FAST FREQUENCY SYNTHESIZER CONCEPT BASED ON DIGITAL TUNING AND I/Q SIGNAL PROCESSING

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Abstract: The design of frequency synthesizers is especially challenging for wireless applications due to the requirements for high spectral purity, high frequency range, and fast tuning together with reasonable power consumption. In this paper, the idea of combining digital and analog synthesis techniques for achieving these goals is discussed and analyzed. The proposed architecture uses I/Q modulation to translate a digitally synthesized tuneable low frequency tone to the final frequency range. In practical implementations, however, unavoidable mismatches between the amplitudes and phases of the I and Q branches result in imperfect sideband rejection degrading the spectral purity of the synthesized signal. A compensation structure based on digital pre-distortion of the low frequency tone is presented to enhance the signal quality. Furthermore, practical algorithms for updating the compensator parameters are proposed based on minimizing the envelope variation of the synthesizer output signal. Simulation results are also presented to illustrate the efficiency of the proposed synthesizer concept.

1. INTRODUCTION

In radio communications, sinusoidal signals are used to convert baseband messages into carrier modulated transmission signals and vice versa. Thus, synthesizing high quality oscillator signals is one of the most crucial parts of any radio transceiver. In general, to communicate over any of the possible frequency channels, the oscillating frequency has to be adjustable across the whole system band with tuning resolution equal to channel spacing [1]-[4].

Traditional analog synthesizer architectures are based on multiplying the frequency of a stable reference source using a phase-locked loop (PLL) with a frequency divider in the feedback path [1][2]. In the basic schemes, the output resolution equals the reference frequency and tunability is obtained by using a programmable divider. Digital synthesis techniques, in turn, generate sinusoidal signal samples in digital domain and use digital-to-analog (D/A) conversion to build up the analog waveform. This kind of procedure is usually referred to as direct digital synthesis (DDS) [3]. While the power consumption of digital techniques is generally higher than that of the corresponding analog solutions, low phase noise, high resolution, and high tuning speed have made DDS an appealing alternative in many applications [3]-[6].

The paper is organized as follows. In Section 2, the frequency synthesizer concept based on digital tuning and I/Q signal processing is presented at an architectural level. Then, in Section 3, the I/Q mismatch problem and its effect on the synthesized signal is considered and analyzed in detail. A digital pre-compensation structure together with iterative coefficient update algorithm are also derived with emphasis on simple implementation. In Section 4, the efficiency of the proposed solution is demonstrated using computer simulations and some practical matters are discussed in Section 5. Finally, conclusions are drawn in Section 6.

2. SYNTHESIZER ARCHITECTURE

In wireless communications, the demands for fast switching and high (typically GHz) operating frequencies make the design of frequency synthesizers a challenging task [1]-[4]. From the whole terminal point of view, integrability and power consumption are also of great concern. With these objectives in mind, the basic approach here is to generate a low frequency tone using digital techniques and then mix this synthesized signal to the final frequency range. This concept is illustrated at a general level in Figure 1. To avoid costly radio frequency (RF) filtering, in-phase/quadrature (I/Q) mixing principle is used for sideband suppression. All the tuning can be made in the digital domain and thus a fixed frequency local oscillator (LO) can be used in the mixing stage. Notice that the frequency range of the digital synthesis part can be kept at a relatively low level resulting in feasible demands for D/A converters and moderate power consumption.

In practical implementations, several non-idealities can distort the spectral purity of the synthesized signal. First of all, due to the finite number of bits in the digital part, some spurious components will appear in the D/A converter outputs, thus also degrading the spectral purity of the actual synthesizer output [3]. Increasing the D/A converter resolution can, however, be used to reduce these spurs to acceptable levels (at the expense of added complexity). More challenging problem in this context is the effect of non-idealities in the analog part. Especially in integrated circuit implementations, the amplitudes and phases of the I and Q branches can only be matched with finite relative accuracy [6][7]. This results in an image tone to appear at the output signal. In addition, the isolation between the mixer LO and output ports is only finite allowing part of the LO signal to leak into the mixer output. However, to enhance the spectral purity of the synthesized signal, digital pre-distortion of the low frequency tone can be applied.

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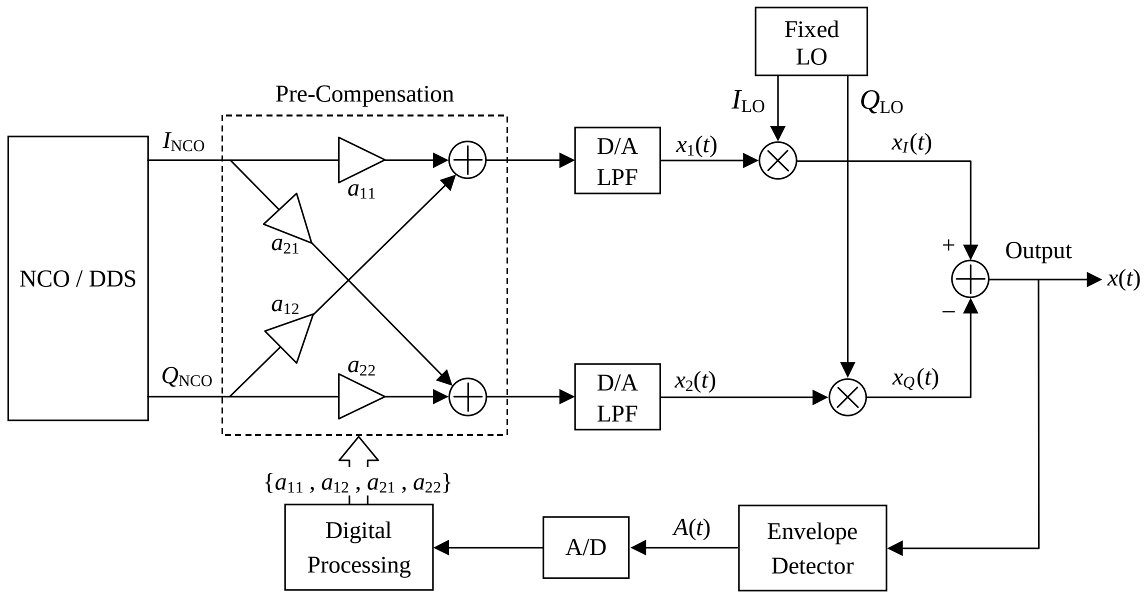

Figure 1: Synthesizer architecture where digital pre-compensation is used to compensate for the non-idealities of the analog part.

3. SIGNAL ANALYSIS

To appreciate the imbalance problem, an expression for the synthesizer output signal is derived next. The effect of LO leakage on the output is self-explaining and thus disregarded in the analysis to simplify the notations.

3.1 I/Q Imbalance Effect and Pre-Compensation

For analysis purposes, we write the low frequency I and Q signals in continuous-time domain as $I_{\mathrm{NCO}}(t) = \cos(\omega_{\mathrm{NCO}}t)$ and $Q_{\mathrm{NCO}}(t) = \sin(\omega_{\mathrm{NCO}}t)$. Furthermore, the I/Q mixer LO signals are written as $I_{\mathrm{LO}}(t) = \cos(\omega_{\mathrm{LO}}t)$ and $Q_{\mathrm{LO}}(t) = g\sin(\omega_{\mathrm{LO}}t + \phi)$ where g and ϕ denote the gain and phase imbalances of the mixing stage. With general pre-compensation parameters a_{ij}, the I/Q mixer input signals, in turn, are given by

$$\begin{aligned} x_1(t) &= a_{11}\cos(\omega_{\mathrm{NCO}}t + \theta_1) + a_{12}\sin(\omega_{\mathrm{NCO}}t + \theta_1) \\ x_2(t) &= a_{21}\cos(\omega_{\mathrm{NCO}}t + \theta_2) + a_{22}\sin(\omega_{\mathrm{NCO}}t + \theta_2) \end{aligned} \tag{1}$$

where θ_1 and θ_2 model the relative phase shifts due to the D/A converters and branch filters. Notice that any gain mismatch between the D/A converters and filters can be modeled already in the mixer gain imbalance g. Now, the synthesizer output signal $x(t) = x_1(t)I_{\mathrm{LO}}(t) - x_2(t)Q_{\mathrm{LO}}(t)$ can be easily shown to be of the form $x(t) = \mathrm{Re}[x_{\mathrm{LP}}(t)\exp(j\omega_{\mathrm{LO}}t)]$ where the lowpass equivalent $x_{\mathrm{LP}}(t)$ is given by

$$x_{\mathrm{LP}}(t) = \alpha e^{j\omega_{\mathrm{NCO}}t} + \beta e^{-j\omega_{\mathrm{NCO}}t}. \tag{2}$$

In other words, the synthesizer output consists of *two* spectral components, the desired tone at $\omega_{\mathrm{LO}} + \omega_{\mathrm{NCO}}$ and the image tone at $\omega_{\mathrm{LO}} - \omega_{\mathrm{NCO}}$ whose relative strengts $|\alpha|^2$ and $|\beta|^2$ depend on g, ϕ, and $\theta = \theta_2 - \theta_1$, as well as on the compensation parameters a_{ij}. With practical imbalance values, the image tone attenuation defined here as

$$L = |\alpha|^2 / |\beta|^2 \tag{3}$$

is only 20-40 dB if no compensation is used (i.e., $a_{11} = a_{22} = 1$ and $a_{12} = a_{21} = 0$). In wireless systems, these levels of synthesizer spurious tones can result in severe interchannel interference (ICI).

In general, image tone attenuations in the order of 50-80 dB are needed in wireless applications [4]. Considering the proposed concept, the image tone at the synthesizer output can be canceled with proper pre-compensation parameters a_{ij}. In fact, setting $\beta = 0$ and $\alpha = \exp(j\theta_1)$ will force the output to be a pure sinusoidal (with relative phase θ_1). These optimum parameters a_{ij} can be easily shown to be of the form

$$a_{11} = 1,\ a_{12} = \tan(\phi),\ a_{21} = -\frac{\sin(\theta)}{g\cos(\phi)},\ a_{22} = \frac{\cos(\theta)}{g\cos(\phi)}. \tag{4}$$

However, if the only motivation is to cancel the image tone (i.e., to set $\beta = 0$), a more simple solution is also available. Using the notation $\psi = \phi - \theta$, the solution can be formulated as

$$a_{11} = 1,\ a_{12} = \tan(\psi),\ a_{21} = 0,\ a_{22} = \frac{1}{g\cos(\psi)} \tag{5}$$

for which also $\alpha \approx \exp(j\theta_1)$ with practical imbalance values. Thus, only two actual compensation parameters a_{12} and a_{22} need to be implemented. For similar discussion related to I/Q data modulators, see [8]-[10]. Notice that only the phase error difference ψ and the gain mismatch g are actually contributing to the image tone, and therefore, on the ideal compensation parameters.

3.2 Update Algorithms

In practice, the imbalances needed to calculate the compensators of (4) or (5) are unknown and need to be measured or estimated somehow. To emphasize implementation simplicity, the approach taken here is to carry out the estimation using only the envelope

$$\begin{aligned} A(t) &= |x_{\mathrm{LP}}(t)| = |\alpha e^{j\omega_{\mathrm{NCO}}t} + \beta e^{-j\omega_{\mathrm{NCO}}t}| \\ &= \sqrt{|\alpha|^2 + |\beta|^2 + 2\,\mathrm{Re}[\alpha\beta^* e^{j2\omega_{\mathrm{NCO}}t}]} \end{aligned} \tag{6}$$

of the synthesizer output. Based on (6), the envelope is flat only if the image tone is completely attenuated ($\beta = 0$). Thus, instead of estimating g and ψ, a more simple approach is to directly adapt a_{12} and a_{22} to minimize the envelope variation. One possibility is to consider the envelope peak-to-peak (PP) value

$$d_A = \max\{A(t)\} - \min\{A(t)\}\,. \tag{7}$$

Given that $|\alpha| > |\beta|$ which is always the case with practical imbalance values, the result of (7) can also be expressed as

$$\begin{aligned} d_A &= 2\,|\,\beta\,| \\ &= \sqrt{1 + g^2 a_{22}^2 + a_{12}^2 - 2 g a_{22} (\cos(\psi) + a_{12}\sin(\psi))}\,. \end{aligned} \tag{8}$$

This gives formal verification that minimizing the envelope peak-to-peak value is really equivalent to minimizing the power of the image tone ($|\beta|^2$).

Though not strictly parabolic, the d_A -surface having a unique minimum lends itself well to iterative minimization. The true gradient of d_A (derivative with respect to a_{12} and a_{22}) depends, however, on g and ψ which are unknown. A practical approach is then to adapt only one parameter (either a_{12} or a_{22}) at a time. The direction (sign) of the needed one dimensional gradient at each iteration can be determined based on observing the behaviour of d_A between two previous adaptations of the corresponding parameter. Assuming that a_{12} and a_{22} are updated at odd and even adaptation instants, respectively, this leads to the following update rule

$$\begin{aligned} \hat{a}_{12}(n) &= \hat{a}_{12}(n-2) + K_{12}(n)\lambda_1 \frac{d_A(n)}{\lambda_2 + d_A(n)} \\ \hat{a}_{22}(n) &= \hat{a}_{22}(n-1) \end{aligned} \tag{9a}$$

for n odd with $K_{12} \in \{+1, -1\}$ and

$$\begin{aligned} \hat{a}_{12}(n) &= \hat{a}_{12}(n-1) \\ \hat{a}_{22}(n) &= \hat{a}_{22}(n-2) + K_{22}(n)\lambda_1 \frac{d_A(n)}{\lambda_2 + d_A(n)} \end{aligned} \tag{9b}$$

for n even with $K_{22} \in \{+1, -1\}$. In the above adaptation scheme, $K_{12}(n) = K_{12}(n-2)$ if $d_A(n-1) \le d_A(n-2)$ and $K_{12}(n) = -K_{12}(n-2)$ if $d_A(n-1) > d_A(n-2)$ with similar reasoning for K_{22}. Step-size parameters $\lambda_1 > 0$ and $\lambda_2 \ge 0$ are used here to control the adaptation speed and steady-state accuracy. Notice that the magnitude of the update term is always between zero and λ_1, depending on the value of d_A.

4. EXAMPLE SIMULATIONS

To demonstrate the proposed synthesizer concept with digital pre-compensation, some computer simulations are carried out. In the basic simulations, imbalance values of $g = 1.05$, $\phi = 6°$, and $\theta = 1°$ ($\psi = \phi - \theta = 5°$) are used, corresponding to an image tone attenuation of approximately 26 dB. Pre-compensation parameters are initialized as $a_{12} = 0$ and $a_{22} = 1$ and are then iteratively updated one-by-one to minimize the peak envelope variation using (9a) and (9b). In general, updating is carried out once per envelope cycle and step-size values of $\lambda_1 = 0.01$ and $\lambda_2 = 0.02$ are used.

With these example values, the d_A -surface is illustrated in Figure 2 as a function of a_{12} and a_{22}. Also shown in the a_{12}, a_{22} -plane is the behaviour of the predistortion parameters during one simulation realization. The corresponding output envelope is depicted in Figure 3 verifying successful synthesizer operation. With these imbalance and step-size values, the steady-state operation is reached in 50 iterations or so and the steady-state image attenuation is in the order of 100-150 dB.

To illustrate the ability to perform the compensation also in time-varying environments, an abrupt change in the imbalances is tested. The gain mismatch coefficient g is changed from 1.05 to 0.98 and the I/Q mixer phase imbalance ϕ from 6° to 4°. The resulting output envelope is presented in Figure 4 evidencing fast and accurate synthesizer operation also in case of time-varying imbalances. This kind of sudden imbalance changes could take place, e.g., in fast frequency hopping (FH) systems, since the phase error due to A/D converters and branch filters can easily be frequency-dependent.

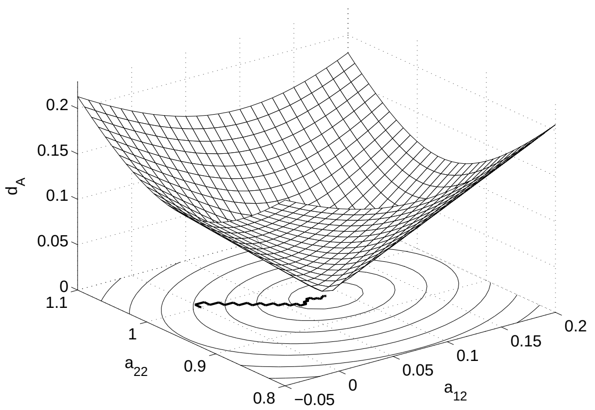

Figure 2: Peak envelope variation d_A as a function of the compensation parameters a_{12} and a_{22} for $g = 1.05$ and $\psi = 5°$. Also shown in the a_{12}, a_{22} -plane is one realization of the compensation parameters during the adaptation.

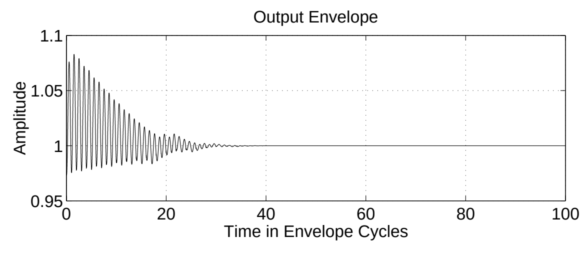

Figure 3: Envelope of the synthesizer output signal during the adaptation (g = 1.05 and ψ = 5°).

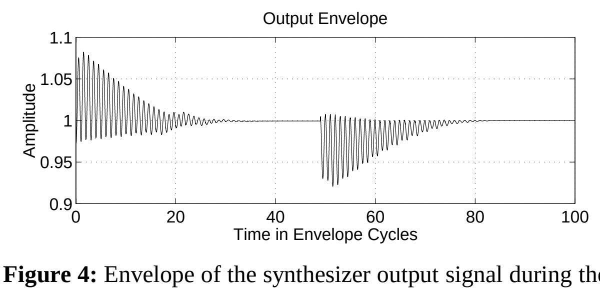

Figure 4: Envelope of the synthesizer output signal during the adaptation with a rapid change in the imbalances (the initial imbalances $g = 1.05$ and $\psi = 5°$ are changed to $g = 0.98$ and $\psi = 3°$, respectively).

5. PRACTICAL MATTERS

Based on the envelope expression given in (6), the output envelope is periodic with fundamental frequency $2\omega_{NCO}$. Thus, if d_A is determined and compensation parameters updated once per envelope cycle (or once per several cycles), this sets some requirements for frequency planning to obtain adequate convergence time with reasonable power consumption. In other words, the minimum frequency of the digital synthesis part should be selected in such a manner that the envelope variation rate does not limit the whole synthesizer's settling time. On the other hand, higher frequencies in the digital part always imply higher sampling rates and higher power consumption. This calls for a proper compromise between these two issues and should be considered individually for any particular application.

Considering the needed digital processing, the number of bits needed in calculations and to represent the data is of primary importance. According to preliminary results, image tone attenuations in the order of 80 dB are achievable with 16 bits. More detailed analysis of the finite wordlength effects is, however, still needed. One important issue in this context is the available accuracy of the envelope variation measurements. Especially with small values of d_A (i.e., close to steady-state), the estimation accuracy and, thus, the compensation performance depend directly on the relative precision of the envelope measurements. Therefore, some alternative ideas other than the direct sampling of analog envelope detector output might yield better performance. This constitutes an interesting topic for further studies. Thinking about the actual implementation of the pre-distortion (two multiplications and one addition) stage, the coefficients a_{12} and a_{22} are always around 0 ($-0.2 \ldots 0.2$) and 1 ($0.9 \ldots 1.1$), respectively, with practical imbalance values. This information can be utilized when determining and optimizing the compensator implementation.

With practical analog mixers, the I/Q mismatch is not the only non-ideality distorting the synthesizer output. One practical effect is the leakage of the local oscillator signals to the I/Q mixer output ports and, thus, to the synthesizer output signal. From the modeling point of view, this creates an additional spectral component to the lowpass equivalent at zero frequency whose strength depends on the leakage levels. Since the digital part is indeed effectively creating and processing this lowpass equivalent, the LO leakage can be compensated simply by modifying the DC levels of the synthesized low frequency signals. In other words, proper constants are added to the digital I and Q signals after the imbalance pre-compensation structure. One possibility to estimate the leakage compensation coefficients during the start-up phase is to set the DDS signals to zero (resulting in zero synthesizer output without leakage) and then minimize the output signal [8]. For an alternative approach related to I/Q data modulators, see the discussion in [9].

6. CONCLUSIONS

In this contribution, frequency synthesizer design for wireless applications was considered. To achieve fast switching capabilities with high operating frequencies and reasonable power consumption, the approach was to use I/Q modulation of digitally tunable low frequency tone. The practical imbalance problem with analog I/Q signal processing was then considered and analyzed. Based on this analysis, a digital pre-compensation structure was presented together with some simple yet efficient approaches to determine the compensator coefficients. The proper operation of the whole synthesizer was demonstrated using computer simulations both in time-invariant and time-varying situations. Future work should be directed to more detailed evaluation of the finite wordlength effects. More generally, hardware prototyping and/or co-simulation of the analog and digital parts are needed to assess the practical performance of the proposed synthesizer concept.

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