Adaptive Filter for Event-Related Bioelectric Signals Using an Impulse Correlated Reference Input: Comparison with Signal Averaging Techniques

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Abstract—**Many bioelectric signals result from the electrical response of physiological systems to an impulse that can be internal (ECG signals) or external (evoked potentials). In this paper an adaptive impulse correlated filter (AICF) for event-related signals that are time-locked to a stimulus is presented. This filter estimates the deterministic component of the signal and removes the noise uncorrelated with the stimulus, even if this noise is colored, as in the case of evoked potentials. The filter needs two inputs: the signal (primary input) and an impulse correlated with the deterministic component (reference input). We use the LMS algorithm to adjust the weights in the adaptive process. First, we show that the AICF is equivalent to exponentially weighted averaging (EWA) when using the LMS algorithm. A quantitative analysis of the signal-to-noise ratio improvement, convergence, and misadjustment error is presented. A comparison of the AICF with ensemble averaging (EA) and moving window averaging (MWA) techniques is also presented. The adaptive filter is applied to real high-resolution ECG signals and time-varying somatosensory evoked potentials.**

I. Introduction

Among the most well-studied bioelectrical signals are the event-related signals that are time-locked to a stimulus. This stimulus is usually external (visual, auditory, or electrical in the case of evoked potentials). In other cases the signal is related to an internal stimulus. In these cases a time-reference point can be defined from a wave of the same signal, as with *QRS* complex when analysing ECG signals.

Bioelectrical signals are often contaminated by noise from various sources. In general, an event-related signal can be considered as a process which can be decomposed into an invariant deterministic signal time-locked to a stimulus, and an additive noise uncorrelated with the signal. The most common signal processing of this type of bioelectric signal separates the deterministic signal from the noise. Several techniques can be considered. Linear filtering is not possible in general, because the spectrums of signal and noise overlap. The classical ensemble averaging (EA) technique [1] is a method for recovering the signal hidden in the noise, but it needs a large number of records to obtain a good estimation of the signal, and cannot show eventual dynamic variations of the signal shape. Such a time-varying property is very common in evoked potentials (EP) and ECG signals.

The adaptive signal processing technique appears to be appropriate for such time-varying situations [2]–[4]. Adaptive filters are self-designing filters based on an algorithm which allows the filter to "learn" the initial input statistics and to track them if they are time-varying. These filters estimate the deterministic signal and remove the noise uncorrelated with the deterministic signal. The closed-loop adaptive filtering technique has been applied to several biomedical signals: ECG [2], [5], [6] and evoked potentials [7], [8]. In particular, *predictors* [2] were applied to detect His-Purkinje signals and ventricular late potentials [9], [10]. *Predictors* consider that the signal is recurrent and the noise is random and Gaussian. Thus, both inputs of the filter (the primary and the reference signals) are the same, but the former is a delayed version of the latter. This filter removes the muscle noise, but not the periodic 50/60 Hz interference. Another adaptive approach which has been applied to bioelectric signals is *interference cancellation* [2]. Here the reference signal must be a correlated version of the noise that is present in the primary signal. This filter was used to cancel the 50-Hz interference [2] and to detect P-waves in the ECG by *QRS-T* cancellation [11].

In this paper we analyse an adaptive impulse correlated filter (AICF) for event-related signals that we proposed recently [11]–[13]. In particular, this filter can be applied

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to evoked potentials and low-amplitude potentials that are time-locked to a high-amplitude wave of the ECG (late potentials and His-Purkinje potentials). The AICF needs two inputs: the signal (primary input) and another input correlated with the deterministic component (reference input). Both the implementation of the filter and its mathematical expressions become especially simple when the reference input is an impulse and the LMS algorithm is used in the adaptation process.

A study of the signal-to-noise ratio (SNR) improvement achieved with the AICF is presented and compared with the classical EA and with moving window averaging (MWA). We show that this AICF filter, when the LMS algorithm is used in the adaptation process, is equivalent to an exponentially weighted averager (EWA), where the filter can be seen as an averager with a forgetting factor.

We illustrate applications of this filter to the study of high-resolution ECG signals, and in particular to the detection of ventricular late potentials. Next we apply the filter to time-varying somatosensory evoked potentials (SEP) recorded before and after the administration of etomidate anesthetic. Results obtained using the AICF and signal averaging are compared.

II. Methods

A. The Adaptive Filter with an Impulse Reference Input

The adaptive filter has two inputs (Fig. 1). The primary input (d_k) is the cosecutive linking of the N recurrences of the event-related signal we want to filter: Each event-related signal extends the interval of interest following the stimulus and is considered as a record of a random process. The primary input is defined by

$$d_k = s_k + n_k$$

with

$$k = (m-1)L + l \quad \begin{cases} m = 1, \cdots, N & \text{Record number index} \\ l = 1, \cdots, L & \text{Record sample index} \\ L & \text{Number of samples in each record} \end{cases} \tag{1}$$

where s_k is a signal formed by the subsequent linking of the deterministic component of the event-related signal ($s_k = s_{k+L}$). The signal n_k is the additive noise not correlated with s_k and hence not correlated with the stimulus that generates s_k.

The reference input (x_k) of the adaptive filter is a unit impulse sequence synchronized with the beginning of each recurrence of s_k. This impulse sequence x_k can be generated in different ways, depending on the signal d_k that we are processing and the origin of the stimulus that triggers the signal occurrence. A signal detector or a more precise alignment method [14] can define the impulse from the high-amplitude waves in case of ECG signal processing. If the signal is an evoked potential, the stimulus can easily be derived from the external stimulus. Thus, the reference input can be defined as follows:

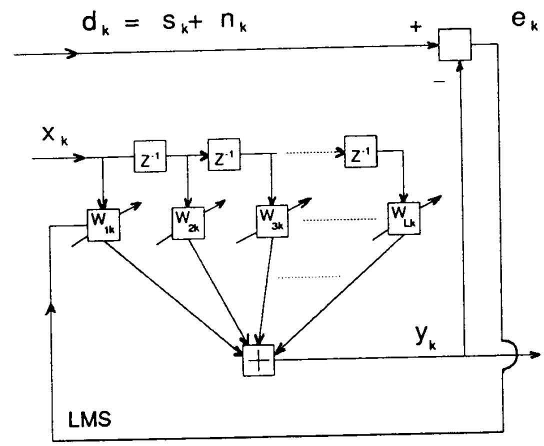

Fig. 1. Block diagram of the adaptive filter to estimate the deterministic component s_k of a signal d_k, using a nonrecursive transversal adaptive filter. x_k is the impulse correlated with the deterministic signal s_k, n_k is noise uncorrelated with s_k, and y_k is the filter output.

$$x_k = x_{(m-1)L+l} = \begin{cases} 1 & l = 1, \forall m \\ 0 & l \neq 1, \forall m \end{cases}. \tag{2}$$

The output of this adaptive filter (y_k) can be expressed, according to the classical notation [2], by

$$y_k = \sum_{i=1}^{L} w_{ik} x_{k-i+1} = \boldsymbol{W}_k^T \boldsymbol{X}_k, \tag{3}$$

where $\boldsymbol{W}_k = [w_{1k} \; w_{2k} \cdots w_{Lk}]^T$ is the weight vector and $\boldsymbol{X}_k = [x_k \; x_{k-1} \cdots x_{k-L+1}]^T$ is the reference vector. The error signal is ϵ_k and the mean-square error (MSE) between the signal under study and the estimated one, can be expressed by

$$\begin{aligned} \xi &= E_t[\epsilon_k^2] = E_t[(s_k - y_k)^2] + E_t[n_k^2] \\ &= E_t[d_k^2] + \boldsymbol{W}^T\boldsymbol{R}\boldsymbol{W} - 2\boldsymbol{P}^T\boldsymbol{W} \\ \boldsymbol{R} &= E_t[\boldsymbol{X}_k\boldsymbol{X}_k^T], \quad \boldsymbol{P} = E_t[d_k\boldsymbol{X}_k] \end{aligned} \tag{4}$$

where $\boldsymbol{R}$ and $\boldsymbol{P}$ are the input correlation matrix and the cross-correlation vector, respectively. In this case x_k is an unit impulse sequence, and so we obtain a simple expression for $\boldsymbol{R}$ and $\boldsymbol{P}$:

$$\boldsymbol{R} = \frac{1}{L}\boldsymbol{I}, \quad \boldsymbol{P} = \frac{1}{L}[s_1 \; s_2 \cdots s_L]^T. \tag{5}$$

Throughout the paper we will use the notation $E[\;]$ in two different ways. When we are referring to a convergence problem, the expectation represents the mean of all the possible values as a function of time for one recurrence of the process (*time* average $E_t[\;]$). This mean does not depend on the chosen recurrence if ergodicity can be assumed; this is the case in (4). On the other hand, when we study the performance of an adaptive algorithm at time instant k (Section II-C), the expectation must be understood as the mean of the possible recurrences of the process at this time instant k (*ensemble* averaging $E_e[\ ]$). The difference between the two expectations lies in the dependence or independence of the result on the time instant k.

The optimum weight vector that minimizes the MSE from (5) is

$$W^* = R^{-1}P = [s_1 \quad s_2 \quad \cdots \quad s_L]^T. \tag{6}$$

In the steady-state the optimum filter output is

$$y_k = W^*X_k = \sum_{i=1}^{L} w_i^* x_{k-i+1} = w_k^* = s_k. \tag{7}$$

When the weight vector $W_k = W^*$ the filter output y_k achieves the deterministic component s_k. Thus, we verify that this AICF filter estimates the component of d_k which is event-related to the stimulus; that is, the component of interest in event-related biomedical signal processing.

From (4), the minimum MSE is

$$\xi_{\min} = E_t[d_k^2] - P^TW^* = E_t[n_k^2]. \tag{8}$$

The error in the steady-state is

$$\xi = \xi_{\min} + \text{Excess MSE} = \xi_{\min}(1 + M), \tag{9}$$

where M is the misadjustment that depends on the adaptive algorithm, which from (5), (8) and (9) can be expressed as

$$M = \frac{E_t[(s_k - y_k)^2]}{E_t[n_k^2]}. \tag{10}$$

B. The Adaptive Algorithm

The least-mean-square (LMS) algorithm [2] is used to adjust the weights of the adaptive filter, in order to minimize the MSE and estimate the deterministic component s_k through the filter output y_k. This algorithm is well known and can be expressed by the following equation:

$$W_{k+1} = W_k + 2\mu\epsilon_k X_k. \tag{11}$$

The condition that assures the convergence of the algorithm is [15]

$$0 < \mu < \frac{1}{3\ \mathrm{tr}[R]} = \frac{1}{3}. \tag{12}$$

The time constant (τ_{mse}) for the convergence of the MSE is [2]

$$\tau_{mse} = \frac{1}{4\mu\lambda} = \frac{L}{4\mu}, \tag{13}$$

where $\lambda = (1/L)$ is the eigenvalue of the matrix R (all the eigenvalues are identical). τ_{mse} is measured in number of sampling periods.

Therefore, the gain constant μ controls the stability and the speed of convergence. Thus, the convergence of weights can be obtained in the first record ($\tau_{mse} < L$) if an appropriate value of μ is selected. This possibility will be very useful for tracking recurrence-to-recurrence variations in the event-related signal.

The misadjustment M can be approximated following the method in [2] as

$$M \simeq \mu \mathrm{tr}[R] = \mu, \quad \text{which implies} \quad \xi = E_t[n_k^2](1 + \mu). \tag{14}$$

Given the special characteristics and simplicity of this filter, we will later consider the exact expression for M and will show that $M \simeq \mu$ is a good approximation to the exact expression, when $\mu << 1$.

The selection of μ becomes a tradeoff between the convergence rate and steady-state MSE. One possibility for resolving this tradeoff, in stationary signals, has been studied in [16], where the optimum μ value selection is made according to the criterion of selecting the μ which yields the smallest misadjustment error at the end of the observation interval. If there are transient changes in the deterministic component, it will be better to select a large μ value to assure the estimation of these transient changes. If the deterministic component changes only progressively, a smaller μ value can be selected to reach a better steady-state estimate.

C. Signal-to-Noise Ratio (SNR) Improvement

In order to determine the performance of this filter, the signal-to-noise ratio (SNR) improvement in the case of a stationary deterministic signal s_k will be studied. First, we define the SNR at the primary input signal d_k as

$$\mathrm{SNR}_d = \frac{E_t[s_k^2]}{E_t[n_k^2]} = \frac{E_t[s_k^2]}{E_t[(d_k - s_k)^2]}. \tag{15}$$

The objective of this section is to find the relationship between the SNR at the input signal d_k and the SNR at the output signal y_k. When the algorithm has not yet converged to the steady-state, the output signal y_k can be considered to be composed of a signal component (s_k') that is correlated with the deterministic signal s_k and a noise component (n_k') uncorrelated with s_k. That is

$$y_k = s_k' + n_k'. \tag{16}$$

Given that the process has not yet converged, the first component s_k' will not be s_k but rather a signal correlated with s_k. Accordingly we define the SNR at the output signal y_k as

$$\mathrm{SNR}_y = \frac{E_t[s_k'^2]}{E_t[n_k'^2]}. \tag{17}$$

Note that $E_t[s_k'^2]$ and $E_t[n_k'^2]$ are local *time* averages; these will be considered later. To find the relation between SNR_y and SNR_d it is first necessary to obtain s_k' and n_k'. We define

$$s_k' = E_e[y_k] \quad \text{and} \quad n_k' = y_k - E_e[y_k] = y_k - s_k', \tag{18}$$

where $E_e[y_k]$ is the expected value of y_k at discrete time k when we repeat the adaptation process for different noise sequences and the same deterministic component. In fact we will see that s_k' defined in this way is the component of y_k correlated with s_k and n'_k is the remainder noise. In Appendix A we will demonstrate that

$$s'_k = \left(1 - \left(1 - 2\frac{\mu}{L}\right)^k\right)s_k. \quad (19)$$

This relation proves that $s'_k = E_e[y_k]$ is related to s_k. In fact, it is proportional to s_k, which is the signal we want to recover. In the steady state ($k \to \infty$) s'_k becomes s_k, since by the convergence condition (12): $\mu < L$; thus $\lim_{k\to\infty} (1 - 2\mu/L)^k = 0$, and $\lim_{k\to\infty} s'_k = s_k$.

Let us now study n'_k. In Appendix B we prove that this residual noise is not correlated with s_k. Thus, n'_k is the noise that contaminates the filter estimation.

To evaluate SNR_y as a function of SNR_d it is necessary to determine the relation of $E_t[s'^2_k]$ and $E_t[n'^2_k]$ with $E_t[s^2_k]$ and $E_t[n^2_k]$, respectively. The dependence of s'_k with s_k has already been discussed, and from (19) we define the local *time* average $E_t[s'^2_k] = (1 - (1 - 2[\mu/L])^k)^2\, E_t[s^2_k]$. In Appendix B we show that if ergodicity is assumed ($E_t[n'^2_k] = E_e[n'^2_k]$), the relation between $E_t[n'^2_k]$ and $E_t[n^2_k]$ is given by

$$E_t[n'^2_k] = \boldsymbol{M} E_t[n^2_k]\left(1 - \left(1 - 2\frac{\mu}{L}\right)^{2k}\right). \quad (20)$$

With this expression, we can now evaluate the SNR_y defined in (17), obtaining

$$\text{SNR}_y = \frac{E_t[s'^2_k]}{E_t[n'^2_k]} = \frac{\text{SNR}_d\left(1 - \left(1 - 2\frac{\mu}{L}\right)^k\right)^2}{\boldsymbol{M}\left(1 - \left(1 - 2\frac{\mu}{L}\right)^{2k}\right)} \quad (21)$$

where SNR_d is defined in (15). From this expression we can define the improvement of SNR ($\Delta\,\text{SNR}_y$) as

$$\Delta\text{SNR}_y = \frac{\text{SNR}_y}{\text{SNR}_d} = \frac{1}{\boldsymbol{M}}\frac{\left(1 - \left(1 - 2\frac{\mu}{L}\right)^k\right)^2}{\left(1 - \left(1 - 2\frac{\mu}{L}\right)^{2k}\right)}. \quad (22)$$

This expression can be reformulated by considering k to be an integral number of occurrences of the event-related signal ($k = NL$) where N is the number of occurrences:

$$\Delta\text{SNR}_y = \frac{1}{\boldsymbol{M}}\frac{\left(1 - \left(1 - 2\frac{\mu}{L}\right)^{NL}\right)^2}{\left(1 - \left(1 - 2\frac{\mu}{L}\right)^{2NL}\right)}. \quad (23)$$

In this expression when $N \to \infty$ the steady-state improvement of SNR is reached. Since the convergence condition assures $|1 - 2\mu/L| < 1$, we have that

$$\lim_{N\to\infty} \Delta\text{SNR}_y = \frac{1}{\boldsymbol{M}} = \frac{1}{\mu}. \quad (24)$$

The convergence condition assumes $2\mu/L << 1$ so we can approximate the expression of ΔSNR_y by a first order Taylor expansion, which gives

$$\Delta\text{SNR}_y \simeq \frac{1}{\boldsymbol{M}}\frac{(1 - (1 - 2\mu)^N)^2}{(1 - (1 - 2\mu)^{2N})}. \quad (25)$$

When N is small enough to satisfy $\mu N << 1$, we can approximate $(1 - 2\mu)^N \simeq (1 - 2\mu N)$ in which case $\Delta\text{SNR}_y \simeq N$. This approximation for small values of N leads to the same results as using a classical EA after averaging a number N of records [1]. However, in classical EA the improvement of SNR increases indefinitely with N, while in this filter the SNR reaches a constant value $1/\boldsymbol{M}$. The advantage of this AICF is the capability of adaptation to dynamic changes in the deterministic signal s_k. EA does not adapt to such changes.

In Fig. 2 we see the evolution of ΔSNR_y for different values of μ as a function of the number N of signal recurrences. We can see that initially ($N \to 0$) all the curves have unit slope $\Delta\text{SNR}_y(N) = N$ and they reach a constant value $1/\boldsymbol{M} = 1/\mu$.

D. The Adaptive Filter, Using the LMS Algorithm, as an Exponentially Weighted Averager (EWA)

The behavior of this adaptive filter using the LMS algorithm can be analysed from a different point of view. We will show that the output y_k is equivalent to an exponentially weighted average of the input signal recurrences, with a forgetting factor that multiplies each recurrence of the signal in an exponentially decreasing fashion.

To show this equivalence, we make use of the LMS algorithm expression (11) for each weight i of the vectorial equality and considering the expression of the error ϵ_k, we can write

$$w_{i(k+1)} = w_{ik} + 2\mu(d_k - y_k)x_{k-i+1} \qquad i = 1, \cdots L. \quad (26)$$

Until now we have specified the discrete time index as extending from $k = 1$ to ∞. The index k is a sample index that can be expressed as a function of the recurrence number m and the sample index in each recurrence l: $k = (m - 1)L + l$. Thus, when we refer to time instants in different recurrences, we will denote by $d^m_l = d_{k=(m-1)L+l}$, $y^m_l = y_{k=(m-1)L+l}$, $w^m_{il} = w_{ik=(m-1)L+l}$, $\cdots$, to refer to values at the time instant l ($l = 1, \cdots, L$) of the mth signal recurrence. As s^m_l is assumed to be independent of m (its value is constant for all m recurrences) we will omit this dependence in these signals and denote them by s_l.

The values of the weights w^m_{il} change only once in each recurrence, when $i = l$, since the adaptation occurs only when $x_{k-i+1} \neq 0$. If we have N recurrences of the signal ($m = 1, \cdots, N$) we can rewrite expression (26) as a function of the recurrence number m for each weight i:

$$w^{m+1}_{ii} = w^m_{ii} + 2\mu(d^m_i - y^m_i) \qquad \begin{cases} i = 1, \cdots, L \\ m = 1, \cdots, N \end{cases}. \quad (27)$$

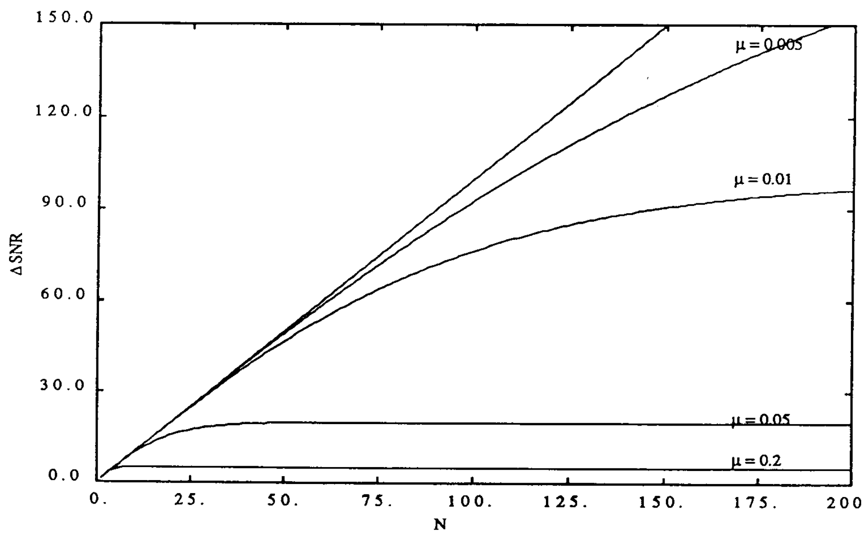

Fig. 2. Improvement of SNR as a function of the number of recurrences (N) for the AICF filter and classical EA. ΔSNR_y as function of N (1, 200) for different values of μ written upon the graph. We can compare the improvement of the adaptive filter with that of classical EA (straight line of unitary slope).

In this expression we know that

$$y_l^m = \sum_{i=1}^{L} w_{il}^m x_{k-i+1} = w_{ll}^m \qquad k = (m-1)L + l. \tag{28}$$

We will write ω_i^m to mean the weight i in the mth recurrence, given that the adaptation of each weight i only occurs once in each recurrence m (when $i = l$). Thus we omit one unnecessary subscript, since w_{il}^m is constant $\forall l$. Composing (27) and (28) we have the adaptation expression as

$$w_i^{m+1} = w_i^m + 2\mu(d_i^m - w_i^m) \tag{29}$$

where i indexes the sample at each of the m recurrences (d_i^m) and the weight under consideration (w_i^m). In this way we have isolated the study of the signal recurrence in the study of the sample i (d_i^m) of each m recurrence. Note how each weight i is now associated only with the $l = i$ sample of the signal recurrences. This is a consequence of the correlated impulse used as a reference input in this adaptive filter.

If we express the recursive relation (29) as a function of the initial weight w_i^1, which is initialized to zero ($w_i^1 = 0$), then

$$w_i^{N+1} = \sum_{m=1}^{N} 2\mu(1 - 2\mu)^{N-m} d_i^m. \tag{30}$$

From (28) and noting that $y_i^m = w_i^m$, we obtain

$$y_l^{N+1} = \sum_{m=1}^{N} 2\mu(1 - 2\mu)^{N-m}(s_l + n_l^m). \tag{31}$$

The index i has been changed to l since now it refers only to samples in each record, and l is the index generally used in this case.

From the expression (31) we see that y_l^N is a weighted average of the lth sample of each signal recurrence m. The weight factor $2\mu(1 - 2\mu)^{N-m}$ decreases when $N - m$ increases as long as $|1 - 2\mu| < 1$ (here again, the convergence condition appears). This equivalence with the weighted averager implies that the adaptive filter with an impulse reference input and using the LMS algorithm is a linear filter (31). However, this fact is not true in general for all adaptive filters.

Regarding (31) we see that the output at time instant l of the $(N + 1)$th recurrence, depends only on the previous $d_l^m = s_l + n_l^m$ ($m = 1, \cdots, N$) signal recurrences at the same time instant l. Thus the requirement on the noise to be filtered is that it must be uncorrelated between the signal recurrences. It does not need to be white noise to satisfy the theoretically predicted filter performance. This is very important in signals like evoked potentials in which it is well known that the noise, usually the background EEG, is a highly correlated signal in each record. Taking the expected value of (31) for each sample l, we obtain

$$E_l[y_l^{N+1}] = 2\mu s_l \sum_{m=1}^{N} (1 - 2\mu)^{N-m}$$

$$= 2\mu s_l \frac{1 - (1 - 2\mu)^N}{1 - (1 - 2\mu)}. \tag{32}$$

If the convergence condition is satisfied then $E_l[y_l^\infty] = s_l$. This means that the weighted average, when the number of recurrences is large enough, converges to the deterministic signal component s_l. This result is the same obtained in Section II-A [see (7)].

Finally, we will deduce the improvement in the SNR with this methodology from (31) and compare the results with the expression obtained through the adaptive formalism (Section II-C). If we take (31) and substitute d_l^m as a function of s_l and the noise n_l^m, we obtain

$$y_l^{N+1} = \sum_{m=1}^{N} 2\mu(1 - 2\mu)^{N-m}(s_l + n_l^m) = s_l'^{N+1} + n_l'^{N+1} \quad (33)$$

where

$$n_l'^{N+1} = \sum_{m=1}^{N} 2\mu(1 - 2\mu)^{N-m} n_l^m \quad (34)$$

is the residual noise, and

$$s_l'^{N+1} = \sum_{m=1}^{N} 2\mu(1 - 2\mu)^{N-m} s_l = 2\mu \frac{1 - (1 - 2\mu)^N}{1 - (1 - 2\mu)} s_l. \quad (35)$$

Taking again the definition of SNR_y and considering the expected values of the signals, and assuming that the noise n_l^m is stationary, has zero-mean and is uncorrelated between recurrences (although it is allowed to be colored), we have

$$E_t[n_l^m n_l^{m'}] = \begin{cases} 0 & m \neq m' \\ E_t[n_l^2] & m = m' \ \forall m, m' \end{cases} \quad (36)$$

from which we obtain that

$$\text{SNR}_y = \frac{E_t[(s_l'^{N+1})^2]}{E_t[(n_l'^{N+1})^2]} = \frac{4\mu^2(1 - (1 - 2\mu)^N)^2(1 - (1 - 2\mu)^2)E_t[s_l^2]}{4\mu^2(1 - (1 - 2\mu))^2(1 - (1 - 2\mu)^{2N})E_t[n_l^2]} \quad (37)$$

and calculating ΔSNR_y yields

$$\Delta\text{SNR}_y = \frac{(1 - \mu)}{\mu} \frac{(1 - (1 - 2\mu)^N)^2}{(1 - (1 - 2\mu)^{2N})}. \quad (38)$$

This expression is the same as (25) except for a factor $(1 - \mu)$. This derivation of SNR_y is exact and does not include any approximation. However in (25) we used the value of $M \simeq \mu tr[\boldsymbol{R}]$ for the LMS algorithm given in [2] which includes the approximation of $\mu << 1$. If we consider now the same approximation, we recover the expression for ΔSNR_y that was obtained in (25). In the steady-state case ($N \to \infty$) we will have

$$\lim_{N \to \infty} \Delta\text{SNR}_y = \frac{1 - \mu}{\mu}. \quad (39)$$

This expression is the exact improvement of SNR in the steady-state, while (24) is an approximated expression for this SNR improvement.

E. Misadjustment

The value of $\boldsymbol{M}$, given approximately in [2] ($\boldsymbol{M} \simeq \mu tr[R]$), can be calculated exactly in this case because of the particular simplicity of the reference input x_k. To calculate exactly the $\boldsymbol{M}$ value we need to obtain the *Excess MSE*. From (4) and (9),

$$\text{Excess MSE} = E_t[(s_k - y_k)^2] = -E_t[s_k^2] + E_t[(\boldsymbol{W}_k^T \boldsymbol{X}_k)^2]. \quad (40)$$

Since $y_k = \boldsymbol{W}_k^T \boldsymbol{X}_k$, and in the steady-state y_k converges to s_k, we have $E_t[s_k y_k] = E_t[s_k^2]$. Considering that x_{k-i+1} is nonzero only for $k = mL + i$, and that in these cases it takes the value of 1, we can write

$$E_t[(s_k - y_k)^2] = -E_t[s_k^2] + E_t[\boldsymbol{W}_k^T \boldsymbol{W}_k]. \quad (41)$$

From expression (30) with $d_i^m = s_i + n_i^m$ we obtain

$$w_i^{N+1} = \sum_{m=1}^{N} 2\mu(1 - 2\mu)^{N-m}(s_i + n_i^m) \quad (42)$$

and now we can calculate

$$E_t[\boldsymbol{W}_k^T \boldsymbol{W}_k] = E_t\left[\sum_{i=1}^{L} (w_i^{N+1})^2\right]. \quad (43)$$

Assuming again that the noise is uncorrelated between recurrences [see (36)], stationary, and zero-mean, we have

$$E_t[\boldsymbol{W}_k^T \boldsymbol{W}_k] = 4\mu^2 \left(\frac{1 - (1 - 2\mu)^N}{1 - (1 - 2\mu)}\right)^2 E_t[s_k^2] + 4\mu^2 \left(\frac{1 - (1 - 2\mu)^{2N}}{1 - (1 - 2\mu)^2}\right)^2 E_t[n_k^2]. \quad (44)$$

In the steady-state ($N \to \infty$) this expression, with the convergence condition ($\mu < 1$), becomes

$$E_t[\boldsymbol{W}_k^T \boldsymbol{W}_k] = E_t[s_k^2] + \frac{\mu}{1 - \mu} E_t[n_k^2]. \quad (45)$$

Substituting this expression into (41) we get

$$E_t[(s_k - y_k)^2] = \frac{\mu}{1 - \mu} E_t[n_k^2] \quad (46)$$

and from (8) and (9), we have

$$\boldsymbol{M} = \frac{\text{Excess MSE}}{\xi_{\min}} = \frac{\mu}{1 - \mu}. \quad (47)$$

This is the exact expression of the misadjustment $\boldsymbol{M}$ for this adaptive filter. We see that if we introduce in (24) this exact value of $\boldsymbol{M}$, we recover the expression for ΔSNR_y in the steady-state given in (39). Finally, if we assume $\mu << 1$ we obtain the result given in [2] where $\boldsymbol{M} \simeq \mu tr[\boldsymbol{R}]$.

F. Comparison of the Adaptive Filter with a Moving Window Averager (MWA)

The AICF and the moving window averager (MWA), which considers a constant number of the most recent recurrences, are two different approaches to tracking dynamic changes in the deterministic signal component. Moreover, the MWA technique is actually implemented in many monitoring devices. So in this section, a comparative study of performance of the AICF and the MWA is presented in terms of signal-to-noise ratio improvement (ΔSNR) and tracking capability.

Using the same notation as in Section II-D, we can express the output of the MWA ($y1_l^{N+1}$) at the $N + 1$ recurrence as

$$y1_l^{N+1} = \frac{1}{A} \sum_{m=N-A+1}^{N} d_l^m \quad (48)$$

where A is the constant number of the most recent recurrences that we consider in the averaging process of the MWA filter.

To compare the capabilities of the AICF and the MWA to track dynamic changes we will assume the deterministic signal to be free of noise. We consider that at the first N recurrences the deterministic signal is s_l, and starting with the $(N + 1)$ recurrence there appears a new and different deterministic component $\tilde{s}_l$ during the next M recurrences.

$$d_l^m = \begin{cases} s_l & m = 1, \cdots, N \\ \tilde{s}_l & m = N + 1, \cdots, N + M \end{cases}. \quad (49)$$

After these $N + M$ recurrences $y1_l^{N+M+1}$ will be

$$y1_l^{N+M+1} = \frac{A - M}{A} s_l + \frac{M}{A} \tilde{s}_l \quad (50)$$

with $A > M$, otherwise $y1_l^{N+M+1} = \tilde{s}_l$.

The output of the AICF ($y2_l^{N+M+1}$) from (31) is

$$y2_l^{N+M+1} = 2\mu(1 - 2\mu)^M \frac{1 - (1 - 2\mu)^N}{1 - (1 - 2\mu)} s_l + 2\mu \frac{1 - (1 - 2\mu)^M}{1 - (1 - 2\mu)} \tilde{s}_l. \quad (51)$$

The outputs $y1_l^{N+M+1}$ and $y2_l^{N+M+1}$ are both linear combinations of s_l and $\tilde{s}_l$. In order to compare the filters (AICF and MWA) we will now consider the situation where the ratio between s_l and $\tilde{s}_l$ is the same in the two output signal filters, $y1_l^{N+M+1}$ and $y2_l^{N+M+1}$, for a given N and M. This situation gives the same dynamic tracking of the deterministic signal, at recurrence $N + M$, in both filters. Under this restriction (same tracking) we study the ΔSNR_y in both cases and we determine which filter has the better SNR improvement with the same tracking capability. From (50) and (51) we see that the same dynamic tracking is achieved in both filters when

$$\frac{A - M}{M} = \frac{(1 - 2\mu)^M(1 - (1 - 2\mu)^N)}{1 - (1 - 2\mu)^M} \quad (A > M) \quad (52)$$

which is a relation between the filter's defining parameters μ and A for a given N and M.

Now we will analyse in this situation what is the ΔSNR_y, in steady-state, for each of the two filters. For the MWA we have the well known SNR improvement of a classical EA with A recurrences, and from (52),

$$\Delta\text{SNR}_y^{\text{MWA}} = A = \frac{M(1 - (1 - 2\mu)^{N+M}}{1 - (1 - 2\mu)^M}. \quad (53)$$

In the steady-state ($N \to \infty$),

$$\Delta\text{SNR}_y^{\text{MWA}} = \frac{M}{1 - (1 - 2\mu)^M}. \quad (54)$$

Note that $\Delta\text{SNR}_y^{\text{MWA}}$ depends on μ and M because of the condition imposed in (52). For the AICF, when $N \to \infty$, we have from (39)

$$\Delta\text{SNR}_y^{\text{AICF}} = \frac{1 - \mu}{\mu}. \quad (55)$$

The difference between $\Delta\text{SNR}_y^{\text{AICF}}$ and $\Delta\text{SNR}_y^{\text{MWA}}$ is a function of μ and M, $f(\mu, M)$:

$$f(\mu, M) = \Delta\text{SNR}_y^{\text{AICF}} - \Delta\text{SNR}_y^{\text{MWA}} = \frac{1 - \mu}{\mu} - \frac{M}{1 - (1 - 2\mu)^M}. \quad (56)$$

In Fig. 3 we plot this function of μ for different values of a required M. We can see that in quickly tracking dynamic changes (small M) the AICF has better performance in the steady-state ($f(\mu, M) > 0$) for a wide range of μ. When M becomes higher, only small values of μ give better performance to the AICF than to the MWA. This is logical since higher M implies that $\tilde{s}_k$ has higher relative contribution in the A recurrences considered by MWA. If we are interested in rapidly tracking changes (small M), the AICF required for this has a better steady-state performance than the equivalent MWA. In addition, the AICF is more efficient and easy to implement than the MWA (which needs to store A signal recurrences).

III. Simulation Study

A simulation study has been carried out to test the performance of the adaptive filter. A signal was synthesized as a sequence of records d_k. Each one consisted of the same *QRS* complex (s_k), taken from a real ECG signal, and additive Gaussian random noise (n_k). Also, a reference signal (x_k) was defined as an impulse at the beginning of each record.

The adaptive filter was applied to this signal d_k. Several SNR values were studied, and different values of the gain constant μ were applied. The adaptive filter and the classical signal averaging technique were compared.

Fig. 4 shows the results for SNR = 10 dB, after different numbers of adaptations (N). At the top, we can see the deterministic component s_k which is present in each beat. The second row shows different records with the same SNR_d. The third row displays the signal estimated by classical EA after processing N beats. Next, the estimation of the deterministic signal by means of the AICF is shown for different values of gain constant μ and after N records. The signals' duration was 200 ms and it was sampled at 1 kHz, which implies a value of $L = 200$.

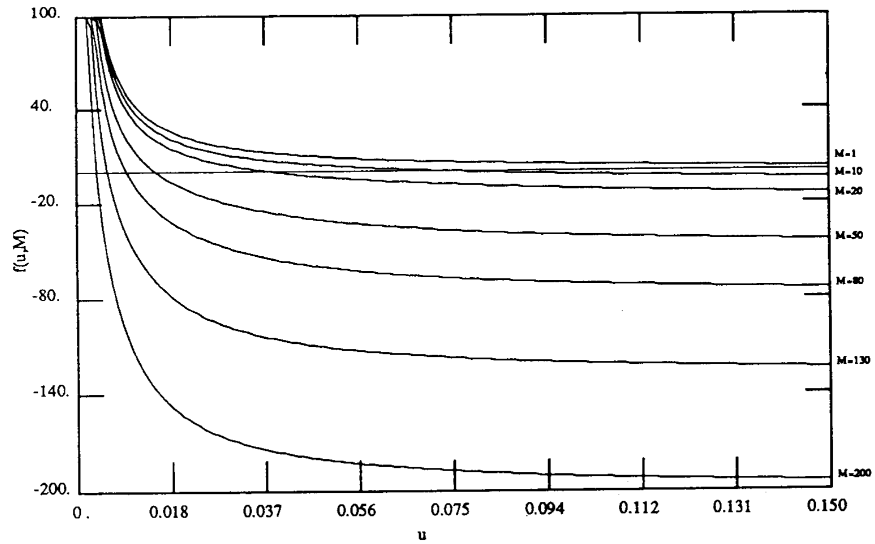

Fig. 3. Comparison of the AICF and MWA filters. Function $f(\mu, M) = \Delta\text{SNR}_y^{\text{AICF}} - \Delta\text{SNR}_y^{\text{MWA}}$ represents the difference in the improvement of SNR for the AICF and MWA filters, with the same dynamic tracking capability as a function of the factor μ and the number of recurrences (M). M is the number of recurrences of the new deterministic signal s_k', beyond that we reach the same dynamic tracking in both filters.

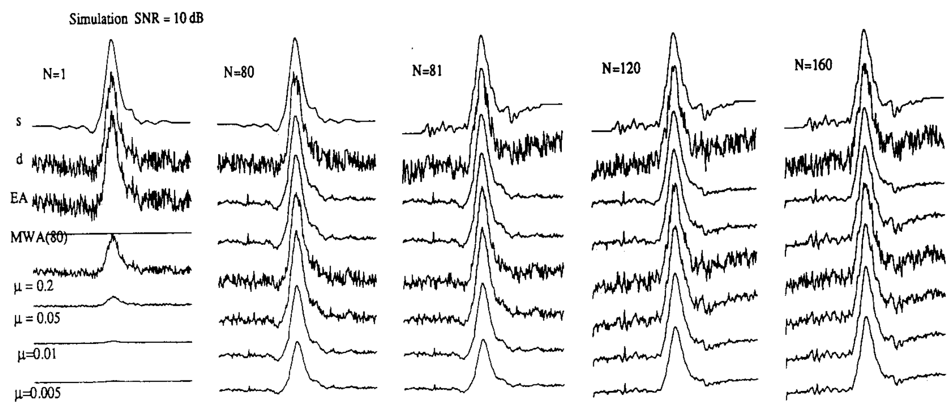

Fig. 4. Results for 160 beats (d_k) with a change of the shape of s_k to s_k' at beat number 81. The first row displays the deterministic components s_k (column $N = 1$, 80) and s_k' (column $N = 81$, 120, 160); in the second row are the 160 records d_k generated by adding noise to s_k and s_k' with a SNR = 10 dB. In the third row are the results after classical EA of N records, and the next row shows the output of the MWA filter with a constant number of recurrences (80). The following rows show the AICF filter output after processing N records for different μ values.

Calculated values of ΔSNR agree with the results obtained in the simulation study. Thus, for example, a value $\mu = 0.01$ leads to a ΔSNR = 99, in the steady-state, with a convergence time of 25 records. In this case, adaptive filtering and classical signal averaging produced comparable results, and thus we can verify that the filter converges to the deterministic component under ideal conditions.

The first 80 records have a deterministic component s_k, and the next 80 records have another s_k' (first row of Fig. 4). Here the AICF performs better than classical EA, because it can learn more quickly the new s_k'. We can observe this by comparing the results after 120 beats with classical EA and AICF. Also comparing results with MWA (fourth row in Fig. 4 with $A = 80$) we see how its performance in this case is similar to that of AICF with $\mu = 0.01$.

IV. Applications

A. Ventricular Late Potentials

The adaptive filter described in this paper has been applied to high resolution ECG signals in order to detect ventricular late potentials (LP). Several μ values were tested and a comparison with classical signal averaging was also carried out. The ECG signals were measured by

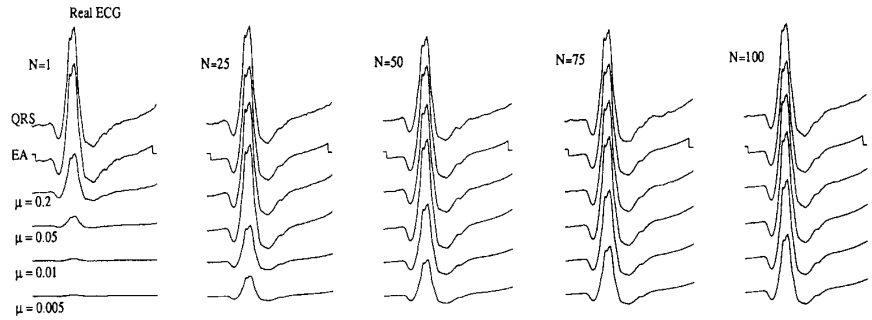

Fig. 5. Application to real high-resolution ECG signals (100 beats). The first row displays the high-resolution ECG signal recurrences. In the second row are the results after classical EA of N records. The following rows show the AICF filter output after processing N records for different μ values.

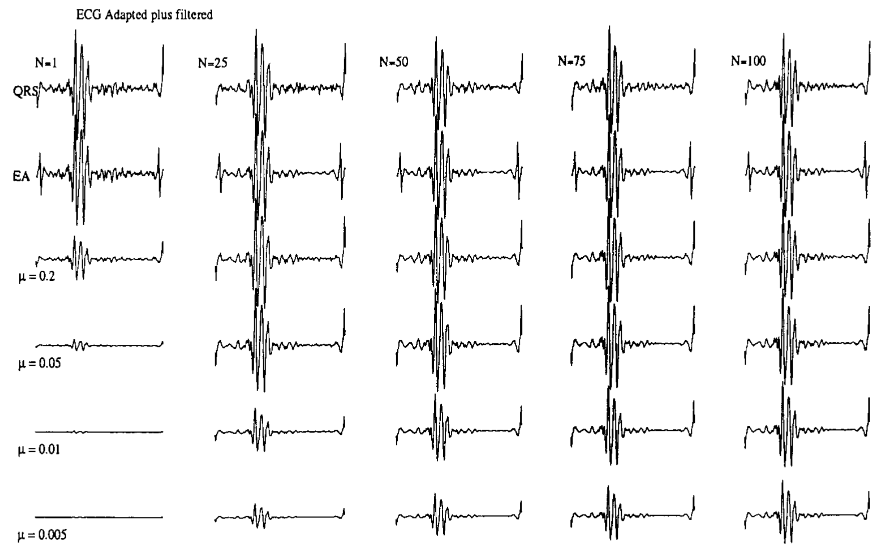

Fig. 6. Band-pass filtered versions of the signals displayed in the previous figure with a FIR filter of bandwidth 50–250 Hz. The time scale is the same as in Fig. 5 and the y-axis scale is renormalized in order to show late potentials.

low-noise high-gain isolated amplifiers and recorded with a PC-based digital acquisition system. The signals were sampled at 5 kHz, with a resolution of 16 bits. The uncorrected orthogonal leads (X, Y, Z) were used in this study. These three bipolar leads were independently processed by both the adaptive filter and classical signal averaging. In both cases, a matched filter [17] was used to align *QRS* complexes and to define the impulse signal x_k synchronized with the beats.

Fig. 5 shows, in the first row, a sequence of records that include the *QRS* complex and the ST segment from an ensemble of 100 cardiac beats. These signals were recorded from the X lead in a patient who previously had had ventricular tachycardia and hence was a candidate to show LP. The results after averaging and adaptive filtering are displayed in the same manner as the simulation results. The records extend 300 ms sampled at 5 kHz, which gives a value of $L = 1500$.

Fig. 6 shows the signals corresponding to the ECG's in Fig. 5, after band-pass filtering with a FIR filter with bandwidth 50–250 Hz. In this filtered signal, we can see remarkable late potentials estimated by EA and AICF filtering. The deterministic component appears practically constant. Thus, classical EA [18] achieves an excellent signal estimation from the first 25 beats. The AICF also obtains comparable estimates. However, the AICF would be more sensitive to dynamic beat-by-beat variations than EA. The slower the signal variations, the more accurate the variable signal estimated by the AICF will be.

B. Somatosensory Evoked Potentials

As in the previous section, the filters have been applied to somatosensorial evoked potentials (SEP). The electrical stimulus is a current of 20 mA given at the rate of 5.9/s. The response is recorded from 40 ms before until 40 ms after the stimulus with a sampling frequency of 3200 Hz. Thus, in each SEP we have the first part with only the EEG signal and the second part with the EEG +

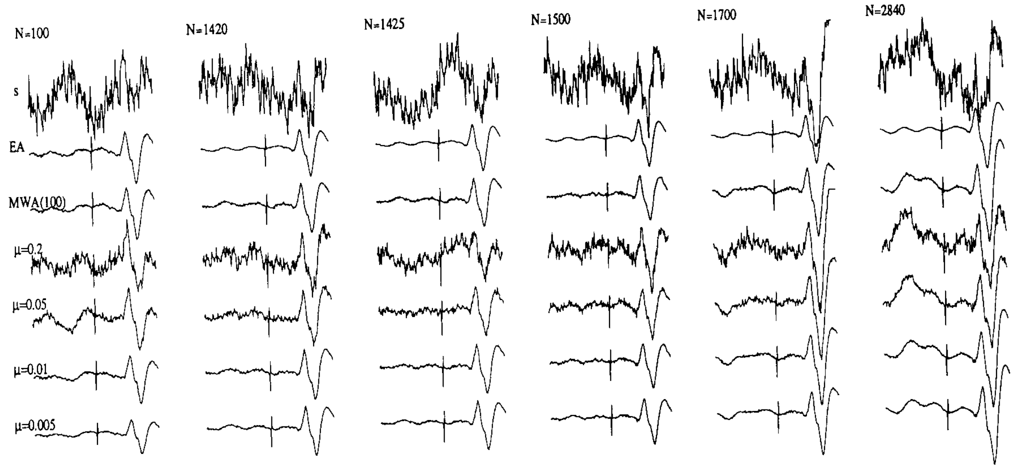

Fig. 7. Application to time-varying somatosensory evoked potentials (2840 recurrences) from a subject with application of etomidate in the 1420 recurrence. The records extend 80 ms, 40 ms previous to the electrical stimulus (20 mA) and 40 ms after the stimulus. The first row displays the SEP recurrences. In the second row are the results after classical EA of N records, and in the third row is the output of the MWA filter with a constant number of recurrences (100). The following rows are the AICF filter output after processing N records for different μ values.

SEP. After 1420 recurrences, etomidate (0.2 mg/kg) was administered to the subject and an additional 1420 SEP recurrences were recorded. The SEP response changed, and we can see in Fig. 7 how the AICF adapts to the dynamic changes. In this figure we have in the first row different recurrences of the SEP. In the second row are the results after classical EA, in the third we have the results obtained with the MWA with $A = 100$ recurrences, and in the following rows are the AICF results for different values of the μ parameter. We note that, in this case, the AICF tracks the dynamic changes in the deterministic signal ($N = 1700$, $\mu = 0.2, 0.05$) better than the MWA (MWA(100)) and the classical EA.

V. Conclusions

An adaptive filter for event-related bioelectric signals has been proposed. This filter estimates the deterministic component of the signal, removing the noise uncorrelated with an impulse time-locked to the deterministic signal. The theoretical analysis of the filter was presented including the convergence, the improvement of the SNR, and the misadjustment, in the case of stationary signals. The adaptive filter AICF using the LMS algorithm has been shown to be equivalent to a weighted averager. If other adaptive algorithms, such as the recursive least squares algorithm [2], were considered then the performance would be different (convergence, misadjustment, . . .) and the result would not be equivalent to an exponentially weighted averager. The equivalence of the AICF to a weighted averager, in the case of the LMS algorithm, shows that this filter is a linear filter.

The misadjustment in this case is calculated exactly, rather than approximately, with the usual notation as given in [2]. For values of μ near 1/3 (the highest allowed value), the difference between the exact $\boldsymbol{M}$ and the approximation could be very important. Hence this exact result is of great interest when estimating the steady-state improvement of the SNR in stationary signals.

The relative improvement in the SNR, or ΔSNR, is a function of the parameter μ. In the steady-state it takes the value $\Delta\text{SNR} = (1/\boldsymbol{M}) = [1 - \mu]/\mu)$. On the other hand, the convergence time ($\tau_{mse} = [L/4\mu]$) also depends on the parameter μ. Thus, the choice of μ must be a compromise between the convergence time and the SNR improvement. In biomedical event-related signals (ECG, EP) many times we are interested in tracking the signal variations. If we have some *a priori* knowledge of the expected change rate, we can select the minimum μ that leads to a τ_{mse} lower than the expected number of recurrences in which the change is supposed to be completed. If we do not expect variations we can select the μ according to the criteria used in [16]. If we do not have any *a priori* information we can select the μ value as a function of the more relevant clinical requirement in each case. In critical care or surgery units, detection of changes in evoked potential response will be critical, hence a μ that gives a quick response time will be desirable. In late potentials detection where the ECG signals are recorded at rest (we do not expect big signal changes), a low steady-state error will be required and then the μ value must be selected according to the desired SNR improvement.

Comparing this adaptive filter with a moving window averager, we find that the adaptive filter AICF, in the case of the LMS algorithm, achieves better steady-state performance (ΔSNR) than the MWA, for the same capability of fast change tracking (small M values in Fig. 3).

The simulation results agree with this theoretical study. The adaptive filter shows better performance than classical EA and MWA when the signal presents dynamic variations. On the other hand, if the signal keeps constant, the different techniques obtain comparable results.

The presented filter appears to be a good way to detect LP, and it also allows us to track the dynamic variations of the signal. The AICF filter was also applied to somatosensory evoked potentials with dynamic changes in the deterministic component. The AICF exhibits better behavior than the MWA for quick detection of these changes. This AICF filter, due to its impulse reference input, has been proved to estimate the deterministic signal, with noise which is colored, zero-mean, and uncorrelated between records. This is the case in EP signals, where the noise is usually colored but uncorrelated between recurrences (background EEG). We have also shown that this AICF reaches a steady-state improvement of SNR which depends on μ. So, if background activity changes the SNR at the input signal (common in EP), the SNR at the output will change in the same ratio.

Another aspect of this filter concerns the impulse reference input. If this impulse x_k is not well synchronized with the onset of the deterministic signal recurrence d_k, a filter effect appears in the signal estimation [19]. In the case of a Gaussian distributed error in the impulse definition, with a standard deviation σ, the filter effect is a low-pass effect [19] which has a cutoff frequency of $f_c = 132.3/\sigma$. In evoked potentials this error can be due to the synchronization with the external trigger or to the physiological delay between the trigger and the transient response (latency). If this physiological delay is random between recurrences, there appears the previously mentioned low-pass effect. If this delay appears at some time and then remains consistent, it must be seen as a signal change, and so, it will be detected if the μ has been correctly selected.

Appendix A

In this Appendix we will show that the signal component s'_k defined in Section II-C, equation (18), is related to the deterministic signal component s_k as described in equation (19).

Using (3) we write s'_k as follows:

$$s'_k = E_e[y_k] = E_e[\boldsymbol{W}_k^T \boldsymbol{X}_k] = E_e[\boldsymbol{W}_k^T]\boldsymbol{X}_k. \quad (57)$$

To calculate $E_e[\boldsymbol{W}_k^T]$ we make use of the recursive equation given in [2] for $E_e[\boldsymbol{W}_k^T]$ in the case of the LMS algorithm:

$$E_e[\boldsymbol{W}_{k+1}] = (\boldsymbol{I} - 2\mu \boldsymbol{R})E_e[\boldsymbol{W}_k] + 2\mu \boldsymbol{R}\boldsymbol{W}^*. \quad (58)$$

Also, this equation can be derived from (11) by taking the expected value and using the expression for the error ϵ. Adding and subtracting $\boldsymbol{W}^*$, and using the value of $\boldsymbol{R} = (1/L)\boldsymbol{I}$, the recursive expression becomes

$$E_e[\boldsymbol{W}_{k+1}] = \boldsymbol{W}^* + \left(1 - 2\frac{\mu}{L}\right)(E_e[\boldsymbol{W}_k] - \boldsymbol{W}^*). \quad (59)$$

If the initial conditions are $\boldsymbol{W}_k = \boldsymbol{W}_0$ we can rewrite the expression as a function of $\boldsymbol{W}_0$:

$$E_e[\boldsymbol{W}_k] = \boldsymbol{W}^* - \left(1 - 2\frac{\mu}{L}\right)^k(\boldsymbol{W}^* - \boldsymbol{W}_0), \quad (60)$$

and if $\boldsymbol{W}_0 = 0$ (the usual initialization), then

$$E_e[\boldsymbol{W}_k] = \boldsymbol{W}^* \left(1 - \left(1 - 2\frac{\mu}{L}\right)^k\right). \quad (61)$$

Using this relation in (57) we can express s'_k as

$$s'_k = E_e[\boldsymbol{W}_k^T]\boldsymbol{X}_k = \left(1 - \left(1 - 2\frac{\mu}{L}\right)^k\right)\boldsymbol{W}^*\boldsymbol{X}_k, \quad (62)$$

and from (7)

$$s'_k = \left(1 - \left(1 - 2\frac{\mu}{L}\right)^k\right)s_k. \quad (63)$$

Appendix B

In this Appendix we will show that the noise component n'_k defined in Section II-C, equation (18), is neither correlated with s'_k nor with s_k. Also we will deduce the relation between $E_t[n_k'^2]$ and $E_t[n_k^2]$ referred to in (20). If ergodicity is assumed, then $E_t[n_k^2] = E_e[n_k^2]$ and $E_t[n_k'^2] = E_e[n_k'^2]$.

Using (18), we obtain

$$E_t[n'_k] = E_e[n'_k] = E_e[y_k] - E_e[y_k] = 0 \quad (64)$$

showing that n'_k is zero-mean. If we now calculate

$$E_t[s'_k n'_k] = E_e[y_k]E_t[n'_k] = 0, \quad (65)$$

we see that n'_k and s'_k are not correlated. This implies that s_k is also not correlated with n'_k, hence s'_k and s_k are proportional (19). The conclusion is that s'_k is the component of y_k correlated with s_k and also the component of interest. The component n'_k is the residual noise that contaminates the estimation.

To derive the relation between $E_t[n_k'^2]$ and $E_t[n_k^2]$, given that n'_k is uncorrelated with s'_k and with s_k, we get from (16)

$$E_t[(y_k - s_k)^2] = E_t[n_k'^2] + E_t[(s'_k - s_k)^2]. \quad (66)$$

From (9) we can write

$$E_t[n_k'^2] = \xi - \xi_{\min} - E_t[(s'_k - s_k)^2]. \quad (67)$$

In this expression we need to know what is the dependence of ξ on each recurrence. The MSE ξ converges to the steady-state solution according to a geometric progression [2] with a time constant $\tau_{mse} = (1/2)\tau_w$. Then we can write the expression (60) for ξ instead of $\boldsymbol{W}$ where the value of ξ in the steady-state is $\xi_{\min}(1 + \boldsymbol{M})$ [this takes the place of $\boldsymbol{W}^*$ in (60)] and the initial value when $k = 0$ ($y_k = 0$) is $E_t[d_k^2]$ [this takes the place of $\boldsymbol{W}_0$ in (60)].

$$\xi = \xi_{\min}(1 + \boldsymbol{M}) + \left(1 - 2\frac{\mu}{L}\right)^{2k} \cdot (E_t[d_k^2] - \xi_{\min}(1 + \boldsymbol{M})), \quad (68)$$

The factor $2k$ is due to the fact that the convergence time of ξ is twice the convergence time of the weight vector.

From (67) and considering (8) and (19) we obtain

$$E_t[n_k'^2] = ME_t[n_k^2]\left(1 - \left(1 - 2\frac{\mu}{L}\right)^{2k}\right). \quad (69)$$

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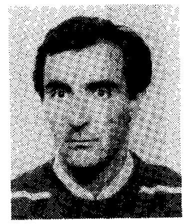

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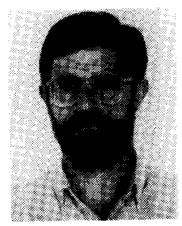
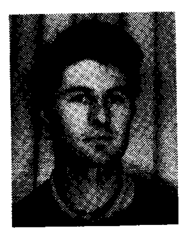


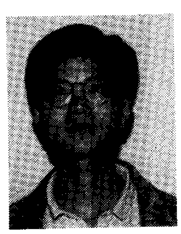
Raimon Jané (M'91) was born in Barcelona, Spain, in 1959. He received the electrical engineering degree and the Doctor Engineer degree from the School of Industrial Engineering of Barcelona at the Polytechnic University of Catalonia (U.P.C.), Spain, in 1983 and 1989 respectively.

From 1984 to 1989 he worked as a Research Assistant at the Biomedical Engineering Division of the Institute of Cybernetics (U.P.C.-C.S.I.C.), where he is now a Researcher. His current research interests are in Digital Signal Processing applied to Biological Signals and in particular to High Resolution ECG. He teaches medical instrumentation and signal processing in the master and Doctoral programs in Biomedical Engineering, at the Polytechnic University of Catalonia.



Olivier Meste was born in Cannes, France, in 1966. He received the M.S. degree in automatic and signal processing from the University of Nice, France, in 1989.

Currently, he is working towards the Ph.D. degree in Scientific engineering at the University of Nice-Sophia Antipolis, France, in the Department of Signals and Systems (LASSY-I3S biomedical team) associated to French National Research Center (URA 1376 CNRS). Research interests include digital signal processing and time-frequency representation in biomedical domain.



Peter W. Poon was born in Hong Kong in 1965. He received the B.Sc. degree in electrical engineering from the University of Calgary, Calgary, Alta., Canada, in 1988 and the M.Sc. degree in biomedical engineering from the The Johns Hopkins University, Baltimore, MD, in 1990.

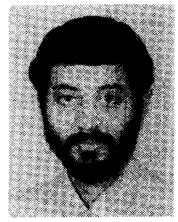
He is currently working as a programmer in the Epileptic Monitoring Unit of the Johns Hopkins Hospital, Baltimore, MD. His research interests include signal processing and microcomputer applications in medicine.

Pere Caminal (M'88) was born in 1952 in Barcelona, Spain. He received the M.S. and Ph.D. degrees in mechanical engineering in 1974 and 1980, respectively, from the Polytechnic University of Catalonia (U.P.C.), Spain.

He is Professor of Automatic Control in the Department of Control Engineering at the same University. Presently he is head of the Biomedical Engineering Division of the Institut of Cybernetics (U.P.C.), and director of the Master in Biomedical Engineering Degree Program at the Polythecnic University of Catalonia. His areas of interest are modelling and simulation of biological systems, and biosignal processing.

Hervé Rix (A'88) received the M.S. degree in applied mathematics in 1967, and the speciality doctoral degree in astrophysics in 1970 from the University of Nice, France, and the Ph.D. degree in 1980.

He was an Assistant Professor at the Institute of Technology and worked in the field of Signal Processing with application to Chromatography. Since 1981 his research interests have been in signal processing, mainly applied to biomedical signal and particulary to high resolution electrocardiography, performed in the Department of Signals and Systems (LASSY). He is associated with the French National Research Center, in collaboration with the Cardiology Department, Pasteur Hospital, Nice and, for the past three years with the Biomedical Engineering Division, Institute of Cybernetics, Barcelona, Spain. Currently, he is Professor at the University of Nice (Institute of Technology). He teaches mathematics and signal processing.



Nitish V. Thakor (S'78–M'81–SM'89) received the bachelor's degree in electrical engineering from the Indian Institute of Technology, Bombay, India, in 1974, and the Ph.D. degree in electrical and computer engineering from the University of Wisconsin, Madison, in 1981.

He previously served on the faculty of the Northwestern University, and is currently on the faculty of the Biomedical Engineering Department of The Johns Hopkins School of Medicine, Baltimore, MD. His research interests are cardiovascular and neurological signal processing, instrumentation, and large-scale computer applications. He currently teaches courses on related subjects and serves a consultant to industries in these specialties. He also serves as an Associate Editor for the IEEE TRANSACTIONS ON BIOMEDICAL ENGINEERING and for the Journal of Ambulatory Monitoring. He is engaged in international collaborative research and educational activities supported in part by a Fulbright Award and NATO.

Dr. Thakor is a previous recipient of a Research Career Development Award from the National Institutes of Health and a Presidential Young Investigator Award from the National Science Foundation.