Least Squares VCG Loop Alignment

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Abstract—**The present paper describes a new least–squares error criterion for the alignment of two vectorcardiographic (VCG) loops. The criterion is developed to handle certain conditions related to changes in body position, large differences in QRS amplitude or high noise levels. Several examples are included which illustrate the performance of the method in terms of, e.g., parameter estimate accuracy.**

Keywords—**Least–squares estimation, loop alignment, VCG loop.**

1 Introduction

In continuous ECG/VCG ischemia monitoring, a change in body position can sometimes be falsely interpreted as a myocardial ischemic event. While this problem obviously reduces the performance of the monitoring system, few recent systems include software targeted to reduce the number of false alarms due to such positional changes.

A number of recent studies draws special attention to this problem by investigating the effect of body position changes on the surface ECG. The effects in the QRS complex and the ST segment were assessed on the standard 12-lead ECG and the derived 12-lead ECG (using a vectorcardiographic lead configuration) [1]. It was concluded that positional changes influence all ECG measurements although those which are related to the QRS complex are more susceptible than those related to the ST segment. Furthermore, the standard 12-lead ECG system seems to be more susceptible to such changes than is the derived 12-lead system. Similar results, showing that the QRS complex is more susceptible to positional changes, were also reported on in [2] and [3].

One of the few papers dealing with this problem was presented by Jager and coworkers [4], who explored the Karhunen-Loève transform (KLT) for detecting ST changes related to ischemia as well as nonischemic episodes due to e.g. body position changes. They developed a trajectory recognition algoritm that studied the trajectories of the feature vectors in the KLT space for both the QRS complex and the ST segment.

In the present paper, detection of body position changes is considered by investigating the properties of successive vectorcardiographic loops. An estimation method is presented for finding the angular time series which reflect how the loop rotates from beat to beat. The method is based on a least-squares criterion for finding the rotation angles; the criterion is especially tailored to handle the large amplitude differences which characterize the P-QRS interval. As mentioned above, this interval is of particular interest since it is susceptible to positional changes while being less sensitive to ischemia than is the ST segment.

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2 Loop alignment

2.1 Signal model

In this paper it is assumed that a VCG loop of the QRS complex, $\mathbf{Z}$, is related to another, "reference" loop, $\tilde{\mathbf{Z}}_R$, but altered by certain geometrical transformations related to body position changes as well as other extracardiac factors, e.g. respiration [5]. Both $\mathbf{Z}$ and $\tilde{\mathbf{Z}}_R$ are matrices, the rows of which correspond to the orthogonal leads X, Y and Z, respectively, in the VCG. The reference loop, $\tilde{\mathbf{Z}}_R$, can be estimated, e.g., by averaging of suitable beats from the onset of the recording or by recursive updating of the averaged, aligned beats.

Rotational changes of the loop are modeled by the orthonormal, 3-by-3 matrix $\mathbf{Q}$; alternatively, this matrix can be represented by three different rotation angles. A scalar amplitude factor α is included to account for loop expansion and contraction (this parameter is not explicitly used for detection of body position changes but indirectly influences the estimation of τ and $\mathbf{Q}$). Although $\mathbf{Z}$ is initially assumed to be reasonably well synchronized in time to $\tilde{\mathbf{Z}}_R$, a refined synchronization is introduced by the shift matrix $\mathbf{J}_\tau$. Assuming that additive Gaussian noise, $\mathbf{V}$, is present, the observation model is defined by,

$$\mathbf{Z} = \alpha \mathbf{Q} \tilde{\mathbf{Z}}_R \mathbf{J}_\tau + \mathbf{V} \tag{1}$$

The matrices $\mathbf{Z}$ and $\mathbf{V}$ are 3-by-N where N denotes the number of samples in the QRS interval. Due to time synchronization, however, the reference loop $\tilde{\mathbf{Z}}_R$ must contain additional samples ($(N+2\Delta)$ samples for each lead). As a consequence, the observed loop $\mathbf{Z}$ can be modeled from any of the $(2\Delta+1)$ possible synchronization positions in $\tilde{\mathbf{Z}}_R$.

2.2 Normalized least–squares estimation

In order to reduce the influence of ischemic events on the angle estimates, the loop alignment is performed over an early part of the QRS complex, see Fig. 1. Due to the time synchronization of the loops by $\mathbf{J}_\tau$ it is necessary to consider an error criterion for alignment which accounts for large differences in amplitude.

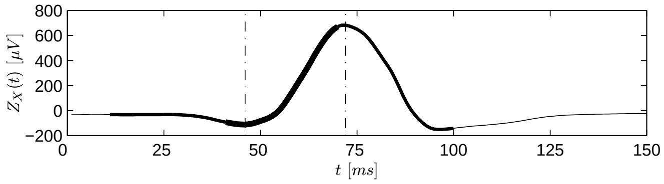

Fig. 1. Example of the interval used for loop alignment. The thick gray line shows the interval that always is included in the loop alignment while the thinner gray line shows the alignment interval. In the figure, the interval between the dashed-dotted marks indicate the initial part of the QRS complex.

The solution presented here is to use a criterion in which the Frobenius norm for the difference between $\mathbf{Z}$ and $\alpha\mathbf{Q}\tilde{\mathbf{Z}}_R\mathbf{J}_\tau$ is normalized with the scaled and rotated reference loop, $\alpha\mathbf{Q}\tilde{\mathbf{Z}}_R\mathbf{J}_\tau$, i.e.,

$$\breve{\epsilon}^2_{min} = \min_{\alpha,\mathbf{Q},\tau} \frac{\left\|\mathbf{Z} - \alpha\mathbf{Q}\tilde{\mathbf{Z}}_R\mathbf{J}_\tau\right\|_F^2}{\left\|\alpha\mathbf{Q}\tilde{\mathbf{Z}}_R\mathbf{J}_\tau\right\|_F^2} \tag{2}$$

The minimization of (2) is done by first rewriting it as

$$\breve{\epsilon}^2 = \frac{\mathrm{tr}\left(\mathbf{Z}^T\mathbf{Z}\right) + \alpha^2\mathrm{tr}\left(\mathbf{J}_\tau^T\tilde{\mathbf{Z}}_R^T\tilde{\mathbf{Z}}_R\mathbf{J}_\tau\right) - 2\alpha\mathrm{tr}\left(\mathbf{Z}^T\mathbf{Q}\tilde{\mathbf{Z}}_R\mathbf{J}_\tau\right)}{\alpha^2\mathrm{tr}\left(\mathbf{J}_\tau^T\tilde{\mathbf{Z}}_R^T\tilde{\mathbf{Z}}_R\mathbf{J}_\tau\right)} \tag{3}$$

and then noting that minimization with respect to $\mathbf{Q}$ is equivalent to maximizing the rightmost term in the numerator. By introducing the matrix

$$\mathbf{D}_\tau = \mathbf{Z}\mathbf{J}_\tau^T\tilde{\mathbf{Z}}_R^T \tag{4}$$

it can be shown [5] that the rotation matrix, for a fixed τ, is estimated by

$$\hat{\mathbf{Q}}_\tau^T = \mathbf{U}\mathbf{V}^T \tag{5}$$

where the matrices $\mathbf{U}$ and $\mathbf{V}$ result from singular value decomposition of $\mathbf{D}_\tau$, i.e., $\mathbf{D}_\tau = \mathbf{U}\Sigma\mathbf{V}^T$.

The parameter α is estimated by differentiating $\breve{\epsilon}^2$ with respect to α and setting the resulting expression equal to zero. The scale factor is estimated by,

$$\hat{\alpha}_\tau = \frac{\mathrm{tr}\left(\mathbf{Z}^T\mathbf{Z}\right)}{\mathrm{tr}\left(\mathbf{Z}^T\hat{\mathbf{Q}}_\tau\tilde{\mathbf{Z}}_R\mathbf{J}_\tau\right)} \tag{6}$$

Finally, the time synchronization parameter τ is obtained by a grid search over all possible values of τ,

$$\hat{\tau} = \arg\min_\tau \frac{\left\|\mathbf{Z} - \hat{\alpha}_\tau\hat{\mathbf{Q}}_\tau\tilde{\mathbf{Z}}_R\mathbf{J}_\tau\right\|_F^2}{\left\|\hat{\alpha}_\tau\hat{\mathbf{Q}}_\tau\tilde{\mathbf{Z}}_R\mathbf{J}_\tau\right\|_F^2} \tag{7}$$

which thus defines the optimal estimates of $\mathbf{Q}$ and α.

2.3 Angle estimation

In order to get an angular time series, the rotation matrix is computed for each loop occurring at time t_i. The corresponding rotation angles can be estimated from $\hat{\mathbf{Q}}(t_i)$ as,

$$\hat{\varphi}_Y(t_i) = \arcsin\left(\hat{q}_{(1,3)}(t_i)\right) \tag{8}$$

$$\hat{\varphi}_X(t_i) = \arcsin\left(\frac{\hat{q}_{(1,2)}(t_i)}{\cos\hat{\varphi}_Y(t_i)}\right) \tag{9}$$

$$\hat{\varphi}_Z(t_i) = \arcsin\left(\frac{\hat{q}_{(2,3)}(t_i)}{\cos\hat{\varphi}_Y(t_i)}\right) \tag{10}$$

where $\hat{q}_{(m,n)}(t_i)$ denotes the element in the m:th row, n:th column in matrix $\hat{\mathbf{Q}}(t_i)$.

2.4 Unnormalized least–squares estimation

In developing a method for detecting body position changes, the estimation problem was initially studied in terms of minimization of an unnormalized least–squares criterion [5],

$$\epsilon^2_{min} = \min_{\alpha,\mathbf{Q},\tau} \left\|\mathbf{Z} - \alpha\mathbf{Q}\tilde{\mathbf{Z}}_R\mathbf{J}_\tau\right\|_F^2 \tag{11}$$

Below, the estimators resulting from minimization of (11) are presented since these are used for comparison. The estimate of $\mathbf{Q}_\tau$ is identical to that in (5) (the optimal value may, of course, be conditioned on a different τ), however, the amplitude factor is instead given by

$$\hat{\alpha}_\tau = \frac{\mathrm{tr}\left(\mathbf{Z}^T\hat{\mathbf{Q}}_\tau\tilde{\mathbf{Z}}_R\mathbf{J}_\tau\right)}{\mathrm{tr}\left(\mathbf{J}_\tau^T\tilde{\mathbf{Z}}_R^T\tilde{\mathbf{Z}}_R\mathbf{J}_\tau\right)} \tag{12}$$

The optimal τ is again found as that value which minimizes the Frobenius norm in (11),

$$\hat{\tau} = \arg\min_\tau \left\|\mathbf{Z} - \hat{\alpha}_\tau\hat{\mathbf{Q}}_\tau\tilde{\mathbf{Z}}_R J_\tau\right\|_F^2 \tag{13}$$

2.5 Rotation matrix constraint

A detailed study of the alignment procedure revealed that large estimation errors are related to the computation of $\hat{\tau}$. When successive beats with similar morphologies are analysed, one can expect that changes in the electrical axis are rather small, alternating around a certain equilibrium, e.g., the rotation angles change within ±15 degrees. This characteristic implies that the rotation matrix should be diagonal dominant.

However, at high noise levels it was found that the optimal estimate $\hat{\mathbf{Q}}$ not always possessed this property. This observation can be interpreted as that the "major part" of one lead in $\mathbf{Z}$ is derived from the other two leads in $\tilde{\mathbf{Z}}_R$. When studying the estimate for other values of τ, especially those adjacent to the optimal τ, it was found that the rotation matrix possessed a diagonal dominant structure.

One technique which corrects this problem is therefore to discard those $\hat{\mathbf{Q}}_\tau$ which are not diagonal dominant and then evaluate the error criterion based on the remaining matrices. A matrix is here defined to be diagonal dominant if its elements fulfil

$$q^2_{\tau,(m,m)} > \sum_{\substack{m,n\ =\ 1,2,3 \\ n\ \neq\ m}} q^2_{\tau,(m,n)} \tag{14}$$

where $-\Delta \leq \tau \leq \Delta$. The resulting set of diagonal dominant matrices are assumed to be contained in the matrix set Ω_τ. The final estimate $\tilde{\tau}$ is obtained by a constrained grid search,

$$\tilde{\tau} = \arg \min_{\tau \in \Omega_\tau} \frac{\|\mathbf{Z} - \hat{\alpha}_\tau \hat{\mathbf{Q}}_\tau \tilde{\mathbf{Z}}_R \mathbf{J}_\tau\|_F^2}{\left\|\hat{\alpha}_\tau \hat{\mathbf{Q}}_\tau \tilde{\mathbf{Z}}_R \mathbf{J}_\tau\right\|_F^2} \tag{15}$$

In the unlikely case of an empty Ω_τ, the beat is excluded from further analysis.

3 Results

The performance of the present method for rotation angle estimation is exemplified by a number of cases taken from an ECG database. The database was recorded from 20 healthy subjects who changed their body position according to the following predefined pattern: supine, right side, supine, left side, supine and so on. Each position was held during one minute.

Obviously, the performance of the alignment method is influenced by a variety of parameters, e.g., the time synchronization interval, the length of the alignment interval as well as its absolute position in the QRS complex. The selection of these parameter values was, for the example presented below, done on a heuristic basis in order to obtain satisfactory performance.

3.1 Normalized error

Two examples are presented which illustrate the benefits of using the normalized error criterion in (2) instead of the criterion in (11). The first example shows the two error norms as calculated over the entire interval $-\Delta \leq \tau \leq \Delta$ for the initial part of a single QRS complex, see Fig. 2. Here, it is evident that a large time synchronization interval together with a short alignment interval from the initial part of the QRS complex results in an improper choice of $\hat{\tau}$ with the original alignment method due to the large differences in amplitude within the alignment interval. Using the normalized loop alignment method, the proper $\hat{\tau}$ is chosen which thus results in reliable estimates of the other alignment parameters.

Based on the initial part in several consecutive QRS complexes, the resulting alignment parameters using the two error criteria are shown in Figs. 3 and 4, respectively, It is evident that artifactual variations occur in the amplitude estimate within the time synchronization interval; the normalized loop error criterion in (7) produces more accurate parameter estimates.

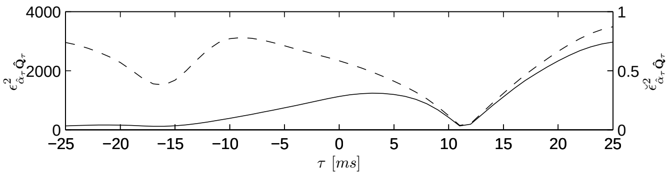

Fig. 2. The error criterion using optimal values for rotation and scaling as a function of τ for a single beat. A solid line is used for the original formulation of the error criterion in (11), $\epsilon^2_{\hat{\alpha}_\tau \hat{\mathbf{Q}}_\tau}$, and a dashed line for the normalized error criterion, $\breve{\epsilon}^2_{\hat{\alpha}_\tau \hat{\mathbf{Q}}_\tau}$, in (2).

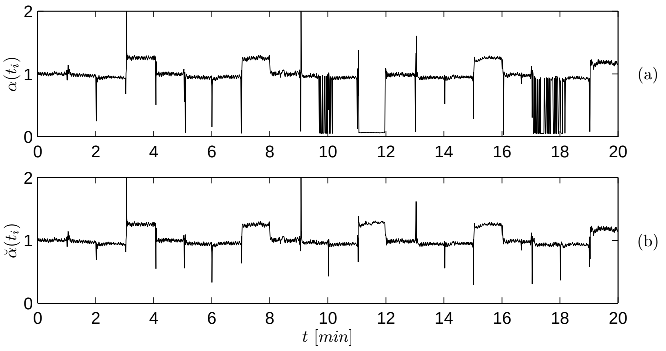

Fig. 3. Example of (a) scaling estimates obtained by (11). Note the small values (approximately zero) of the scaling estimates around 10 minutes, 11–12 minutes and 17–18 minutes respectively. In (b) the same example as in (a) is shown using the normalized error in (2). At the same instants the scaling estimates are approximately unity implying correct estimation.

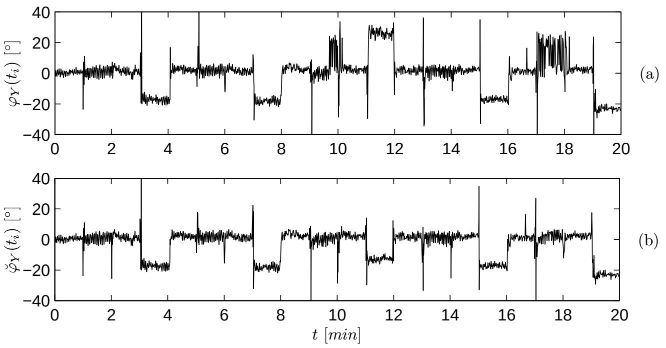

Fig. 4. Example of (a) rotation angle estimates obtained by (11). Note the large variation in rotation angle estimates around 10 minutes and 17–18 minutes as well as the erroneous estimates around 11–12 minutes respectively. In (b) the same example as in (a) is shown using the normalized error in (2). The large variation of the estimates are now eliminated at the previously mentioned time instants.

3.2 Diagonal dominance

In order to illustrate the effect of the constraint on diagonal dominance, another example is presented. The combination of a high noise level and leads with similar shapes sometimes results in the interchange of two leads, see Fig. 5. Using the alignment method with a constraint obviously does not produce the minimum error, however, the result is more accurate in terms of parameter estimates.

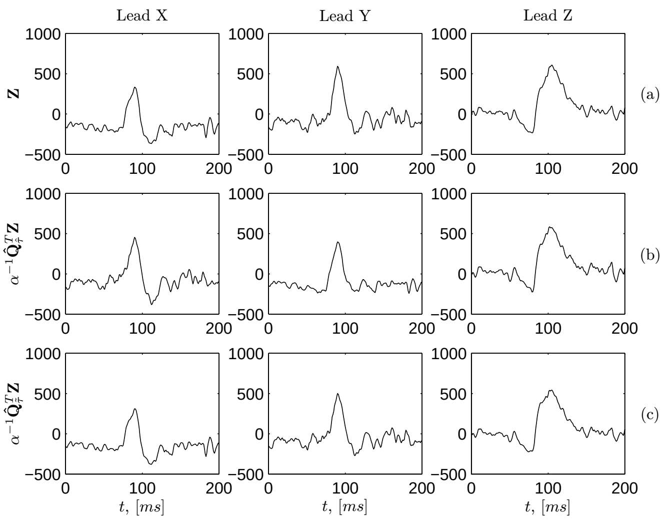

Fig. 5. Example of diagonal dominance. In (a), the original VCG is shown , in (b) the nondiagonal dominant aligned beat and in (c) the constrained aligned beat obtained with the algoritm in (14) and (15). Note the interchange of signal in leads X and Y in (b), while in (c), this change is not present.

4 Discussion

The normalized loop alignment formulation was introduced due to problems identified when using only a subinterval of the QRS complex as alignment interval. Its properties suit the case of alignment using a subinterval of the QRS complex for which results in better estimates of the alignment parameters. However, it should be pointed out that the normalized alignment only extends the effective range regarding the length of the alignment interval and the time synchronization interval up to a certain degree. Extending the alignment outside of this interval reducing the part of the QRS complex utilized in the alignment further, erroneous estimates will result. Also, the size of the alignment interval and the amplitude of the signal in that interval will affect the reliability of the alignment parameter estimates from a statistical point of view. In the case where the whole QRS complex is used as alignment interval, the difference compared to the original formulation is negligible.

The constrained estimation of the rotation matrix is primarily dependent on the shape of the QRS complex. In the case of a beat with either a low amplitude in one lead or two or three leads with similar shapes, the likelihood of the rotation matrix to be nondiagonal dominant increases. The constrained estimation results in more accurate parameter estimates but also a larger mean square error.

The present method is intended for use in a detector which finds changes in body position based on tracking of loop–related rotation angles. However, this method only describes that part of the detector which extracts the decision signal; work is in progress to develop the over–all detector structure.

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