ADAPTIVE ESTIMATION OF EVENT-RELATED BIOELECTRIC SIGNALS: EFFECT OF MISALIGNMENT ERRORS

Raimon Jané, Pablo Laguna, Nitish V. Thakor and Pere Caminal

Institut de Cibernètica (UPC-CSIC), Diagonal 647, 08028 Barcelona, Spain

ABSTRACT

We present a general adaptive system to estimate deterministic recurrent transient signals time-locked to a stimulus. This system estimates the components of the deterministic signal using a function base and was applied to evoked potentials and ECG signals. In this work we analyse for this adaptive system the effect of misalignment errors in each signal recurrence. The theoretical analysis shows that the errors in the occurrence time estimation produce a low-pass filtering effect with a cutoff frequency function of the error distribution. A simulation study with real ECG signals agree with theoretical results.

INTRODUCTION

Adaptive estimation of recurrent signals allows to reduce noise uncorrelated with the signal and to track dynamic variations in these signals. This technique was used in ECG signals [1,2] and evoked potentials [3].

This adaptive technique makes use of the recurrent property of the signal and is based on the adaptive linear combiner [4]. The adaptation starts with each recurrence and extends the time period where the signal is supposed to be defined. This is equivalent to consider a signal composed by the succession of the recurrences that appear after the occurrence time instants. In this adaptive system the reference inputs used in the estimation of the deterministic signal are elements of a base of the vectorial space where the signal can be represented. The adaptive system dynamically estimates the component of the deterministic recurrent signal in each base element considered. In [1] these reference inputs were the orthonormal Hermite functions, in [2] were unit impulses and in [3] the base elements were sine and cosine functions. All these works made the assumption that the occurrence time given by the stimulus is well known, and this is true in evoked potentials where the signal is the response to an external stimulus, but in the ECG the occurrence time is determined through a wave occurrence time detector.

In this work we analyze how the errors in the determination of the occurrence time affect in the estimation of the deterministic signal component. We will see in simulation how a Gaussian distribution of the occurrence time determination around the exact time instant generates a low-pass filtering effect on the estimated signal.

THE ADAPTIVE ESTIMATION MODEL

The adaptive estimation model (figure 1) is based on the adaptive linear combiner [4]. The primary input d_k is formed by the succession of each signal recurrences following the stimulus. This signal d_k is considered as composed of the deterministic signal component of interest s_k, correlated with the stimulus, and the noise n_k that is supposed not correlated with the stimulus. If the occurrence time

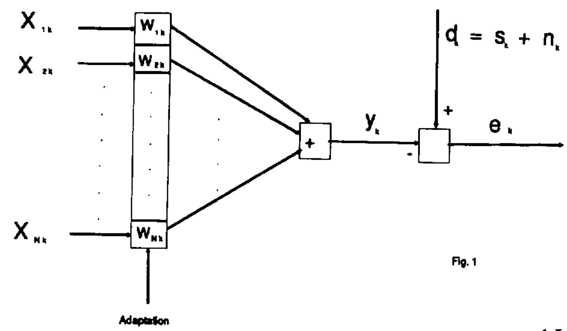


Fig. 1

is well determined in each recurrence, that is supposed to extend L samples, the deterministic signal component s_k satisfies $s_k = s_{k+L}$. The reference inputs X_{ik} ($i = 1, ..., N$) ($N \leq L$) are the succession of the ith base element of the vectorial space where the deterministic signal is represented, and then $X_{ik} = X_{i\ k+L}$.

The signal d_k in this vectorial space representation with all the base functions can be expressed as

$$d_k = \sum_{i=1}^{L} c_i\, X_{ik} + n_k \, , \tag{1}$$

where c_i are the coefficients that characterize the deterministic signal s_k in the base.

The output y_k is the signal that we want to be an estimation of s_k. and e_k is the error signal $e_k = s_k + n_k - y_k$ with

$$y_k = \sum_{i=1}^{N} w_{ik}\, X_{ik} \tag{2}$$

If $\mathbf{X}_k$ denotes the reference vector and $\mathbf{W}_k$ the weight vector

$$\mathbf{X}_k = [X_{1k}, X_{2k}, ..., X_{Nk}]^T \qquad \mathbf{W}_k = [w_{1k}, w_{2k}, ..., w_{Nk}]^T \tag{3}$$

then

$$y_k = \mathbf{X}_k^T \mathbf{W}_k = \mathbf{W}_k^T \mathbf{X}_k . \tag{4}$$

Minimizing $\xi = E[e_k^2]$ with any adaptive algorithm and according the adaptive formalisme [4] we obtain that the weight vector converges to the optimal solution $\mathbf{W}^*$, that has the value $\mathbf{W}^* = \mathbf{R}^{-1}\mathbf{P}$ [4], where

$$\mathbf{R} = E[\mathbf{X}_k \mathbf{X}_k^T] \quad \text{and} \quad \mathbf{P} = E[d_k \mathbf{X}_k] \tag{5}$$

In this case, given the orthonormality conditions of the base elements of a vectorial space, this $\mathbf{R}$ matrix and $\mathbf{P}$ vector take the values

$$\mathbf{R} = \frac{1}{L}\mathbf{I} \quad \text{and} \quad \mathbf{P} = \frac{1}{L}[c_1, c_2, ..., c_N]^T , \tag{6}$$

and then the optimal weight vector solution $\mathbf{W}^*$ is

$$\mathbf{W}^* = [c_1, c_2, ..., c_N]^T . \tag{7}$$

*This work was supported by grant TIC88-0204, from CICYT (Spain), and by NATO grant CGR 900058

It means that each weight w_i is an estimation of the component of s_k in the base element X_{ik}. Then the weight vector is a characterization of the deterministic signal component, and when the weight vector is the optimum, the output signal y_k takes the value

$$y_k = \sum_{i=1}^{N} w_i^* X_{ik} = \sum_{i=1}^{N} c_i X_{ik} \, , \tag{8}$$

that is the projection of s_k on the subspace generated by X_{ik} ($i = 1, ..., N$) with $N \leq L$. In case where all the base elements were considered ($N = L$) it is obtained $y_k = s_k$.
Given that the weight vector oscillates around this optimal value, y_k will be an unbiased estimation of s_k. The remaining noise due to the misadjustment M [4] will depend of the adaptive algorithm used to adjust the weight.

EFFECT OF ERRORS IN THE OCCURRENCE TIME ESTIMATION

The previous study considers that the occurrence time of each recurrence is well estimated ($s_k = s_{k+L}$). If there are some errors in the estimation of this occurrence time we will have some delay ($\pm\delta$) between the recurrence adaptation start and the deterministic signal start. This fact will be reflected in a delay between the function base elements ($X_{ik} = X_{i\ k+L}$) and the deterministic signal ($s_k = s_{k+L\pm\delta}$). The effect of these errors on the estimated signal will be reflected through the effect on the **P** vector. The optimum weight vector $\mathbf{W}^* = \mathbf{R}^{-1}\mathbf{P}$ will be affected throught **P** vector modifications (**R** does not change). Then we will analyze the **P** vector in this case

$$\mathbf{P} = E[d_k \mathbf{X}_k] = E[s_k \mathbf{X}_k] + E[n_k \mathbf{X}_k] \tag{9}$$

As noise n_k are supposed to be not correlated with the stimulus, in (9) the second term of **P** is null and **P** is reduced to be $\mathbf{P} = E[s_k \mathbf{X}_k]$. If we assume that the errors of the occurrence time determination (δ) have a probability distribution $p(\delta)$, the **P** vector can be expressed as

$$\mathbf{P} = E[s_k \mathbf{X}_k] = \int_{-\infty}^{\infty} \left[\frac{1}{L} \sum_{k=1}^{L} s_{k+\delta} \mathbf{X}_k \right] p(\delta) \; d\delta \tag{10}$$

$$\mathbf{P} = \frac{1}{L} \sum_{k=1}^{L} \mathbf{X}_k \int_{-\infty}^{\infty} s_{k+\delta} \; p(\delta) \; d\delta \tag{11}$$

From this result we observe that the **P** vector elements are (c_i'/L) where c_i' are the components of a signal s_k' that takes the value

$$s_k' = \int_{-\infty}^{\infty} s_{k+\delta} \; p(\delta) \; d\delta \tag{12}$$

Calculating the Fourier transform of this signal s_k' ($S'(f)$) we have

$$S'(f) = S(f) \int_{-\infty}^{\infty} e^{j2\pi f \delta} p(\delta) \; d\delta \tag{13}$$

where $S(f)$ is the Fourier transform of s_k. Then the effect of the error in the occurrence time estimation makes a filtering effect on the signal s_k in the estimated y_k. The transfer function $C(f)$ of this filter ($S'(f) = C(f)S(f)$) is the characteristic function of the δ distribution [5]

$$C(f) = \int_{-\infty}^{\infty} e^{j2\pi f \delta} p(\delta) \; d\delta \tag{14}$$

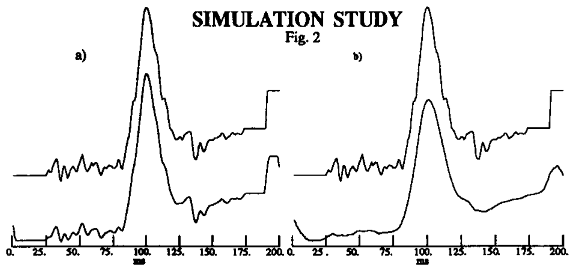
In case that $p(\delta)$ is a Gaussian distribution with standard deviation σ, the characteristic function is

$$C(f) = e^{-2\pi^2 \sigma^2 f^2} \tag{15}$$

that consists in a low-pass filter with a cutoff frequency (f_c) at -3 dB of $f_c = 132.5/\sigma$, where f_c is expressed in Hz and σ in ms. Then the estimation of $\mathbf{W}^*$ will be the coefficients c_i' of a low-pass filtered deterministic signal component, which cutoff frequency depends of the error distribution.

SIMULATION STUDY

Fig. 2



In order to test the previous results we have taken 150 ms of a deterministic signal that belongs to a real QRS complex sampled at 1000 Hz. We have extended this signal to 200 ms with a 25 ms flat line on the left and other 25 ms on the right. The time domine extension includes an step at the 190 ms of the total signal (upper signals of figure 2). With this signal we have generated the signal in study d_k as composed of the succession of 500 recurrences of the same signal. In this way we can consider that all the signal is deterministic and there is not noise (n_k=0). We have estimated the deterministic signal with the adaptive model, the LMS adaptive algorithm and a base of unit impulses [2], considering the L elements of the base. Then all the deterministic signal can be represented (in this case L = 200). In the estimation we have started the adaptation of each recurrence with a Gaussian distributed delay (σ) respect the exact occurrence point of each realization. In figure 2 we have the original deterministic signal and below the estimated after adaptation of 500 recurrences for σ=1 ms (Fig. 2a) and for σ=5 ms (Fig. 2b). From the cutoff frequency reached in the theoretical study we have that f_c=132.5 Hz for σ=1 ms, and f_c=26.5 Hz for σ=5 ms. Figures 2a and 2b agree with these results, where we can see that filtering effect (f_c) occurs at lower frequencies for higher σ, according to the expression $f_c = 132.5/\sigma$.

CONCLUSIONS

We have presented a general method to adaptively estimate a recurrent transient signal through its representation on a specific base of a vectorial function space. We have analyzed the effect of errors in the determination of the occurrence time of each transient signal recurrence. This effect has been theoretically shown to be a low-pass filtering effect with a cutoff frequency inversely proportional to the error desviation. The simulation with a stationary deterministic signal free of noise and with artificially introduced errors in the occurrence time (ideal case where only these errors will affect the estimation) agrees the theoretical results.

REFERENCES

[1] Laguna, P., Caminal P., Thakor N.V. and Jané R. (1989), "Adaptive QRS shape estimation using Hermite model", *Proc. of the 11th Int. Conf. of the IEEE Eng. in Med. and Biol. Soc.*, pp. 683-684.
[2] Jané R., Laguna P. and Caminal P. (1990), "Adaptive filtering of event-related bioelectric signals", *Proc. of the 12th Int. Conf. of the IEEE Eng. in Med. and Biol. Soc.*, pp. 864-865.
[3] Vaz C. A. and Thakor N.V. (1989), "Adaptive Fourier estimation of time-varying evoked potentials" *IEEE Trans. Biomed. Eng.* Vol. BME-36, No. 4, pp. 448-455.
[4] Widrow. B. and Stearns S. D. (1985). "Adaptive signal processing" *Prentice-Hall,* New Jersey.
[5] Bendat J.S. and Piersol A.G. (1986), "Random Data. Analysis and Measurements Procedures", *John Wiley & Sons*, New York.