BOUNDS ON CAPACITY OVER GAUSSIAN MIMO MULTIACCESS CHANNELS WITH CHANNEL STATE INFORMATION MISMATCH

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ABSTRACT

In this paper, we deal with the capacity of Gaussian MIMO multiaccess fading channels when the channel knowledge available at the receiver and the transmitters is erroneous. First, upper and lower bounds on the multiaccess capacity region are presented when the channel state information is estimated by means of a MMSE criterion, and consequently, there exist some uncertainty on the channel knowledge. Then, the difference between these bounds for the joint mutual information is analyzed. A final expression for the ergodic sum-capacity gap is obtained as a function of the number of users, the quality of the channel estimates and the number of receive and transmit antennas. Computer results show that adding more users or transmit antennas reduces this gap in equal factor at high SNR.

1. INTRODUCTION

The capacity region of multiaccess (MAC) channels has been extensively studied in the literature (see [1][2][3] and references therein). Recently, an iterative water-filling algorithm has been found optimal to compute the input covariance matrices in a Gaussian MIMO MAC channel [1]. In fact, it has been shown this iterative water-filling procedure converges after one iteration to only 1/2 nats per user per output dimension away from the sum-capacity limit. On the other hand, as analyzed in [3][4], the erroneous channel state information (CSI) available in practical systems bounds the achievable data rate in fading channels. In this paper, we first analyze the capacity region of Gaussian MIMO MAC fading channels with channel knowledge mismatch, establishing upper and lower bounds on the individual and sum-capacity. Next, a simple limit for the gap between the bounds of the ergodic sum-capacity is presented. This allows us to state that this difference becomes negligible when the total sum of transmit antennas is large.

2. ON CAPACITY OF MIMO GAUSSIAN MULTIACCESS CHANNELS

A K-user Gaussian MIMO MAC channel can be modeled as

$$\mathbf{y} = \sum_{i=1}^{K} H_i \mathbf{x}_i + \mathbf{z},$$

where $\mathbf{x}_i \in \mathcal{C}^{t_i \times 1}$ is the vector input signal transmitted by a user i with t_i transmit antennas, $\mathbf{y} \in \mathcal{C}^{r \times 1}$ is the output vector signal at the receiver with r antennas, $\mathbf{z} \in \mathcal{C}^{r \times 1}$ is the noise vector and $H_i \in \mathcal{C}^{r \times t_i}$ is the time-invariant channel between user i and the receiver. We assume both H_i and $\mathbf{z}$ are ergodic and stationary random variables (r.v.), whose entries are independent, identically distributed (i.i.d.) and zero-mean circularly symmetric complex Gaussian (ZM-CSCG) with unit variance [5], i.e., $\mathbf{z}$ and $H_i \sim \mathcal{CN}(0, \mathbf{I})$, where $\mathbf{I}$ denotes the identity matrix of proper size. The input signals are assumed to be also i.i.d. with each user satisfying an individual power constraint $\mathrm{tr}(S_i) \leq P_i$, where $S_i = E[\mathbf{x}_i \mathbf{x}_i^H]$ is the covariance matrix of input $\mathbf{x}_i$. Both the input signals and the noise are uncorrelated.

Assuming that CSI at the receiver (CSIR) is perfectly acquired, the capacity region for a K-user vector MAC channel is given by maximizing $\sum_{i=1}^{K} \mu_i R_i$, with $\mu_i \geq 0$. When the goal is to maximize the sum rate, we impose that $\mu_1 = \ldots = \mu_K = 1$, and the sum-capacity is achieved as follows [1]:

$$\begin{array}{ll} \text{maximize} & \log \left| \mathbf{I} + \sum_{i=1}^{K} H_i S_i H_i^H \right| \\ \text{subject to} & \mathrm{tr}\,(S_i) \leq P_i \qquad i = 1, \ldots, K \\ & S_i \geq 0 \qquad i = 1, \ldots, K \end{array}$$

This objective function is concave because $\log|\cdot|$ is concave and the constraints are convex in the space of positive semidefinite matrices [1]. Any point of the MAC capacity region can be achieved by successive decoding. It is well-known [5] that for a single-user Gaussian MIMO channel, the mutual information between the input vector and the output vector is maximized when the input covariance matrix is $S = M\hat{S}M^H$, where M are the right singular vectors of the channel matrix singular value decomposition (SVD), i.e, $H = U\Sigma M^H$. In this way, the transmit directions align with the right eigenvectors of the effective channel, decomposing the vector channel into a set of parallel independent scalar subchannels. Then, the matrix $\hat{S}$ is a diagonal matrix whose diagonal elements are computed via water-filling [2]. In [1], it is shown that this idea of water-filling can be generalized to the multiuser MAC case if the objetive is to maximize the sum data rate. In essence, the iterative water-filling algorithm can be described as [1]:

Initialization: $S_i = 0, i = 1, \ldots, K$.
repeat
  for i=1 to K **do**
    Compute $S'_z = \mathbf{I} + \sum_{j=1, j \neq i}^{K} H_j S_j H_j^H$.
    Calculate $S_i = \arg\max_S \quad \log|S'_z + H_i S H_i^H|$.
      Compute Eigenvalue Decomposition of $S'_z = Q\Delta Q^H$.
      Define $\hat{H}_i = \Delta^{-1/2} Q^H H_i$.
      Compute SVD of $\hat{H}_i = F\Sigma M^H$.
      Water-filling to obtain the diagonal values of $\hat{S}_i$ from Σ.
      Compute optimal $S_i = M\hat{S}_i M^H$.
  end for
until Convergence.

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Note that the above algorithm is equal to single-user capacity maximization procedure, but considering that the modified noise covariance matrix S'_z is the sum of the covariance matrices of the other users received signals plus the noise covariance matrix.

3. MAC MUTUAL INFORMATION BOUNDS WITH CHANNEL ESTIMATION ERROR

The iterative water-filling algorithm assumes perfect CSI at the receiver (CSIR) and the transmitter (CSIT). However, this assumption is far from being possible. Let's consider the situation where the receiver carries out Minimum Mean Square Error (MMSE) estimation of the user channels. Particularly, consider for user i that $H_i = \widehat{H}_i + E_i$, where $\widehat{H}_i$ is the MMSE estimation of the channel H_i available at the receiver and E_i is the corresponding channel estimation error. According to the orthogonality principle, $\widehat{H}_i$ and E_i are uncorrelated [4], and the entries of E_i are ZMCSCG r.v. with variance $\sigma^2_{E,i} = E[H^2_{i,j}] - E[\widehat{H}^2_{i,j}]$. Henceforth, we will denote a set of matrices as $\mathcal{A} = \{A_i\}_{i=1}^K$. For example, $\mathcal{H} = \{H_i\}_{i=1}^K$ denotes the set of actual user channels. Similarly, the set of transmitted signals will be denoted as $\mathbf{x} = \{\mathbf{x}_i\}_{i=1}^K$, with corresponding set of covariance matrices $\mathcal{S} = \{S_i\}_{i=1}^K$.

To achieve sum-capacity requires Gaussian inputs with covariance matrices obtained by an iterative water-filling algorithm [1] and successive interference cancellation with maximum-likelihood (ML) decoding at the receiver. In presence of channel uncertainty, although decoding a user without error is possible, then cancelling the complete interference term from that user is not possible, resulting in some additional error term [3]. In order to study the influence of the channel estimation error on the capacity region of Gaussian MIMO MAC fading channels, we derive upper and lower bounds on the mutual information $I(\mathbf{x};\mathbf{y}|\widehat{\mathcal{H}})$, given the set of estimated channels $\widehat{\mathcal{H}}$. Without loss of generality, we shall focus on a two-user scenario in order to clarify the exposition.

3.1. Lower bounds on MAC mutual information

As developed in [3][4], to obtain a lower bound we expand the mutual information in terms of differential entropies as follows:

$$I(\mathbf{y};\mathbf{x}_1|\mathbf{x}_2,\widehat{H}_1,\widehat{H}_2) = h(\mathbf{x}_1|\mathbf{x}_2,\widehat{H}_1,\widehat{H}_2) - h(\mathbf{x}_1|\mathbf{y},\mathbf{x}_2,\widehat{H}_1,\widehat{H}_2) \quad (1)$$

$$= h(\mathbf{x}_1) - h(\mathbf{x}_1 - \alpha(\mathbf{y} - \widehat{H}_2\mathbf{x}_2)|(\mathbf{y} - \widehat{H}_2\mathbf{x}_2),\mathbf{x}_2,\widehat{H}_2) \quad (2)$$

$$\geq h(\mathbf{x}_1) - h(\mathbf{x}_1 - \alpha(\mathbf{y} - \widehat{H}_2\mathbf{x}_2)|\mathbf{x}_2), \quad (3)$$

where we have used: in (2) the transmitted vector signals are independent from each other and the corresponding channel, and adding a constant does not modified differential entropy; in (3) conditioning always reduces the differential entropy [2, 3]. Choose α so that $\alpha(\mathbf{y} - \widehat{H}_2\mathbf{x}_2)$ is the linear MMSE estimate of $\mathbf{x}_1$,

$$\alpha \triangleq E\left[\mathbf{x}_1(\mathbf{y} - \widehat{H}_2\mathbf{x}_2)^H\right] E\left[(\mathbf{y} - \widehat{H}_2\mathbf{x}_2)(\mathbf{y} - \widehat{H}_2\mathbf{x}_2)^H\right]^{-1}$$
$$= S_1\widehat{H}_1^H\left(\widehat{H}_1 S_1 \widehat{H}_1^H + \Sigma_{E_1\mathbf{x}_1} + \Sigma_{E_2\mathbf{x}_2} + \mathbf{I}\right)^{-1}, \quad (4)$$

where $\Sigma_{\mathbf{v}}$ denotes the covariance matrix of a vector $\mathbf{v}$.

Assuming $\mathbf{x_1}$ is Gaussian distributed, even though the Gaussian distribution may not be the capacity achieving distribution with CSI mismatch, the first term in (2) is given by $h(\mathbf{x}_1) = \log\left|\pi e S_1\right|$. On the other hand, the second term is upper bounded by the entropy of a Gaussian random variable whose covariance is $\Sigma_{\mathbf{m}|\mathbf{x_2}}$, where $\mathbf{m} \triangleq \mathbf{x}_1 - \alpha(\mathbf{y} - \widehat{H}_2\mathbf{x}_2)$. Employing (4) to compute the MSE covariance matrix, $\Sigma_{\mathbf{m}|\mathbf{x_2}}$ becomes

$$\Sigma_{\mathbf{m}|\mathbf{x_2}} = S_1 - S_1\widehat{H}_1^H(\widehat{H}_1 S_1 \widehat{H}_1^H + \Sigma_{E_1\mathbf{x}_1} + \sigma^2_{E,2}\|\mathbf{x}_2\|^2 + \mathbf{I})^{-1}\widehat{H}_1 S_1,$$

therefore, $h(\mathbf{m}|\mathbf{x}_2) \leq \log\left|\pi e \Sigma_{\mathbf{m}|\mathbf{x_2}}\right|$.

Since the entries of the channels matrices H_i are i.i.d. and linear MMSE estimation is carried out, the entries of the channel estimation error matrices E_i are also Gaussian i.i.d. random variables. Thus, for a particular channel estimation error matrix E_i, $E[E_{i,m}E_{j,n}] = \sigma^2_{E,i}\delta_{i-j,m-n}$ and $\Sigma_{E_i\mathbf{x}_i} = \sigma^2_{E,i}P_i\mathbf{I}$ [4]. Finally, the lower bound of the MAC mutual information for user 1 is:

$$I\left(\mathbf{y};\mathbf{x}_1|\mathbf{x}_2,\widehat{H}_1,\widehat{H}_2\right) \geq \log\left|\mathbf{I} + \frac{\widehat{H}_1 S_1 \widehat{H}_1^H}{1 + \sigma^2_{E,1}P_1 + \sigma^2_{E,2}\|\mathbf{x}_2\|^2}\right|$$
$$\triangleq I_{low}\left(\mathbf{y};\mathbf{x}_1|\mathbf{x}_2,\widehat{H}_1,\widehat{H}_2\right).$$

Equivalently, regardless the number of users K, the lower bound for the mutual information of user i and the joint mutual information can be written as follows:

$$I_{low}(\mathbf{y};\mathbf{x}_i|\{\mathbf{x}-\mathbf{x}_i\},\widehat{\mathcal{H}}) = \log\left|\mathbf{I} + \frac{\widehat{H}_i S_i \widehat{H}_i^H}{1 + \sigma^2_{E,i}P_i + \sum_{\substack{j=1\\ j\neq i}}^K \sigma^2_{E,j}\|\mathbf{x}_j\|^2}\right| \quad \text{for} \quad i = 1,\ldots,K, \quad (5)$$

and

$$I_{low}(\mathbf{y};\mathbf{x}|\widehat{\mathcal{H}}) = \log\left|\mathbf{I} + \frac{\sum_{i=1}^K \widehat{H}_i S_i \widehat{H}_i^H}{1 + \sum_{i=1}^K \sigma^2_{E,i}P_i}\right|. \quad (6)$$

3.2. Upper bounds on MAC mutual information

In order to obtain an upper bound for the mutual information, we can expand (1) in different manner:

$$I(\mathbf{y};\mathbf{x}_1|\mathbf{x}_2,\widehat{H}_1,\widehat{H}_2) = h(\mathbf{y}|\mathbf{x}_2,\widehat{H}_1,\widehat{H}_2) - h(\mathbf{y}|\mathbf{x}_1,\mathbf{x}_2,\widehat{H}_1,\widehat{H}_2)$$
$$= h(\mathbf{y}|\mathbf{x}_2,\widehat{H}_1,\widehat{H}_2) - h(E_1\mathbf{x}_1 + E_2\mathbf{x}_2 + \mathbf{z}|\mathbf{x}_1,\mathbf{x}_2,\widehat{H}_1,\widehat{H}_2). \quad (7)$$

Since the Gaussian distribution maximizes the entropy over all distributions with the same covariance [2], we obtain an upper bound of the first term on the right hand side (RHS) as

$$h(\mathbf{y}|\mathbf{x}_2,\widehat{H}_1,\widehat{H}_2) \leq \log\left|\pi e \Sigma_{\mathbf{y}|\mathbf{x_2},\widehat{\mathbf{H}}_1,\widehat{\mathbf{H}}_2}\right|$$
$$= \log\left|\pi e\left((1 + \sigma^2_{E,1}P_1 + \sigma^2_{E,2}\|\mathbf{x}_2\|^2)\mathbf{I} + \widehat{H}_1 S_1 \widehat{H}_1^H\right)\right|. \quad (8)$$

Since all the input signals $\mathbf{x}_i$ and the channel estimation errors E_i are i.i.d. Gaussian r.v, the second term on the RHS in (7) becomes

$$h(E_1\mathbf{x}_1 + E_2\mathbf{x}_2 + \mathbf{z}|\mathbf{x}_1,\mathbf{x}_2) = \log\left|\pi e(1 + \sigma^2_{E,1}\|\mathbf{x}_1\|^2 + \sigma^2_{E,2}\|\mathbf{x}_2\|^2)\mathbf{I}\right|. \quad (9)$$

Similarly to lower bounds, combining (7)-(9), the upper bound of the mutual information for user i and the joint mutual information are given by (10) and (12), shown at the top of the next page.

$$I_{up}(\mathbf{y};\mathbf{x}_i\big|\{\mathbf{x}-\mathbf{x}_i\},\widehat{\mathcal{H}}) = \log\left|\frac{\widehat{H}_i S_i \widehat{H}_i^H + (1+\sigma_{E,i}^2 P_i + \sum_{\substack{j=1\\ j\neq i}}^{K}\sigma_{E,j}^2\|\mathbf{x}_j\|^2)\mathbf{I}}{1+\sum_{i=1}^{K}\sigma_{E,i}^2\|\mathbf{x}_i\|^2}\right| \tag{10}$$

$$= I_{low}(\mathbf{y};\mathbf{x}_i\big|\{\mathbf{x}-\mathbf{x}_i\},\widetilde{\mathcal{H}}) + \log\left|\frac{(1+\sigma_{E,i}^2 P_i + \sum_{\substack{j=1\\ j\neq i}}^{K}\sigma_{E,j}^2\|\mathbf{x}_j\|^2)\mathbf{I}}{1+\sum_{i=1}^{K}\sigma_{E,i}^2\|\mathbf{x}_i\|^2}\right| \tag{11}$$

$$I_{up}\left(\mathbf{y};\mathbf{x}\big|\widehat{\mathcal{H}}\right) = \log\left|\frac{\sum_{i=1}^{K}\widehat{H}_i S_i \widehat{H}_i^H + (1+\sum_{i=1}^{K}\sigma_{E,i}^2 P_i)\mathbf{I}}{1+\sum_{i=1}^{K}\sigma_{E,i}^2\|\mathbf{x}_i\|^2}\right| = I_{low}\left(\mathbf{y};\mathbf{x}\big|\widehat{\mathcal{H}}\right) + \log\left|\frac{(1+\sum_{i=1}^{K}\sigma_{E,i}^2 P_i)\mathbf{I}}{1+\sum_{i=1}^{K}\sigma_{E,i}^2\|\mathbf{x}_i\|^2}\right| \tag{12}$$

4. APPROACHING ERGODIC SUM-CAPACITY BOUNDS

Traditionally, the Shannon capacity of a fading channel is referred in terms of ergodic capacity. Ergodic capacity can be defined for a fading channel with a long-term delay constraint. Assuming an ergodic channel, we can communicate at the rate defined by the average mutual information with vanishing error assuming we use asymptotically optimal codewords that cover all channel states according to the channel's probability distributions [6]. In this sense, the average mutual information is called ergodic capacity.

4.1. Difference between upper and lower bounds

In Section 3, the upper and lower bounds of mutual information of Gaussian MIMO MAC channels have been derived in presence of channel uncertainty. Particularly, in a K-user MAC channel, the objective function can be to maximize the sum-data rate instead of the rate of one particular user. As shown in (12), the upper bound of the joint mutual information is related to its correponding lower bound given by (6). The difference between the upper and lower bounds of the ergodic sum-capacity is given by the expectation of the last term in (12) as follows [4]:

$$\Delta \triangleq E\left[I_{up}\left(\mathbf{y};\mathbf{x}\big|\widehat{\mathcal{H}}\right) - I_{low}\left(\mathbf{y};\mathbf{x}\big|\widehat{\mathcal{H}}\right)\right] \tag{13}$$

$$= r\cdot E\left[\log\left(\frac{1+\sum_{i=1}^{K}\sigma_{E,i}^2 P_i}{1+\sum_{i=1}^{K}\sigma_{E,i}^2\|\mathbf{x}_i\|^2}\right)\right], \tag{14}$$

where $E[\cdot]$ denotes the expectation operator. In [4], a limit for Δ is obtained for the single user case ($K=1$) for Gaussian inputs in the high SNR regime and a large number of antennas. Here, a simple approximation is adopted to compute the expected value of the sum-capacity gap. To begin with, let us write Δ as follows:

$$\Delta = rE\left[\log\left(1+\sum_{i=1}^{K}\sigma_{E,i}^2 P_i\right) - \left(1+\sum_{i=1}^{K}\sigma_{E,i}^2\|\mathbf{x}_i\|^2\right)\right]$$
$$= r\Big[\log\big(1+\sum_{i=1}^{K}\sigma_{E,i}^2 P_i\big) - E\Big[\log\big(1+\sum_{i=1}^{K}\sigma_{E,i}^2\|\mathbf{x}_i\|^2\big)\Big]\Big]. \tag{15}$$

Assuming gaussian input vectors, the norm $\|\mathbf{x}_i\|^2$ follows a chi-square distribution $\chi^2(t_i)$ with $2t_i$ degrees of freedom (d.f.) [1] and $E[\|\mathbf{x}_i\|^2] \leq P_i$ [4]. Therefore, $\chi^2(t_i)$ is scaled by a factor P_i/t_i.

[1] Here, a chi-square r.v. $\chi^2(m)$ with $2m$ d.f. is defined as $\frac{1}{2}\sum_{j=1}^{m}|x_j|^2$, which corresponds to the sum of the squared-magnitudes of m independent ZMCSCG r.v. with unit variance [7]. The mean of $\chi^2(m)$ is $E[\chi^2(m)] = m$ and its variance is $\mathrm{var}[\chi^2(m)] = 2m$.

Defining $\xi \triangleq 1+\sum_{k=1}^{K}\sigma_{E,i}^2\|\mathbf{x}_i\|^2$, observe that ξ is the weighted sum of K independent chi-square r.v. with different d.f.. Employing a classical technique in statistics [8], ξ can be approximated by a single chi-square r.v. with l d.f. and a proper scaling factor α/l,

$$\xi = 1+\sum_{k=1}^{K}\sigma_{E,i}^2\frac{P_i}{t_i}\chi^2(t_i) \approx \frac{\alpha}{l}\chi^2(l).$$

The parameters α and l should be chosen such that both terms have identical mean and variance. This is given by

$$\alpha = 1+\sum_{k=1}^{K}\sigma_{E,i}^2 P_i,$$
$$\frac{\alpha^2}{l} = \sum_{k=1}^{K}\sigma_{E,i}^4\frac{P_i^2}{t_i},$$

and, therefore, the number of d.f. is

$$l = \frac{(1+\sum_{k=1}^{K}\sigma_{E,i}^2 P_i)^2}{\sum_{k=1}^{K}\sigma_{E,i}^4\frac{P_i^2}{t_i}}. \tag{16}$$

Note that l is rounded to the nearest integer if necessary. Following Lemma 1 in [4], the expected logarithm of $\frac{\alpha}{l}\chi^2(l)$ is given by [7]:

$$E\Big[\log\big(\frac{\alpha}{l}\chi^2(l)\big)\Big] = \log\frac{\alpha}{l} + \left(\sum_{i=1}^{l-1}\frac{1}{i} - \gamma\right),$$

where γ is the Euler's constant. Therefore, Δ becomes

$$\Delta = r\Big[\log l + \gamma - \sum_{i=1}^{l-1}\frac{1}{i}\Big]. \tag{17}$$

In the limit of a large value of l, Δ is shown to achieve [4]

$$\lim_{l\to\infty}\Delta = \frac{r}{2l}. \tag{18}$$

Clearly, (18) is similar to the limit obtained in [4], but substituting the number of single-user transmit antennas t into the parameter l. Observe that when channel knowledge is perfect and $\sigma_{E,i}^2$=0, l tends to ∞ and logically Δ reduces to zero.

In order to fairly compare ergodic sum-capacities of scenarios with different number of users K, assume that users are provided with equal number of transmit antennas $t_i = t$ and a total amount of power must be shared out between all users so that the individual power constraints are $P_i = P/K$ for $i = 1,\ldots,K$. Usually, accurate power control algorithms are employed at the receiver so

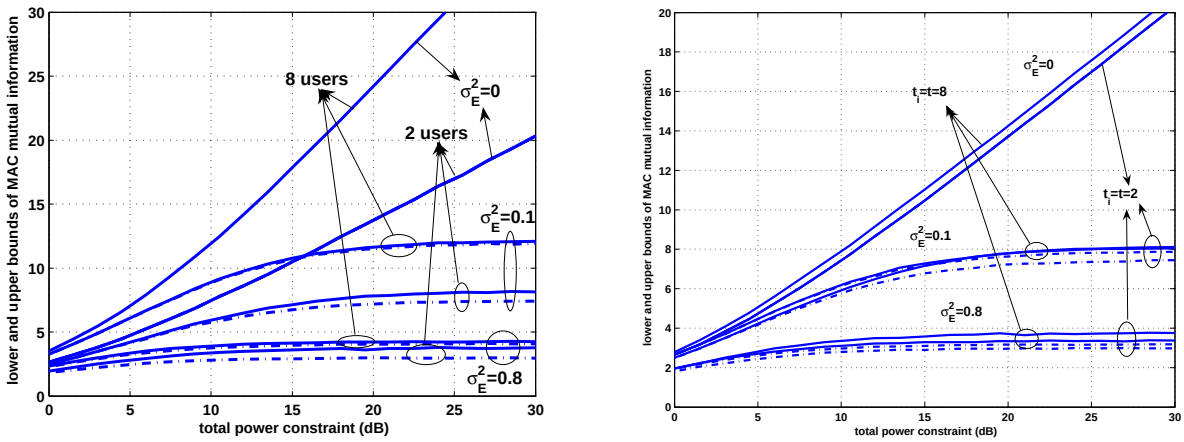

Fig. 1. Lower (-.-) and upper (-) bounds of joint mutual information (r=4). (a) $t_i = t = 2$, (b) K=4. The received SNR is P/K.

that user signals impinge with similar SNRs. Operating under these assumptions, if channel MMSE estimation is carried out, it is reasonable to assume that the error variances are identical, i.e., $\sigma^2_{E,i} = \sigma^2_E$ for $i = 1, \ldots, K$. Therefore, the parameter l is given by

$$l = \frac{(1 + \sum_{k=1}^{K} \sigma^2_{E,i} P_i)^2}{\sum_{k=1}^{K} \sigma^4_{E,i} \frac{P_i^2}{t_i}} = \frac{(1 + \sigma^2_E P)^2}{\sigma^4_E \frac{P^2}{Kt}}. \quad (19)$$

In the high SNR regime, l limits to $\lim_{P\to\infty} l = Kt$, and the gap between the upper and lower bounds achieves

$$\lim_{l\to\infty} \lim_{P\to\infty} \Delta = \frac{r}{2Kt}. \quad (20)$$

Thus, in this simple scenario with identical user parameters, Δ is proportional to the quotient r/t, and it is inversely proportional to K at the high SNR regime. Similar to [4], this result can be interpreted as if one user would have the total sum of transmit antennas Kt. Observe that the difference between the lower and upper bound of ergodic sum-rate decreases with increasing number of users K or transmit antennas t. This relationship between the number of transmit antennas and the number of users is in agreement with the results shown in [9] for the ergodic sum-capacity in MAC channels with no CSIT available at high SNRs.

In Figure 1 the lower and upper bounds of the joint mutual information are depicted for $r = 4$ as a function of the total power constraint, the channel estimation qualities and the number of users. As illustrated, increasing the number of users from K=2 to K=8 (left) or the number of transmit antennas from t_i=2 to t_i=8 (right) reduces the gap between the bounds. Note that the gap is relatively small for any SNR values. Obviously, more channel estimation error always decreases the feasible sum-data rate. On the other hand, notice that increasing the number of transmit antennas for fixed number of receive antennas (right) does not have the same effect in terms of absolute sum-data rate as adding users (left), particularly for low channel estimation errors. An scenario with more users achieves larger values of sum-data rate thanks to the multiuser diversity gain [9]. The number of available signalling degrees of freedom (subchannels) depends on the number of receive antennas. These subchannels must be distributed between the users according to their channel qualities, with a maximum number of subchannels per user which is limited to the corresponding number of transmit antennas. Finally, the figure illustrates a linear increase in the joint mutual information in ideal scenarios without channel estimation error, but is bounded with channel mismatch.

5. CONCLUSIONS

In this work, the lower and upper bounds of Gaussian MIMO MAC fading channels have been established in presence of channel mismatch at the receiver and the transmitters. Then, a further analysis on the difference between the bounds of the joint mutual information has been carried out. As a result, we introduce a closed expression for this gap, which states that the bounds tend to overlap for larger number of users or transmit antennas at high SNR. Simulation results demonstrate that the multiuser diversity gain obtained by adding more users is still beneficial in terms of absolute sum-data rate, specially when the channel estimation quality is good enough. Future work should address this CSI mismatch analysis on outage capacity.

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