# QT-RR Adaptation Time Lag Estimation and its Dependence on Heart Rate Trend Frequency Content in Exercise Stress Testing

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***Abstract*—A new model for the estimation of the QT-RR adaptation time lag using exercise ECG stress testing has been proposed, assuming a linear heart rate trend during the test. In this work, simulated ECGs have been generated based on heart rate patterns with oscillations at different frequencies to demonstrate that the assumption can be relaxed so that the QT-RR adaptation time lag can be adequately estimated for any heart rate trend whose frequency content is below a certain frequency, which depends on the QT-RR time lag.**

## I. Introduction

The QT adaptation time lag in response to sudden changes in heart rate (HR) can be computed using model-based estimation of the memory parameters describing the time lag between RR and QT changes. A method was proposed by Pueyo et al. [1] to model the QT-RR dependence by two blocks: a first-order system that models the QT memory lag after RR followed by an instantaneous (typically nonlinear) transformation that models the stationary QT-RR relation.

Recently, we proposed a model-based time lag estimator that is suitable for ECGs recorded during exercise stress testing (EST) [2], where the gradual HR changes observed serves as the input to the estimator. This method is supported by the theoretical definition that a linear trend input to a first-order system generates as output a delayed version of the linear input, with this delay being the system's time constant. However, the observed gradual changes do not follow a perfectly linear trend.

The novelty of this study includes the demonstration that the requirement of a linear HR trend can be relaxed to any change in the trend as long as its frequency content is below a certain frequency $F_c = 1/(2\pi\tau_s)$, where $\tau_s$ is the system time constant in seconds and $F_c$ in Hz. The performance of the estimator is evaluated using a recently proposed simulator to generate a dataset of exercise ECGs.

## II. Hypothesis

The QT-RR model displayed in Fig. 1 describes the proposal in [2] to estimate the QT-RR adaptation time lag. The output $d^i_{\mathrm{QT}}(n)$ of the memoryless transformation, which is derived from the observed RR interval time series $d_{\mathrm{RR}}(n)$ and is obtained from a hyperbolic regression model, is fed to a linear,

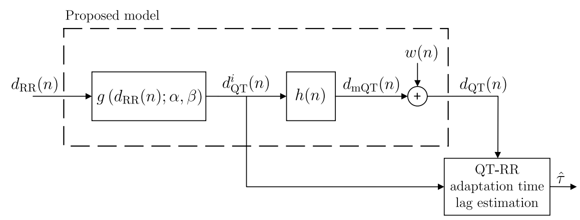


Fig. 1: The model underlying time lag estimation, relating the observed RR series $d_{\mathrm{RR}}(n)$ to the observed QT series $d_{\mathrm{QT}}(n)$.

time-invariant, first-order filter $h(n)$ whose impulse response is given by

$$h(n) = \kappa e^{-n/\tau} u(n), \tag{1}$$

where $\tau$ is the memory time constant, expressed in samples, here considered as the QT-RR adaptation time lag, the constant $\kappa$ is chosen so that the gain is unitary, and $u(n)$ is the unit step function. The output of $h(n)$ is the modeled QT series $d_{\mathrm{mQT}}(n)$, resulting in $d_{\mathrm{QT}}(n)$ once noise $w(n)$ is added accounting for modeling and delineation errors.

The estimated delay between the observed $d_{\mathrm{QT}}(n)$ and the instantaneous series $d^i_{\mathrm{QT}}(n)$, which is assumed to follow a linear trend, is taken as the time lag $\hat{\tau}$.

When $d_{\mathrm{RR}}(n)$ is better characterized by a low-frequency trend, denoted $s(n)$, than by a linear trend, it can be shown that the first-order system $h(n)$ still behaves as a time-delay system provided that the spectral content of $s(n)$ is below a certain frequency.

The discrete-time Fourier transform of $h(n)$ in (1) is

$$H(\omega) = \frac{\kappa}{1 - e^{-1/\tau} e^{-j\omega}}. \tag{2}$$

For healthy subjects $\tau_s \approx 25$ s [1] and, accordingly, the system has a cut-off frequency $F_c = (2\pi\tau_s)^{-1} \approx 0.006$ Hz ($\omega_c \approx 0.01$), so the magnitude function of $H(\omega)$ can be approximated for $\omega \ll 0.01$ by

$$|H(\omega)| = \frac{\kappa}{\sqrt{1 - 2e^{-1/\tau}\cos(w) + e^{-2/\tau}}} \approx \frac{\kappa e^{1/\tau}}{e^{1/\tau} - 1}. \tag{3}$$

For $1/\tau \ll 1$, the phase function $\angle H(\omega)$ is approximated by

$$\angle H(\omega) = -\arctan\left(\frac{\sin(w)}{e^{1/\tau} - \cos(w)}\right) \approx -\frac{\omega}{e^{1/\tau} - 1} \approx -\omega\tau, \tag{4}$$

resulting in the following approximate expression of $H(\omega)$:

$$H(\omega) \approx \frac{\kappa e^{1/\tau}}{e^{1/\tau} - 1} e^{-j\omega\tau}, \tag{5}$$

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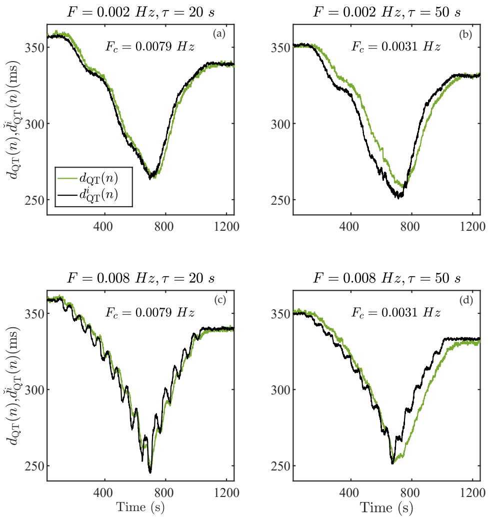


Fig. 2: Examples of $d_{\mathrm{QT}}(n)$ and the instantaneous series $d^i_{\mathrm{QT}}(n)$ for different $\tau$ and $F$.

which is a pure delay for frequencies below $F_c$.

Therefore, in order to estimate $\tau$ by measuring the delay between $d_{\mathrm{QT}}(n)$ and $d^i_{\mathrm{QT}}(n)$, the trend $s(n)$ does not need to be a linear ramp, but it is sufficient that its frequency content is below $F_c$. Then $d^i_{\mathrm{QT}}(n)$ and $d_{\mathrm{QT}}(n)$ can be model as:

$$\begin{aligned} d^i_{\mathrm{QT}}(n) &= s(n) + v^i(n), \\ d_{\mathrm{QT}}(n) &= s(n-\tau) + v(n), \end{aligned} \qquad n = 0, ..., N-1, \tag{6}$$

where both noise components $v^i(n)$ and $v(n)$ account for beat-to-beat uncertainty in R-wave position and delineation errors in Q-wave onset and T-wave end, respectively, with these being statically independent. The integer $N$ is the length of the interval, containing either the exercise or the recovery trend, where the estimation of $\tau$ is to be performed.

## III. Dataset

The dataset contains simulated ECGs defined by a linear trend template of an RR interval pattern mimicking typical EST trends [2] plus an added low-frequency oscillation $F$ during both exercise and recovery. Simulated noise is added to the ECG with a signal-to-noise ratio (SNR) of 40 dB [3]. Ten simulated ECGs for every combination of $\tau$ and oscillation frequency $F$, and with a mean duration of 37 min (basal phases at the beginning and at the end of the EST have a length of 10 min each one) are obtained with values:

$$\tau_s \in \{20, 30, 40, 50\} \text{ s}, \tag{7}$$

$$F \in \{0.002, 0.004, 0.006, 0.008, 0.01\} \text{ Hz}. \tag{8}$$

The range of $\tau_s$ is determined from healthy and pathological subjects, and the range of $F$ is below and slightly above the $F_c$ imposed by $\tau_s$. The method described in [2] is used to estimate $\tau$.

## IV. Results and conclusions

Examples of $d_{\mathrm{QT}}(n)$ and $d^i_{\mathrm{QT}}(n)$ for different $\tau$ and $F$ are shown in Fig. 2. For the case of Fig. 2(d), $F > F_c$ the effect of $h(n)$ results in a smoothed $d_{\mathrm{QT}}(n)$. In such cases, the model in (6) is inappropriate since $s(n)$ is distorted.

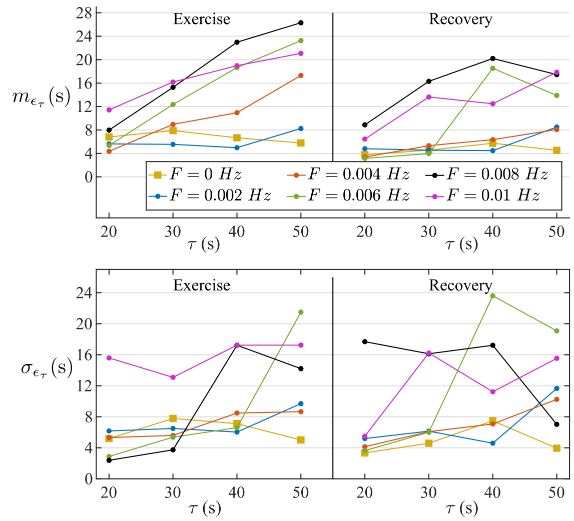


Fig. 3: Error $m_{\epsilon_\tau}$, and $\sigma_{\epsilon_\tau}$, for different $\tau$ and $F$ pairs. Results based on oscillations with full and half amplitude are displayed with solid and dashed lines, respectively.

Defining $\epsilon_\tau$ as the error between the known, simulated time lag $\tau$ and the estimated $\hat{\tau}$, the mean absolute error $m_{\epsilon_\tau}$ and the standard deviation $\sigma_{\epsilon_\tau}$ are computed separately for each pair $(\tau, F)$ during exercise and recovery, see Fig. 3. We can observe that the lowest $m_{\epsilon_\tau}$, corresponding to $\tau_s = 20$s, has the highest cut-off frequency $F_c$; then, for a fixed $\tau$, $m_{\epsilon_\tau}$ increases as the oscillation frequency $F$ is higher than $F_c$. When $\tau$ increases, the error $m_{\epsilon_\tau}$ increases as $F$ becomes larger, influenced by decreasing $F_c$. This behavior almost vanished for $F = 0.002$ Hz, always lower than any $F_c$. The same conclusions, extracted after evaluating $m_{\epsilon_\tau}$, can be made for the analysis in terms of $\sigma_{\epsilon_\tau}$, see Fig. 3.

In a previous work [2], we observed this nonlinear HR trend in series from clinical ECGs. So, the evaluation of the method in the present study shows that estimate the QT adaptation time from EST signals is feasible. Moreover, we have only assessed the methodology for nonlinear HR trend from simulated ECGs with low SNR since we observed a minor effect of the SNR value in the estimation of $\tau$ [4].

In short, the present study demonstrates that the QT adaptation time lag can be estimated from varying HR trend with low-frequency content below $F_c = 1/(2\pi\tau_s)$, which fits well observed HR trends in exercise stress testing.

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