Spatio-Temporal Linear Expansions for Repolarization Analysis

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Abstract

In this work we propose a multichannel signal model based on linear expansions to analyze the cardiac repolarization. Our hypothesis is that a joint spatio-temporal signal description which takes into account both temporal and spatial features provide a more compact signal representation, i.e. the signal energy is packed into a smaller number of coefficients. In this work we also deal with the problem of estimating optimal basis functions in two different situations: when no information of the noise source is available and when the noise statistics is known or estimated from especific signals or intervals, like the TP segment.

1. Introduction

Myocardial ischemia is reflected in the ECG by amplitude changes in the ST segment and the T wave. The conventional ST level measurement (typically obtained at J+60 ms) represents a local measurement which unfortunately is vulnerable to various noise sources such as baseline wander and muscle; in addition, the measurement is rendered even more difficult due to heart rate related repolarization changes. In order to obtain more robust measurements, additional information can be introduced by making use of information from previous beats. For example, signal averaging [1] and its many variants relies on the observation that ischemia-induced beat-to-beat changes in ECG morphology are relatively slow.

Another approach is to analyze the complete repolarization waveform (STT complex) by means of linear expansions to get a global characterization of the repolarization waveform in each lead [2]. A desired property of the basis functions is that they should characterize the relevant features in a small subspace. Then, a few subset of expansion coefficients will characterize the dominant signal waveform. The trends defined by the subset of expansion coefficients reflect the main beat-to-beat evolution of repolarization changes, in the same way as the ST trends.

In this study we explore the spatial information available in multichannel ECG recordings for use in repolarization analysis. The repolarization waveforms from all leads are jointly analyzed by a truncated linear expansion model where the basis functions are matrices. Our hypothesis is that multichannel expansions may achieve a better packing of the signal energy than single-channel (temporal) expansions, taking into account the joint spatio-temporal information of the repolarization process.

Linear expansions is a well-known technique for signal analysis and modelling. It is based on the decomposition of the signal as a linear combination of simple and elementary basis functions which define a new signal representation domain [3]. The selection of the domain is a key factor and should be done according to the properties of the analyzed signal and the application. The optimal (in the mean squared error sense) linear and unitary transform for signal coding is the Karhunen-Loève transform (KLT) [4]. This transformation is data-dependent and it is estimated from a data training set. Two different kinds of training sets can be used for ECG analysis: a unique training set formed by a large number of signals containing a wide range of waveforms, or smaller patient-specific training sets. The latter are often much more homogeneous, providing a better energy packing performance. However, if the training signals are heavily contaminated by noise and only a small number of occurrences are available, the KL basis functions may be greatly affected by noise. This may be the case of exercise test recordings.

The aim of this work is twofold. Firstly, to introduce a multichannel signal model based on linear expansions which gives a joint spatio-temporal description of the signal. Secondly, to define a procedure to estimate optimal basis functions from a learning set of signals, considering two situations: when there is no available information from the noise source (or the observed signals have a good signal-to-noise ratio, SNR) and when information of the noise source is available.

2. Methods

2.1. Multichannel signal model

The information conveyed by a multichannel signal can be represented by a matrix $\mathbf{D}\in\mathbb{R}^{N\times L}$, N being the number of samples and L the number of sensors. The signal $\mathbf{D}$ can be decomposed as a linear combination of $N \times L$ linearly independent spatio-temporal functions (elementary matrices) $\mathbf{B}_{ij}$

$$\mathbf{D} = \sum_{i=1}^{N} \sum_{j=1}^{L} w_{ij} \, \mathbf{B}_{ij} \; . \tag{1}$$

The linear coefficients, w_{ij}, give information about the strength of the contribution of every function $\mathbf{B}_{ij}$ in the signal. Each $\mathbf{B}_{ij}$ carries spatial as well as temporal characteristics of the signal.

As these two characterstics are often decoupled, we can assume that the basis functions $\mathbf{B}_{ij}$ are separable (rank-one matrices)

$$\mathbf{B}_{ij} = \mathbf{t}_i \, \mathbf{s}_j^T \; , \tag{2}$$

where the temporal and spatial elementary vectors $\mathbf{t}_i$ and $\mathbf{s}_j$ are the *i*-th and *j*-th column of two matrices denoted respectively by $\mathbf{T}$ and $\mathbf{S}$. The only restriction for these matrices is that they must be full rank. The linear expansion (1) can then be written in matrix form as

$$\mathbf{D} = \mathbf{T} \, \mathbf{W} \, \mathbf{S}^T \; , \tag{3}$$

where $\mathbf{T} \in \mathbb{R}^{N \times N}$ contains the temporal information of the basis functions, $\mathbf{W}$ is the linear coefficient matrix formed by w_{ij}, and $\mathbf{S} \in \mathbb{R}^{L \times L}$ contains the spatial information of the basis functions. The particular case of channel-by-channel signal expansion can be obtained from (3) by setting $\mathbf{S} = \mathbf{I}$.

Truncated expansions are usually needed in several applications, such as data compression, feature extraction or filtering and can be interpreted as a restriction of the signal to a given subspace. Truncation in the linear model (3) is achieved by selecting $p < N$ basis functions from $\mathbf{T}$ and/or $q < L$ basis functions from $\mathbf{S}$ yielding the model $\mathbf{D} = \mathbf{T}_p \, \mathbf{W} \, \mathbf{S}_q^T$, where the columns of $\mathbf{T}_p$ and $\mathbf{S}_q$ are the truncated basis functions. For simplicity, the subscripts p and q will be dropped in the rest of the paper.

2.2. KLT of noiseless multichannel signals

The KLT basis functions are given by the dominant eigenvectors of the signal covariance matrix. However, several limitations make the application of the KLT difficult: it must be defined for each signal set, the computation of the basis functions is intensive, and no fast transformation algorithms can be easily defined because of the lack of structure in the basis functions. The last two limitations motivate lower-complexity approximations to the optimal transform, especially when long signals are analyzed, such as multichannel signals.

2.2.1 Global optimum.

The optimal 2D-KLT can be obtained by representing data matrices as piled vectors. The data matrix $\mathbf{D} \in \mathbb{R}^{N \times L}$ can be represented by

$$\widetilde{\mathbf{d}} \equiv \mathrm{vec}\left(\mathbf{D}\right) = \left[\mathbf{d}_1^T \, \mathbf{d}_2^T \, \cdots \mathbf{d}_L^T\right]^T \; , \tag{4}$$

where $\mathbf{d}_i$ are the columns of $\mathbf{D}$ and $\mathrm{vec}\left(\cdot\right)$ denotes the matrix-to-vector mapping [5]. The optimal basis matrices are given by the dominant eigenvectors of the $NL \times NL$ correlation matrix $\mathbf{R}_{\mathbf{d}} = E\left\{\widetilde{\mathbf{d}}\widetilde{\mathbf{d}}^T\right\}$ [3, 4]. Finally, the 2D-basis functions $\mathbf{B}_{ij}$ are the vector-to-matrix mapping of each eigenvector.

2.2.2 KLT of signals with separable autocorrelation

If the observed signal $\mathbf{D}$ has a separable autocorrelation function, i.e.,

$$E\left\{d_{ij} \, d_{kl}\right\} = r_d^t\left(i,k\right) \, r_d^s\left(j,l\right) \; , \tag{5}$$

the correlation matrix can be written as the Kronecker product

$$\mathbf{R} = \mathbf{R}^t \otimes \mathbf{R}^s \tag{6}$$

where $\mathbf{R}^t$ and $\mathbf{R}^s$ denote the $N \times N$ and $L \times L$ temporal and spatial covariance matrices respectively. When the assumption (6) holds, the eigenvectors of $\mathbf{R}$ (KLT basis functions) can be computed as the outer product of the eigenvectors of $\mathbf{R}^t$ and $\mathbf{R}^s$ [5, p. 423]. Now, two eigenproblems must be solved to compute the basis functions with a total complexity $O\left(N^3 + L^3\right)$, which is smaller than in the previous case, $O\left(N^3 L^3\right)$.

In addition, the basis matrices have a structure (separable matrices) which can be used to reduce the complexity to compute each transform coefficient from $O\left(NL\right)$ to $O\left(N + L\right)$.

When the assumption (6) does not hold, this transformation can be understood as an approximation to the optimal KLT with a lower computational complexity and with a lower energy packing peformance because each basis matrix is a rank-one approximation to the optimal eigenvectors. When the assumption (6) holds, both transformations are identical.

2.3. Optimal expansions for noisy signals

We consider now the problem of finding optimal linear reduced rank approximations when the noise statistics is known a priori (or estimated from specific time intervals, such as the TP interval, or specific channels).

The observed signal[1], $\mathbf{x} = \mathbf{d} + \mathbf{n}$ is composed of two terms: the desired signal $\mathbf{d}$ with correlation matrix $\mathbf{R}_{\mathbf{d}}$ and an uncorrelated noise $\mathbf{n}$ which is assumed to be stationary (in the temporal domain) and with correlation matrix $\mathbf{R}_{\mathbf{n}}$. The problem is to find the optimal reduced rank approximation of the observed signal which minimizes the MSE between the desired signal $\mathbf{d}$ and its approximation $\hat{\mathbf{d}}$. Figure 1 illustrates this problem, where $\mathbf{A} \in \mathbb{R}^{p \times N}$ and $\mathbf{B} \in \mathbb{R}^{N \times p}$ are reduced rank matrices with $p \leq N$. Note that no constraints are applied to $\mathbf{A}$ and $\mathbf{B}$, such as orthogonality.

[1] Vector notation is used to simplify expressions. For multichannel signals, the matrix-to-vector mapping, $\mathrm{vec}\left(\cdot\right)$ must first be used.

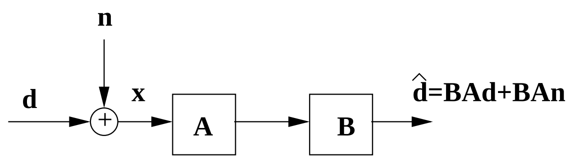

Figure 1. Linear reduced rank approximation of noisy signals.

The cost function can be written as

$$J = E\left\{ \left\| \mathbf{d} - \hat{\mathbf{d}} \right\|^2 \right\} = \mathrm{tr}\left\{\mathbf{R_d}\right\} + \mathrm{tr}\left\{\mathbf{BA}\left(\mathbf{R_d}+\mathbf{R_n}\right)\mathbf{A}^{\mathrm{T}}\mathbf{B}^{\mathrm{T}}\right\} - 2\,\mathrm{tr}\left\{\mathbf{BAR_d}\right\}. \quad (7)$$

Differentiation yields the equations

$$\mathbf{A} = \left(\mathbf{B}^T\mathbf{B}\right)^{-1}\mathbf{B}^T\mathbf{R_d}\left(\mathbf{R_d}+\mathbf{R_n}\right)^{-1} \quad (8)$$

$$\mathbf{B} = \mathbf{R_d}\mathbf{A}^T\left[\mathbf{A}\left(\mathbf{R_d}+\mathbf{R_n}\right)\mathbf{A}^T\right]^{-1}. \quad (9)$$

The reduced rank approximation operator is then given by

$$\mathbf{BA} = \mathbf{R_d}\mathbf{A^T}\left[\mathbf{A}\left(\mathbf{R_d}+\mathbf{R_n}\right)\mathbf{A^T}\right]^{\mathbf{-1}}\mathbf{A} \quad (10)$$

In the case of full-rank decompositions (square matrices $\mathbf{B}$ and $\mathbf{A}$), $\mathbf{BA}$ is equivalent to the full-rank Wiener filter because

$$\mathbf{BA} = \mathbf{R_d}\left(\mathbf{R_d}+\mathbf{R_n}\right)^{-1} \quad (11)$$

The minimum cost function is obtained by using (8) and (9) in (7), which, after some algebraic manipulations, can be written as

$$J_{\min} = \mathrm{tr}\left\{\mathbf{R_d}\right\} - \mathrm{tr}\left\{\mathbf{W}^T\mathbf{R_d}\left(\mathbf{R_d}+\mathbf{R_n}\right)^{-1}\mathbf{R_d}\mathbf{W}\right\}, \quad (12)$$

where $\mathbf{W} = \mathbf{B}\left(\mathbf{B}^T\mathbf{B}\right)^{-1/2}$. If complete expansions are used, $\mathbf{B}$ is a $N \times N$ full rank matrix, and $\mathbf{W}$ is unitary. If the expansion is truncated, then $\mathbf{A}$ and $\mathbf{B}$ are nonsquare, but $\mathbf{W}$ still has orthogonal columns. To find the optimal basis functions the term $\mathrm{tr}\left\{\mathbf{W}^T\mathbf{R_d}\left(\mathbf{R_d}+\mathbf{R_n}\right)^{-1}\mathbf{R_d}\mathbf{W}\right\}$ in (12) must be maximized with the orthogonality restriction on $\mathbf{W}$. Again, the solution is given by the dominant eigenvectors, but now of the matrix $\mathbf{R_d}\left(\mathbf{R_d}+\mathbf{R_n}\right)^{-1}\mathbf{R_d}$.

It may be noted that the optimal reduced rank approximation for noisy signals considers statistical information from both sources (signal and noise), in the same way as the Wiener filter does. More specifically, the same solution is obtained by the classical KLT when the training set is prefiltered with the Wiener filter.

In actual ECG recordings, the noise covariance matrix can be estimated from an interval with essentially no electrical activity of the heart, such as the TP segment, assuming that noise statistics is similar from the STT complex to the TP segment.

3. Materials and methods setup

ECG recordings from 114 non-selected subjects referred for myocardial scintigraphy were used in this work. The ECG signals were acquired during rest for five minutes using a standard 12 lead configuration, sampling rate of 1 kHz and amplitude resolution of 0.6 μV. Preprocessing analysis included removal of baseline wander, QRS detection and classification, selection of normal beats and finally STT complex segmentation to a fixed distance of the QRS fiducial point. A total of 37246 heartbeats were used in the training set. Before estimation of temporal and spatial covariance matrices, the STT data matrices were energy normalized in order to give the same representation strength to each complex.

Temporal and spatial noise covariance matrices $\mathbf{R}_\mathbf{n}^t$ and $\mathbf{R}_\mathbf{n}^s$ were estimated from the TP segment after highpass filtering for detrending purposes.

Three different implementations of the KLT were used to illustrate the benefits of the joint spatio-temporal analysis:

- global KLT of long-piled vectors $\widetilde{\mathbf{x}}$ as in section 2.2.1, (denoted as 2-D KLT). The number of basis functions can be any integer number $1 \leq i \leq NL$. The length of the basis functions is NL samples.
- KLT assuming separability with rank-one basis functions as in section 2.2.2 (denoted as ST-KLT). The number of basis functions is the product of two integer numbers $i = p\,q$ where $1 \leq p \leq N$ and $1 \leq q \leq L$. The basis functions are given by $N + L$ values.
- channel-by-channel KLT (denoted as 1D-KLT) where the matrix transformation $\mathbf{T}$ is formed by the dominant eigenvectors of $\mathbf{R}_\mathbf{x}^t$ and $\mathbf{S} = \mathbf{I}$. As as consequence, the number of coefficients is a multiple of the number of channels, $L = 9$ in our case. The length of the basis functions is N samples.

4. Results

The signal energy distribution in the training set is reflected in the eigenvalues of the signal covariance matrix. Figure 2 illustrates the percentage of signal energy represented by truncated expansions when the training set is formed by all subjects. 2D-KLT and ST-TKL get similar energy packing performance, suggesting that the dominant waveform of the STT complex can be accurately represented by a small subset of separable matrices. However, the computational complexity of the ST-KLT is much lower. In contrast, the energy packing peformance of the 1D-KLT is much lower because the inter-channel correlation is not considered.

The number of basis functions is a useful performance measure for some rank reduction applications like feature extraction or filtering, but not for others such as coding. A better performance measure for coding would be the date rate (measured on bits per second) of the coded signal. In

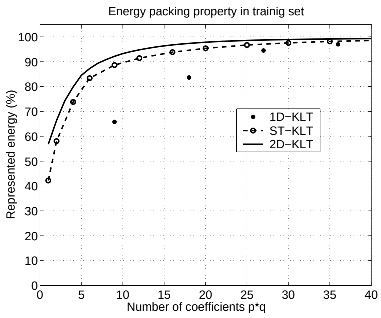

Figure 2. Energy packing property in a big training set.

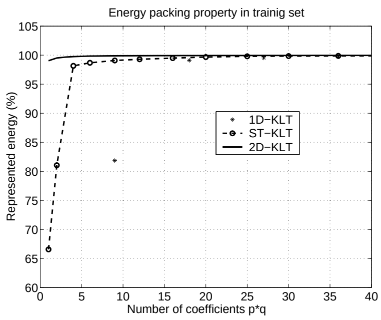

Figure 3. Energy packing property in a small training set.

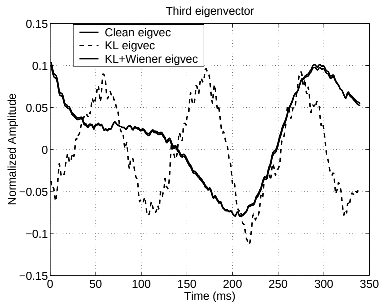

Figure 4. First dominant temporal eigenvector. Note that the two solid lines are overlaid.

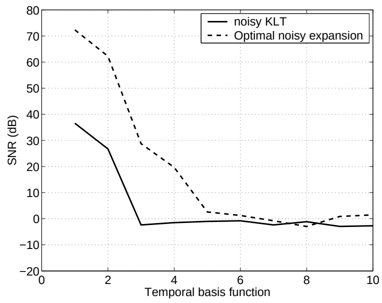

Figure 5. Noise effect on each dominant eigenvector.

that case, the side-information needed for coding the basis functions can be relevant.

If the training set is formed by only one subject (404 heartbeats), a much better energy packing is obtained due to the higher homogeneity of the training set (see Fig. 3).

Noisy simulated ECG signals were generated by adding noise to actual ECG recordings with a SNR of 10 dB. Noise was obtained by filtering Gaussian white noise in order to get the covariance matrices estimated from the TP segment. Figure 4 illustrates one example of the noise effect in the estimation of the basis functions when a small and noisy training set is used (150 STT complexes). The third dominant temporal eigenvector is presented as obtained by KLT of the clean signal (as a reference), KLT of the noisy signal and the optimal expansion of noisy signals. For this homogeneous training set, the signal subspace is reduced to the very few eigenvectors (as reflected in the eigenvalues diagram). Figure 5 illustrates the SNR of each eigenvector (in dB), showing that the dominant signal subspace is much better estimated when the noise source information is included in the expansion estimation.

5. Conclusions

Multichannel linear expansions provide a tool for joint spatio-temporal repolarization waveform analysis. The problem of estimation of multichannel optimal basis functions, in the mean squared error sense, was considered. ST-KLT provides an energy packing performance very close to the optimal 2D-KLT with a much lower complexity. A priori information of the noise source, when available, makes the estimation of the basis functions much more robust.

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