Robust array beamforming with sidelobe control using support vector machines

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Abstract — **Robust adaptive beamforming is a challenging task in wireless communications due to the strict restrictions in the number of available snapshots, signal mismatches, or calibration errors. In this paper, we present a new approach to adaptive beamforming that provides increased robustness against the mismatch problem as well as some control over the sidelobe level. We modify the conventional Capon cost function by including a regularization term that penalizes differences between the actual and the target (ideal) array responses. By using the so-called e-insensitive loss function as the penalty term, the cost function adopts the form of a support vector machine for regression. In particular, the resulting cost function is convex with a unique global minimum that can be efficiently found using quadratic programming techniques. Simulation examples show the performance of the proposed SVM-based beamformer when it is compared with traditional and other robust beamforming techniques.**

I. Introduction

Robust array beamforming has received considerable attention in the past years due to its importance for wireless communications, radar, medical imaging and other fields. To achieve high interference suppression and signal of interest (SOI) enhancement, an adaptive array must introduce deep nulls in the directions of arrival (DOA) of strong interferences while keeping the desired signal distortionless. This design criterion yields the well-known minimum variance distortionless response (MVDR) or Capon beamformer [1].

In practice, however, the knowledge of the array response to the desired signal can be imprecise, which often occurs due to estimation errors in the DOA of the desired signal or imperfect array calibration. In these situations, the performance of the MVDR beamformer is known to degrade substantially. Furthermore, when the number of snapshots used for covariance matrix estimation is insufficient, the MVDR beamformer can present unacceptably high sidelobes, which reduces its performance in the presence of powerful noise or unexpected interferences. A number of techniques have been proposed to improve the robustness of the MVDR beamformer for the signal look mismatch problem [2][3][4][5]. Recently, an approach with sidelobe control has been presented in [6]: the MVDR beamforming problem is modified to incorporate multiple quadratic inequality constraints outside the mainlobe beampattern. The corresponding optimization problem can be written as a second-order cone (SOC) programming problem. Although using this approach the sidelobe levels are guaranteed to be under a certain prescribed value (as long as a feasible solution exists), this technique does not consider the signal mismatch problem.

In this paper, we consider the application of support vector machines (SVM's) [7] [8] to adaptive beamforming. Based on the principle of structural risk minimization, the theory of SVM's was first introduced by Vapnik [7] and has recently found application in a number of communications problems such as blind equalization/identification [9][10] or multiuser detection [11]. Here we reformulate the MVDR beamforming problem by incorporating additional constraints that penalize sidelobe levels while, at the same time, allow a certain error in the desired signal direction. The resulting cost function adopts the form of a support vector machine for regression (SVM-R). The proposed SVM-based beamformer is a regularized solution which can be appropriate for rank-deficient scenarios. Unlike [6], which requires a feasible problem, the proposed SVM-based formulation always provides an approximate solution close to the prescribed sidelobe level. We present simulation examples where the performance of the proposed SVM beamformer is compared with the conventional Capon beamformer and with other robust beamforming techniques.

II. Background

The output of a narrowband beamformer composed by M sensors is given by

$$y(k) = \mathbf{w}^H \mathbf{x}(k),$$

where k is the time index, $\mathbf{x}(k) = \mathbf{x}_k = [x_1(k), \ldots, x_M(k)]^T \in C^{M\times 1}$ is the complex vector of array observations, $\mathbf{w} = [w_1, \ldots, w_M]^T \in C^{M\times 1}$ is the complex vector of beamformer weights, and $(.)^T$ and $(.)^H$ denote the transpose and Hermitian transpose, respectively. The observation (snapshot) vector is given by

$$\begin{aligned}\mathbf{x}(k) &= \mathbf{s}(k) + \mathbf{i}(k) + \mathbf{n}(k)\\ &= s(k)\mathbf{a}(\theta_s) + \sum_{j=1}^{N_i} i_j(k)\mathbf{a}(\theta_j) + \mathbf{n}(k),\end{aligned}$$

where $\mathbf{s}(k)$, $\mathbf{i}$(k), $\mathbf{n}$(k) are the desired signal, interference and noise components, respectively. The number of interference signals is N_i. Here, $s(k)$ and $i_j(k)$ are the signal and interference waveforms. θ_s and θ_j, $j = 1, \ldots, N_i$ denote the signal and interference directions of arrival, respectively, with corresponding steering vectors $\mathbf{a}(\theta_s)$ and $\mathbf{a}(\theta_j)$.

The classical formulation for the MVDR beamformer is

$$\min_{\mathbf{w}} E\left[|y(k)|^2\right] \qquad \text{subject to} \quad \mathbf{w}^H\mathbf{a}(\theta_s) = g \tag{1}$$

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whose solution is given by

$$\mathbf{w}_0 = \frac{g^* \mathbf{R}_x^{-1} \mathbf{a}(\theta_s)}{\mathbf{a}(\theta_s)^H \mathbf{R}_x^{-1} \mathbf{a}(\theta_s)}, \qquad (2)$$

where * denotes conjugate. The $M \times M$ matrix $\mathbf{R}_x$ is the theoretical covariance matrix of the array output vector. We assume that $\mathbf{R}_x \succ 0$ is a positive semidefinite matrix that has the following form

$$\mathbf{R}_x = \sigma_s^2 \mathbf{a}(\theta_s)^H \mathbf{a}(\theta_s) + \sum_{j=1}^{N_i} \sigma_j^2 \mathbf{a}(\theta_j)^H \mathbf{a}(\theta_j) + \mathbf{Q}, \qquad (3)$$

where σ_s^2 and σ_j^2, $j = 1, \ldots, N_i$ are the powers of the uncorrelated impinging signals $s(k)$ and $i(k)$, respectively, and $\mathbf{Q}$ is the noise covariance matrix.

In practice, the exact covariance matrix is not available and is replaced by the sampled covariance matrix $\hat{\mathbf{R}}_x$,

$$\hat{\mathbf{R}}_x = \frac{1}{N} \sum_{k=1}^{N} \mathbf{x}(k) \mathbf{x}(k)^H, \qquad (4)$$

where N is the number of observed snapshots.

III. MVDR beamforming with Sidelobe Control using SVMs

In this section we modify the basic MVDR beamforming problem by incorporating additional constraints in order to control the sidelobe level. The basic idea is similar to that proposed in [6], but here we use the constraints as a regularization term of the array output power. In this way, we end up with a quadratic programming (QP) problem, which is equivalent to that obtained for a support vector machine for regression (SVM-R), whereas in [6] a convex conic optimization problem is obtained. Unlike [6], which guarantees the prescribed sidelobe levels if the problem is feasible, the SVM technique is able to relax the sidelobe constraints but always provide an approximate solution. We feel this is an advantage of the proposed technique in comparison to [6]. Moreover, later we show that our procedure can be generalized to account for errors in the look direction.

Let us consider a grid of directions of arrival θ_i, $i = 1, \cdots, P$; which sample the beampattern in [-90°, 90°]. Without loss of generality we assume that θ_P is the direction of the desired signal. Based on the theory of SVMs [7], we consider the following regularized MVDR problem

$$\min_{\mathbf{w}} \quad \frac{1}{2} \mathbf{w}^H \mathbf{R}_x \mathbf{w} + C \sum_{i=1}^{P} |d_i - \mathbf{w}^H \mathbf{a}(\theta_i)|_\epsilon, \qquad (5)$$

where

$$|d_i - \mathbf{w}^H \mathbf{a}(\theta_i)|_\epsilon = \max\left\{0, |d_i - \mathbf{w}^H \mathbf{a}(\theta_i)| - \epsilon\right\}, \qquad (6)$$

is the so-called Vapnik's ϵ-insensitive loss function and d_i is the desired beamformer output

$$d_i = \begin{cases} 0, & i = 1, \cdots P-1, \\ g_R + j g_I, & i = P. \end{cases} \qquad (7)$$

The regularized cost function (5) establishes a trade-off between the array output power and a term that penalizes mismatches larger than ϵ between the actual and desired array responses for the given angle grid. The parameter ϵ defines the maximum gain level outside the mainlobe beampattern and therefore acts as a sidelobe control parameter. Note also that, unlike the conventional MVDR formulation and the method in [6], with the proposed formulation errors smaller that ϵ are allowed in the array response for the assumed signal arrival angle.

In order to apply the standard methodology for support vector regression the first term in (5) must be written as a complexity term for the SVM structure. Also, the overall problem must be rewritten in terms of real variables. To this end we use the eigenvalue decomposition of the sampled covariance matrix, $\hat{\mathbf{R}}_x = \mathbf{U}_x \mathbf{D}_x \mathbf{U}_x^H$; then, the power at the beamformer output can be written as

$$\mathbf{w}^H \hat{\mathbf{R}}_x \mathbf{w} = \hat{\mathbf{w}}^H \hat{\mathbf{w}} = \widetilde{\mathbf{w}}^T \widetilde{\mathbf{w}}, \qquad (8)$$

where $\hat{\mathbf{w}} \in C^{M \times 1}$ and $\widetilde{\mathbf{w}} \in R^{2M \times 1}$ are given by

$$\hat{\mathbf{w}} = \mathbf{D}_x^{1/2} \mathbf{U}_x^H \mathbf{w}, \qquad (9)$$

$$\widetilde{\mathbf{w}}^T = \begin{bmatrix} \hat{\mathbf{w}}_R^T & \hat{\mathbf{w}}_I^T \end{bmatrix}. \qquad (10)$$

On the other hand, the beamformer output for each DOA, $\mathbf{w}^H \mathbf{a}(\theta_i)$, can be written as

$$\mathbf{w}^H \mathbf{a}(\theta_i) = \hat{\mathbf{w}}^H \mathbf{D}_x^{-1/2} \mathbf{U}_x^H \mathbf{a}(\theta_i) = \hat{\mathbf{w}}^H \hat{\mathbf{a}}(\theta_i) \qquad (11)$$

where the new set of transformed steering vectors $\hat{\mathbf{a}}(\theta_i) \in C^{M \times 1}$ is defined as

$$\hat{\mathbf{a}}(\theta_i) = \mathbf{D}_x^{-1/2} \mathbf{U}_x^H \mathbf{a}(\theta_i). \qquad (12)$$

Likewise, it can be shown that

$$\hat{\mathbf{w}}^H \hat{\mathbf{a}}(\theta_i) = \widetilde{\mathbf{w}}^T \widetilde{\mathbf{a}}(\theta_i) + j \widetilde{\mathbf{w}}^T \widetilde{\mathbf{a}}'(\theta_i) \qquad (13)$$

where $\widetilde{\mathbf{a}}(\theta_i)$ and $\widetilde{\mathbf{a}}'(\theta_i) \in R^{2M \times 1}$ are given by

$$\begin{aligned} \widetilde{\mathbf{a}}(\theta_i)^T &= \begin{bmatrix} \hat{\mathbf{a}}_R^T(\theta_i) & \hat{\mathbf{a}}_I^T(\theta_i) \end{bmatrix} \\ \widetilde{\mathbf{a}}'(\theta_i)^T &= \begin{bmatrix} \hat{\mathbf{a}}_I^T(\theta_i) & -\hat{\mathbf{a}}_R^T(\theta_i) \end{bmatrix}. \end{aligned} \qquad (14)$$

Clearly, the regression problem can be expressed independently for the real and imaginary terms, as

$$\begin{aligned} \widetilde{\mathbf{w}}^T \widetilde{\mathbf{a}}(\theta_i) &= Re(d_i), \\ \widetilde{\mathbf{w}}^T \widetilde{\mathbf{a}}'(\theta_i) &= Im(d_i), \end{aligned}$$

for $i = 1, \cdots, P$, where d_i is the desired array response, which is given by (7).

For notational simplicity, we define the following compact variable $\bar{\mathbf{a}} \in R^{2M \times 2P}$,

$$\bar{\mathbf{a}}(i) = \begin{cases} \widetilde{\mathbf{a}}(\theta_i), & i = 1, \cdots P, \\ \widetilde{\mathbf{a}}'(\theta_{i-P}), & i = P+1, \cdots 2P. \end{cases} \qquad (15)$$

With these transformations, the initial complex formulation (5) can be written in terms of real variables as

$$J(\widetilde{\mathbf{w}}) = \frac{1}{2} \|\widetilde{\mathbf{w}}\|^2 + C \sum_{i=1}^{2P} |y_i - \widetilde{\mathbf{w}}^T \bar{\mathbf{a}}(i)|_\epsilon \qquad (16)$$

where the real variable $y_i = Re(d_i)$ for $i = 1, \cdots, P$, and $y_i = Im(d_i)$ for $i = P+1, \cdots, 2P$, represents the desired output for each product $\widetilde{\mathbf{w}}^T \bar{\mathbf{a}}(i)$.

Introducing a set of slack positive variables ξ and $\tilde{\xi}$, the cost function (16) can be written as the following optimisation problem with constraints [7]: minimize

$$L(\widetilde{\mathbf{w}}, \xi, \tilde{\xi}) = \frac{1}{2} \|\widetilde{\mathbf{w}}\|^2 + C \sum_{i=1}^{2P} (\xi_i + \tilde{\xi}_i) \qquad (17)$$

subject to

$$\widetilde{\mathbf{w}}^T\bar{\mathbf{a}}(i) - y_i \leq \epsilon + \xi_i, \quad (18)$$
$$y_i - \widetilde{\mathbf{w}}^T\bar{\mathbf{a}}(i) \leq \epsilon + \tilde{\xi}_i, \quad (19)$$
$$\xi_i, \tilde{\xi}_i \geq 0 \quad (20)$$

for $i = 1, \cdots, 2P$.

IV. SVM-based solution

The solution of the optimization problem with constraints is a saddle point of the Lagrange functional [7][8]

$$L(\widetilde{\mathbf{w}}, \xi, \tilde{\xi}, \alpha, \tilde{\alpha}, \gamma, \tilde{\gamma}) = \frac{1}{2}\|\widetilde{\mathbf{w}}\|^2 + C\sum_{i=1}^{2P}(\xi_i + \tilde{\xi}_i) - \sum_{i=1}^{2P}\left(\tilde{\gamma}_i\tilde{\xi}_i + \gamma_i\xi_i\right)$$
$$-\sum_{i=1}^{2P}\alpha_i\left(y_i - \widetilde{\mathbf{w}}^T\bar{\mathbf{a}}(i) + \epsilon + \xi_i\right) - \sum_{i=1}^{2P}\tilde{\alpha}_i\left(\widetilde{\mathbf{w}}^T\bar{\mathbf{a}}(i) - y_i + \epsilon + \tilde{\xi}_i\right) \quad (21)$$

minimum with respect to the primal variables $\widetilde{\mathbf{w}}$, ξ_i and $\tilde{\xi}_i$; and maximum with respect to the Lagrange multipliers $\alpha_i \geq 0$, $\tilde{\alpha}_i \geq 0$, $\gamma_i \geq 0$ and $\tilde{\gamma}_i \geq 0$, for $i = 1, \cdots, 2P$.

Differentiating the above Lagrangian with respect to $\widetilde{\mathbf{w}}$, ξ_i and $\tilde{\xi}_i$ yields to

$$\widetilde{\mathbf{w}} = \sum_{i=1}^{2P}(\tilde{\alpha}_i - \alpha_i)\,\bar{\mathbf{a}}(i), \quad (22)$$
$$\gamma_i = C - \alpha_i, \qquad i = 1, \cdots, 2P, \quad (23)$$
$$\tilde{\gamma}_i = C - \tilde{\alpha}_i, \qquad i = 1, \cdots, 2P. \quad (24)$$

Similarly to other SVM-based problems, here the optimal beamformer can be expanded in term of a set of steering vectors (those corresponding to $\tilde{\alpha}_i - \alpha_i \neq 0$), which are the support vectors for the problem. Substituting (22), (23) and (24) into (21), the Lagrange multipliers $\alpha_i, \tilde{\alpha}_i$ are the coefficients which maximize the following quadratic functional

$$W(\alpha, \tilde{\alpha}) = -\frac{1}{2}\sum_{i,j=1}^{2P}(\tilde{\alpha}_i - \alpha_i)(\tilde{\alpha}_j - \alpha_j)\langle(\bar{\mathbf{a}}(i), \bar{\mathbf{a}}(j)\rangle - \epsilon\sum_{i=1}^{2P}(\tilde{\alpha}_i + \alpha_i) + \sum_{i=1}^{2P}(\tilde{\alpha}_i - \alpha_i)\,y_i, \quad (25)$$

subject to $0 \leq \alpha_i, \tilde{\alpha}_i \leq C$; where $\langle\mathbf{x}, \mathbf{y}\rangle = \mathbf{x}^T\mathbf{y}$ denotes inner product.

Once α_i and $\tilde{\alpha}_i$ are computed by using quadratic programming techniques [12], the coefficients $\widetilde{\mathbf{w}}$ can be calculated with (22). Then, we construct $\hat{\mathbf{w}}$ by means of the inverse transformation of (10), i.e., each complex coefficient is formed as $\hat{w}(i) = \widetilde{w}(i) + j\widetilde{w}(i + M)$, for $i = 1, \ldots, M$. Finally, in order to derive the beamformer weights, the coefficients $\hat{\mathbf{w}}$ are mapped into the original base

$$\mathbf{w} = \mathbf{U}_x\mathbf{D}_x^{-1/2}\hat{\mathbf{w}}. \quad (26)$$

V. Robust array beamforming

It is well known that the MVDR beamformer suffers a severe performance degradation when the array response vector for the desired signal is not known exactly [3]-[5]. This degradation is especially noticeable at high SNR's or when a scarce number of snapshots is available to estimate the signal covariance matrix.

Many approaches has been proposed to improve the robustness of the MVDR beamformer (for a good review, see [1]). In [5], a Bayesian approach is derived where θ_s is assumed to be a random variable with a known *a priori* pdf that characterizes the level of uncertainty. The solution is a weighted sum of MVDR beamformers pointed at a set of candidates DOA's, whose relative contribution is determined from the *a posteriori* pdf of the DOA conditioned on observed data [5]. Diagonal loading methods and its extended versions [2][3][4] impose additional quadratic constraints either on the Euclidean norm of the weights vector or in its difference from a desired weight vector. In [2], the actual steering vector ranges between a given uncertainty set of the source steering vector, such as spherical or flat ellipsoidal.

In the previous section, exact knowledge of the desired DOA was assumed in the regression procedure. Obviously, the proposed technique can easily be extended to deal with source steering vector mismatches. Instead of a single steering vector corresponding to the assumed signal arrival angle, we can consider now a number of θ_i's around the assumed source DOA, for which the desired beamformer output is $g \neq 0$. In this way, we increase the mainlobe beampattern area to account for a possible mismatch error. Specifically, the desired output for the beamforming problem is defined now as

$$y_i = \begin{cases} 0 & \text{if } |\theta_i - \theta_s| > \triangle, \\ g & \text{if } |\theta_i - \theta_s| \leq \triangle, \end{cases} \quad (27)$$

where $\triangle$ denotes the angular mainlobe beamwidth centered at θ_s. The choice of $\triangle$ depends on the accuracy of the desired DOA measure. Any *a priori* information about the reasonable range of the source DOA error can be utilized to optimize the approach with similar principles as in [5].

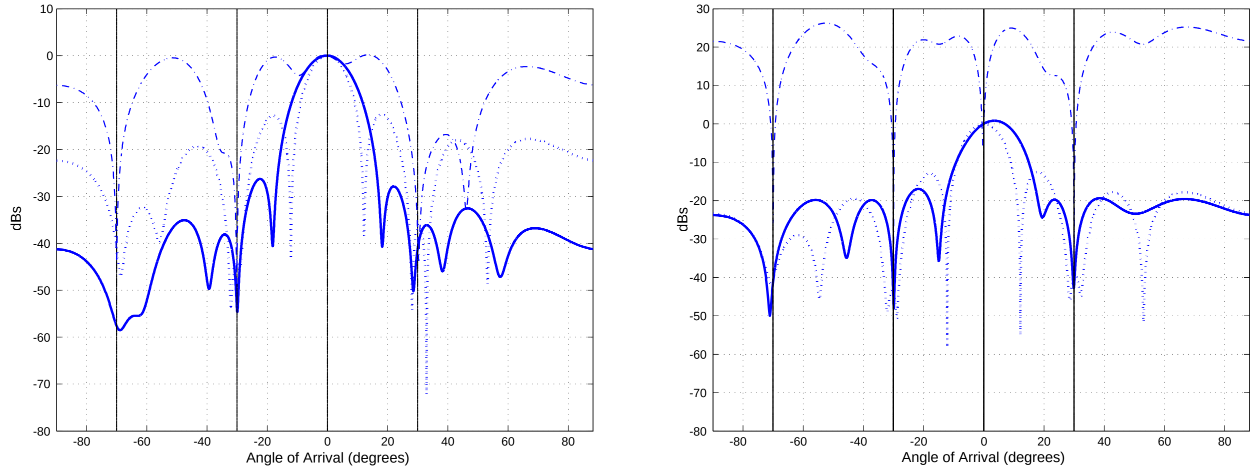
VI. Simulation results

Some computer simulations are carried out to demonstrate the performance of the proposed beamforming technique. In all the simulations we assume a uniform linear array with $M = 10$ sensors and half-wavelength sensor spacing.

Example 1

In the first simulation we consider the ideal scenario where the source steering vector is exactly known. All signal waveforms are i.i.d. QPSK. We assume a spatially white Gaussian noise whose covariance matrix is given by $\mathbf{Q} = \sigma_n^2\mathbf{I}$. The power of SOI is $\sigma_s^2 = 10$ dB, and the power of the interferences is $\sigma_j^2 = 30$ dB, $\forall j$. We assume that the SOI direction of arrival is $\theta_s = 0^o$, and that the DOAs of the interferences are $\theta_1 = -30^o$, $\theta_2 = 30^o$ and $\theta_3 = 70^o$. In order to compute $\hat{\mathbf{R}}_x$, $N = 50$ snapshots are used.

Fig.(1) shows the beampatterns, for signal-to-noise ration SNR = 5 and 30 dB, of the proposed SVM approach compared to the Capon beamformer and the robust array beamformer with spherical constraint proposed in [2] with control parameter $\epsilon = 4.5$ (in the following denoted as SpheRCB). In [2] is proven that the SpheRCB beamformer and the SOC method proposed in [4] provide the same solution. In our SVM approach, an uniform grid with $P = 11$ and $\triangle = 0^o$ is used to obtain the angles θ_i ($i = 1, \ldots, P$) between the range [-90^o,90^o], including θ_s. The support vector regression parameters are $C = 1$ and $\epsilon = 0.001$ for both SNR cases. The vertical lines in the figure indicate the DOAs of the SOI as well as the interferences. Note that the beampatterns have been scaled in order to achieve unity gain in the SOI direction.



(a) **(b)**

Fig. 1: No-mismatch case: SVM (solid line) with $\triangle = 0^o$, $P = 11$, $C = 1$ and $\epsilon = 0.001$; SpheRCB (dotted line) with $\epsilon = 4.5$, and Capon (dash-dotted line). **(a)** SNR = 5 dB, **(b)** SNR = 30 dB.

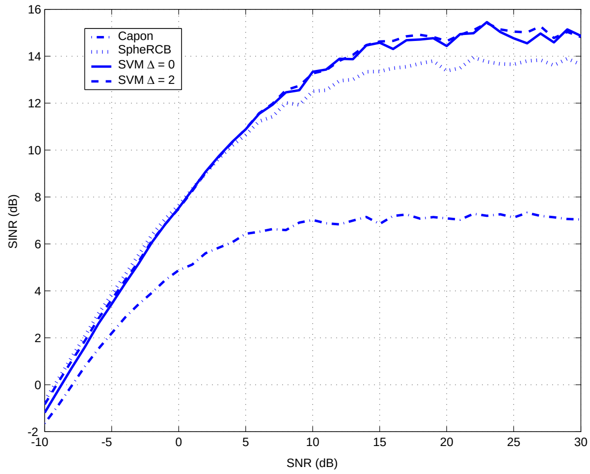


Fig. 2: No-mismatch case: SINR versus SNR, assuming three interferences with 30 dB of power at -30^o, -70^o and 30^o.

From Fig.(1) we see that, when the signal steering vector is exactly known, all the beampatterns have nulls at the DOAs of the interferences. On the other hand, at high SNR's the sidelobe level increases considerably for the Capon beampattern: this can result in deep degradations in case of unexpected interferences. Although the SVM beamformer does not present deeper nulls than the rest, it has a lower sidelobe level, particularly for low SNR's.

The performance of the proposed SVM beamformer depends on the prescribed angular mainlobe beamwidth $\triangle$ or, equivalently, on the number of regression points defined inside the mainlobe P_1 and outside the mainlobe P_2. To illustrate this point, Fig.(2) depicts the average (from 200 Montecarlo simulations) signal-to-interference-noise ratio (SINR) versus the SNR for the beamforming methods under comparison. For the SVM method with $\triangle = 0^o$ we choose $P_1 = 1$ and $P_2 = 10$, (i.e., the total number of θ_i is $P = 11$), whereas for $\triangle = 2^o$, we use $P_1 = 20$ and $P_2 = 40$. As can be seen, since we are considering a no-mismatch scenario, similar results are achieved with $\triangle = 0^o$ and $\triangle = 2^o$. On the other hand, both SVM procedures obtain better results than the Capon and SpheRCB approaches.

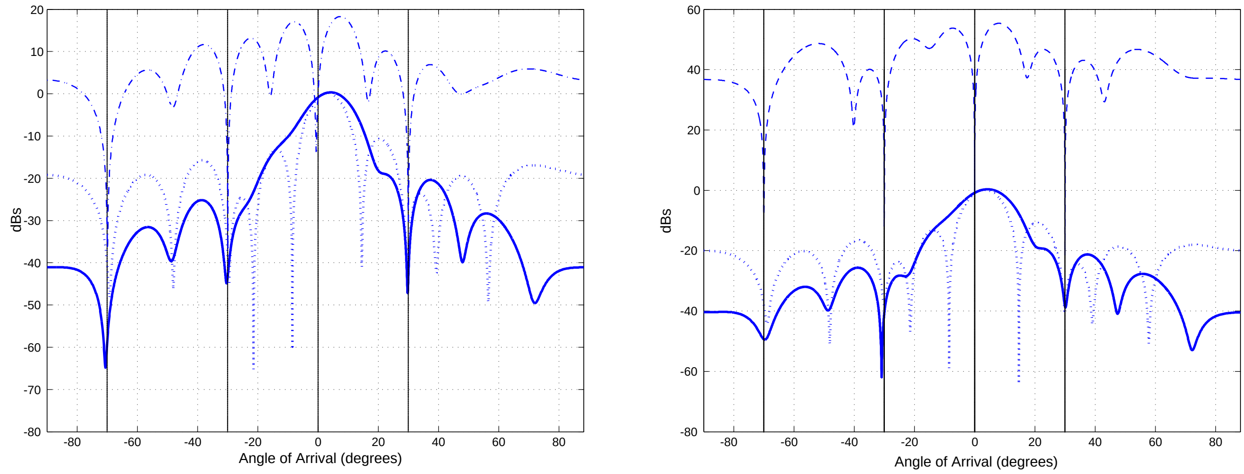
Example 2

In this example a scenario with some error in the SOI steering vector is considered. We assume that both the presumed and actual desired DOAs are $\theta_s = 2^o$ and $\theta_a = 0^o$, respectively. The rest of parameters are equal to those of the previous example. Fig.(3) shows a single realization of the beampatterns. As can be seen, the SOI is considered to be an interference by the Capon beamformer. On the other hand, the SOI is preserved by the SVM and the SpheRCB approaches. However, note that the SVM beampattern also has a lower sidelobe level than the SpheRCB solution.

Fig.(4.a) shows the averaged SINR versus SNR for this example: the SVM method behaves similar to the SpheRCB beamformer in the low SNR region, but the former achieves better results for high SNR. In Fig. (4.b), we vary the number of interferences from $N_i = 1$ to $N_i = 9$ with a fixed SNR of 10 dB. The power of the SOI is $\sigma_s^2 = 10$ dB, and the interference powers are $\sigma_1^2 = \ldots = \sigma_9^2 = 30$ dB. The SOI and interference DOAs are $\theta_s = 0^o$, $\theta_1 = -75^o$, $\theta_2 = 60^o$, $\theta_3 = -45^o$, $\theta_4 = 30^o$, $\theta_5 = -10^o$, $\theta_6 = -25^o$, $\theta_7 = 35^o$, $\theta_8 = -50^o$ and $\theta_9 = 70^o$. From Fig.(4.b) we see that the SVM-based beamformer provides better results than the SpheRCB regardless of the number of interferences.

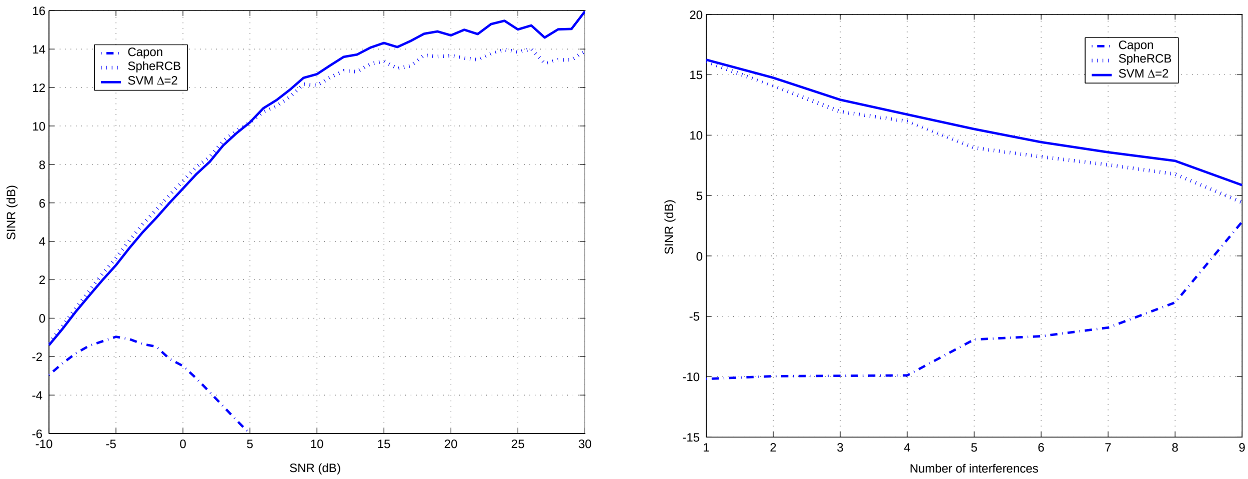
VII. Conclusions

In this paper, robust adaptive beamforming was reformulated as a support vector regression problem. The proposed approach modifies the traditional Capon beamformer with the goals of: a) increasing the beamformer robustness against errors in the desired signal array response and b) providing some additional control over the sidelobe level. By using the ϵ-insensitive loss function in the regression problem we end up with a convex function that can be efficiently minimized. The satisfactory performance of the proposed SVM beamformer was demonstrated through computer simulations, both in no-mismatch and mismatch situations. The proposed method was shown to provide suitable results, specially for high SNR scenarios and when the number of available snapshots is scarce. Future work should be directed to optimize the selection of the SVM parameters (loss function, C and ϵ), as well as to reduce the complexity of the QP optimization (for instance using the IRWLS procedure described in [13]) of the proposed beamforming approach.



(a) **(b)**

Fig. 3: Look direction mismatch of 2°: SVM (solid line) with $\triangle = 2^o$, $P_1 = 20$, $P_2 = 40$, $C = 1$ and $\epsilon = 0.001$; SpheRCB (dotted line) with $\epsilon = 4.5$ and Capon (dash-dotted line). **(a)** SNR = 5 dB, **(b)** SNR = 30 dB.



(a) **(b)**

Fig. 4: Look direction mismatch of 2°: **(a)** SINR versus SNR, assuming three interferences with 30 dB of power at -30°, 30° and 70°. **(b)** SINR versus number of interferences with SNR = 10 dB.

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