Through-the-Earth Magnetic Field Propagation: Modelling and Experimental Validation[1]

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Introduction

The study of the electromagnetic wave propagation in a dissipative medium is of high importance in applications such as emergency communications in confined environments (tunnels, mines, parking, caves), radiolocation and electromagnetic methods in geophysics. This topic has received considerable attention in the past. The work of Sommerfeld in 1909 about electromagnetic field propagation for wireless telegraphy is considered the earliest serious study of this topic [1]. In 1968, after a dramatic mining accident in Farmington (West Virginia) important funds were assigned by the USA government to this work line. During these years the most relevant papers on the subject were published, remarkably those of Wait [2]. Later, the decrease in mine accidents during the eighties caused the withdrawal of the funds and a reduction of the scientific studies. Unfortunately, the main works have focused to particular operation cases determining non-general formal solutions. Furthermore, the limited computational power of that epoch have reduced numerical analysis to special cases. Finally, very few field experiments supporting theory have been reported so far in the literature [3][4]. Since 1998, the GTE (Group of Technologies in hostile Environments) has focused on this research line.

In this paper we present a comparison between analytical and experimental results for a vertical magnetic dipole source. Free-space, dissipative infinite, semi-infinite and three-layered models for linear, homogeneous and isotropic horizontal mediums are considered. The exact analytical solutions for magnetic field expressions are reformulated in a suitable form and numerically solved. Moreover, a complex field experiment has been undertaken showing that, unexpectedly, the most complex model does not fit reality the best.

Analytical Solution

The formal expressions for propagated field in presence of a conducting earth are attributed to Sommerfeld [1]. The existence of boundaries adds extra contributions to the primary field present in infinite mediums: the secondary field. It is possible to derive the electromagnetic field from a single vector function, the Hertz vector $\mathbf{\Pi}$, and by taking into account the influence of the primary and secondary excitation this could be written as $\mathbf{\Pi} = \mathbf{\Pi}^{\mathbf{prim}} + \mathbf{\Pi}^{\mathbf{sec}}$ [5]. The exact solutions found in the literature only offer particular field expressions depending on the considered operational conditions.

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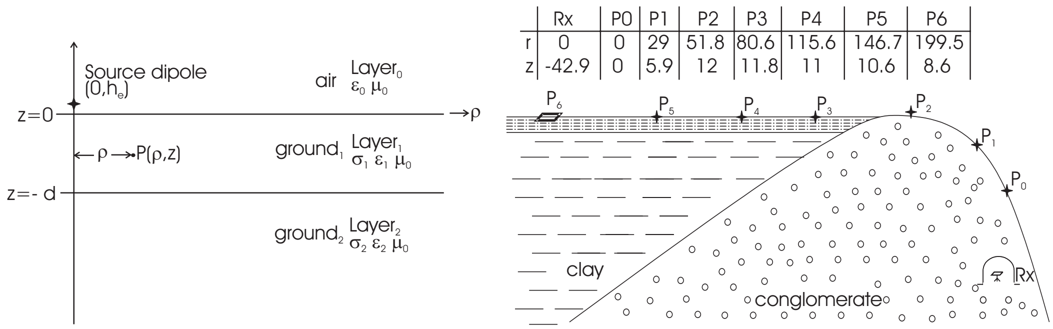

(a) (b)

Figure 1: (a) Layers and parameters (b) Geology schema and point positions

After establishing a common reference system, the exact magnetic field solutions have been reformulated for a magnetic dipole with arbitrary orientation and for the different stratified flat models. See Figure 1(a) for a description of the models and properties concerned in the calculations. For this work, the dipole is placed over a 1 or 2-layered half space in the z-axis at $z = h_e$ and the fields are calculated in the ground region for a radial distance and depth determined by the field experiment. The different theoretical models are taken into account by proper selection of medium properties.

Using the integral representation of the primary stimulation by means of the Fourier-Bessel transforms and applying the suitable boundary conditions, the field expressions have been obtained. We follow the work of Jones that clearly splits the contributions of the different mediums [6]. These expressions are programmed in MATLAB. The computation of the integral expression needs setting the integration parameters very carefully and also transforming the expression into a dimensionless form due to the oscillating nature of integrand [7].

Experimental validation and results

The Chaves cave in Sierra de Guara (Huesca), Spain, has been selected as the experimental site due to its inner and outer accessibility. It is a karstic cave that opens in a limestone conglomerate of Oligocene Age, located near the lateral transition with clays and sandstones of the same age. See Figure 1(b).

The experiment consists in positioning a vertical magnetic dipole at different emission points (P0 to P6) generating a magnetic field at 1, 3, 9.5, 28.4 and 97.8 KHz. The radial and axial magnetic field intensities are measured in the cave where the signal to noise ratio is expected to be much better than in surface. This is due to the earth shielding effect to atmospheric and man-made noise. It is important to note that several field work days were necessary to prepare battery-powered instrumentation able to work in the hostile environment surrounding the cave. Also, a vertical electric sounding (VES) to determine conductivity was made, as well as an accurately positioning of the inner and the outer points (differential GPS plus a total station of topography). In addition, nine people work during the test day.

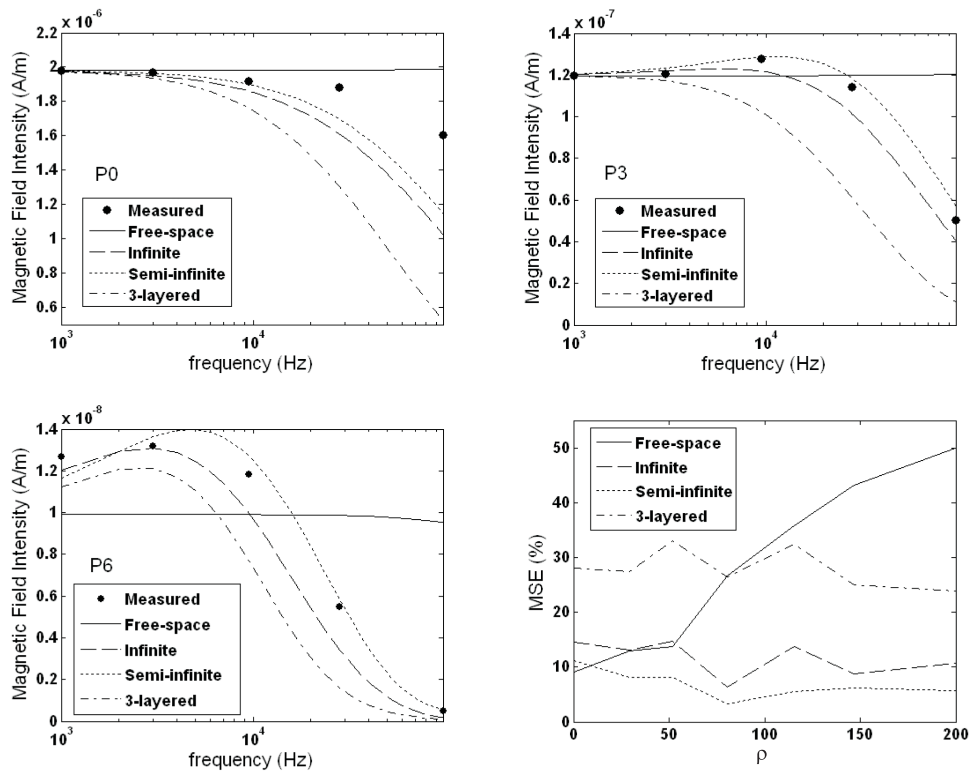

Figure 2: Computed and measured magnetic field intensity for P0, P3 and P6. The figure at the bottom right corner presents the mean squared error (MSE).

The emitter equipment consists in a precision function generator, a power amplifier, a tuning capacitor set for every selected frequency, and a square $1.5m$ side multi-turn loop antenna mounted on a regulable structure to guarantee a horizontal plane. The receiver is an active square loop that includes a voltage-current transformer to force a flat frequency response with high gain in all the band of interest. The output is feed to a Lock-In amplifier that allows a precise measurement for low signals in presence of noise. A $500m$ fiber optic link has to be used to transmit the reference signal from the distant emitter to the Lock-In in order to avoid interference conduction through the metallic cables that would otherwise invalidate the experiment.

The sensed area in the VES covers a range of $250m$ around P6. The interpretation of the measurements agrees with the geological studies and allows to establish the conductivities of the infinite and semi-infinite models as $\sigma_1 = 5.13mS/m$ and the 3-layered model as $\sigma_1 = 14.18mS/m$ and $\sigma_2 = 1mS/m$, with $54m$ depth for the central layer.

In Figure 2, the computed and experimental magnetic field intensity for three significant points are presented. The remaining figure compares computed and experimental results established through the mean squared error (MSE) defined as:

$$MSE = \frac{1}{N}\sum_{f}\left(\frac{\mathbf{H}_{calculated}\left(f\right) - \mathbf{H}_{measured}\left(f\right)}{\mathbf{H}_{calculated}\left(1KHz\right)}\right)^{2}; f = 1, ..., 97.8KHz; N = 5$$

As shown in Figure 2, the semi-infinite model agrees the best with the measured field intensity. Its MSE falls under 10 % for all points, being it lower for the farther ones. The uphill where the nearest points are placed causes the apparent earth conductivity to be lower than the value estimated by the VES. So, in these points the free-space model is also in good agreement with measurements. As expected, at the farther points the MSE of the free-space model raises up to 50 %. Both infinite and the 3-layered models have similar qualitative behavior but the 3-layered performs worse. This is probably due to the difficulty to match a 3-layered flat model to the complex 3D geometry shown by the geological structure.

Conclusions

The comparison of the computed results of the free-space, infinite dissipative, semi-infinite and 3-layered models with a field experiment is presented. Result analysis shows that the semi-infinite model describes better the reality than the 3-layered one due to the difficulty to obtain a good layered model of the experimental site. Thus, as a help in designing through-the-earth communication equipment a simple semi-infinite model seems to be the best choice. A detailed study of the phenomena observed in the field experiment suggests us the need to model the actual site geometry by means of a finite element description.

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