information for predictor coefficients. If an adaptive GAL predictor is used (quantizer Q3) this overhead can be avoided. Moreover, the adaptive GAL predictor achieves lower values of prediction error standard deviation $\sigma_e$ than fixed predictor for most of the KL transform coefficients of record 106 as it can be seen in figure 6.

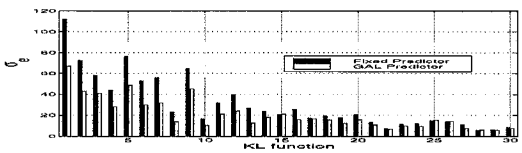


Figure 6: *Improvement of adaptive GAL predictor vs fixed predictor.*

If we apply the GAL adaptive predictor with factor $\beta$=0.97, and then the same uniform adaptive quantizer as Q2 (based on transmitted prediction errors), the performance of the on line system is a bit improved, considering that no overhead information is needed for the predictor (see $\sigma_q$ in table 1).

Finally, we applied quantizers off line Q1, Q2 and Q3 to the first lead of all records from MIT database in order to evaluate their performance and to give more consistent results. For coding the ECG signal $N$=30 KL functions were used for normal beats, 25 for left bundle branch block beats and 20 for ectopic beats. A total of $B$=150 bits/beat were allocated using the greedy algorithm explained in section 4. The KL coefficient series were quantized with the quantizers Q1 on line, Q2 and Q3. The mean results of compression ratio and distortion (values of mean MSE and RMS value measured in adquisition resolution LSB=$5\mu V$) are collected in table 2. Mean absolute error values (rms) are given because the signal energy can be very different for each record.

| | Q1 off line | Q2 | Q3 |
|---|---|---|---|
| $CR$ | 21.5 | 21.5 | 22.4 |
| $MSE(\%)$ | 1.1±1.5 | 1.3±1.0 | 1.2±1.7 |
| $RMS(LSB)$ | 6.2±3.5 | 7.5±4.7 | 6.7±4.2 |

Table 2: *Mean results of the compression system.*

## 6. Conclusion

In this paper we have analyzed several scalar quantizers for transform coding the ECG signal. Beat to beat variability of ECG signal leads to non stationary transform coefficient series with sudden abrupt changes. The selection of the step factor for off line-fixed uniform quantizers is a trade off between granular and overload noise. Quantization error in on line-fixed quantizers was mainly due to overload errors produced in the fast changes of the signal. It has been corroborated that for these situations adaptive quantizers are more appropriate because they can be adapted to the input signal. A bit performance improvement is obtained with an adaptive GAL predictor considering that predictor overhead information can be avoided.

Results obtained from whole MIT-BIH Arrythmia database show that an adaptive quantizer with ADPCM is the best scheme for on line coding the KLT coefficients of ECG signal obtaining similar results than for off line quantizers.


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