Robust Array Beamforming With Sidelobe Control Using Support Vector Machines

César C. Gaudes, *Student Member, IEEE*, Ignacio Santamaría, *Senior Member, IEEE*, Javier Vía, *Student Member, IEEE*, Enrique Masgrau Gómez, *Senior Member, IEEE*, and Talesa Sesé Paules

***Abstract*—Robust beamforming is a challenging task in a number of applications (radar, sonar, wireless communications, etc.) due to strict restrictions on the number of available snapshots, signal mismatches, or calibration errors. We present a new approach to adaptive beamforming that provides increased robustness against the mismatch problem as well as additional control over the sidelobe level. We generalize the conventional linearly constrained minimum variance cost function by including a regularization term that penalizes differences between the actual and the target (ideal) array responses. By using the so-called ϵ-insensitive loss function for the penalty term, the final cost function adopts the form of a support vector machine (SVM) for regression. In particular, the resulting cost function is convex with a unique global minimum that has traditionally been found using quadratic programming (QP) techniques. To alleviate the computational cost of conventional QP techniques, we use an iterative reweighted least-squares (IRWLS) procedure, which also converges to the SVM solution. Computer simulations demonstrate an improved performance of the proposed SVM-based beamformer, in comparison with other recently proposed robust beamforming techniques.**

***Index Terms*—Adaptive arrays, linearly constrained minimum variance beamformer, robust beamforming, robust Capon beamforming, sidelobe control, steering vector errors, support vector machines (SVMs) for regression.**

I. INTRODUCTION

ROBUST array beamforming has drawn considerable attention in the past years due to its importance for wireless communications, sonar, medical imaging, astronomy, and other applications. Providing robustness against model mismatches and tracking possible environment changes calls for robust adaptive beamforming techniques. In order to achieve high interference suppression and signal-of-interest (SOI) enhancement, an adaptive array must introduce deep nulls in the directions of arrival (DOA) of strong interferences, while keeping the desired signal distortionless. This design criterion yields the well-known minimum-variance distortionless response (MVDR) beamformer [1].

In practice, the knowledge of the desired steering vector can be imprecise, which often occurs due to estimation errors in the DOA of the desired signal or imperfect array calibration. In these situations, the SOI is considered as an interference and the performance of the MVDR beamformer is known to degrade substantially. This undesired behavior results in a reduction of the array output signal-to-interference-plus-noise-ratio (SINR) or erroneous SOI power estimation. The performance degradation due to the signal nulling effect and the resolution of the MVDR beamformer in the presence of steering vector errors are analyzed in [2] and [3]. Similar degradation occurs when the number of snapshots used for covariance matrix estimation is insufficient. In this situation, the MVDR beamformer can present unacceptably high sidelobes, which reduce its performance in the presence of high noise or unexpected interferences. A complete study about the performance of the MVDR beamformer considering these practical drawbacks is presented in [4], where an expression for the output SINR is obtained when only one interference signal impinges into the array. An extension of this analysis for scenarios with steering vector mismatch is developed in [5].

A large number of approaches have been presented in the array processing literature in order to improve the robustness of the MVDR beamformer (see [1] and [6] for an extensive review, and the references therein). Among them, diagonal loading (DL) has been widely employed due to its simplicity and relatively acceptable performance [7], [8]. Probably the first DL approach for robust beamforming is the classic work by Cox *et al.* [7], where a white noise gain constraint (i.e., a quadratic constraint on the beamformer weights) is incorporated into the cost function. However, a serious drawback of the diagonal loading method is that it is not clear how to select the diagonal loading factor based on the uncertainty of the desired steering vector or the number of available snapshots [9]. Recently, a number of techniques have been proposed to improve the robustness of the minimum variance beamformer [6], [10]–[13]. These algorithms make explicit use of the uncertainty set of the array steering vector in order to derive DL solutions. An online version of this robust beamformer using a constrained Kalman

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C. C. Gaudes was with the Communications Technology Group (GTC), Aragón Institute for Engineering Research (I3A), University of Zaragoza. He is now with the School of Computer Science and Information Technology, University of Nottingham, Jubilee Campus, NG8 1BB, Nottingham, U.K. (e-mail: cxc@cs.nott.ac.uk).

I. Santamaría and J. Vía are with the Communications Engineering Department (DICOM), University of Cantabria, Santander 39005, Spain (e-mail: nacho@gtas.dicom.unican.es; jvia@gtas.dicom.unican.es).

E. Masgrau Gómez is with the Communications Technology Group (GTC), Aragón Institute for Engineering Research (I3A), University of Zaragoza, 50018, Zaragoza, Spain (e-mail: masgrau@unizar.es).

T. Sesé Paules was with the Communications Technology Group (GTC), Áragon Institute for Engineering Research (I3A), University of Zaragoza. She is now with DMR Consulting (e-mail:tsese@unizar.es).

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filter is presented in [14]. A different scenario occurs when the source signal is far from being a point source. In this case, multirank beamformers must be considered to enhance the SOI [2], [15], [16]. Other robust beamforming approaches that do not use diagonal loading are the eigenspace beamformers [17]–[19], the covariance tapering methods [20]–[23], the Bayesian beamformer [24], the Hung–Turner adaptive beamformers [25], [26] or the fuzzy-inference-based beamformer [27]. From another point of view, a general statistical analysis based of the DL beamformer is carried out in [28] based on random matrix theory. This analysis sheds light on the array performance when the number of available snapshots and the number of sensors have the same order of magnitude, and it is useful to derive array signal processing architectures.

With regard to sidelobe control, a new approach has been presented in [29], where the MVDR beamforming problem is modified to incorporate multiple quadratic inequality constraints outside the mainlobe beampattern. The corresponding optimization problem can be written as a second-order cone (SOC) programming problem. Although using this approach the sidelobe levels are guaranteed to be under a certain prescribed value (as long as a feasible solution exists), this technique does not consider the signal mismatch problem. Another method for sidelobe control is presented in [30], where semidefinite programming is considered for array pattern design in both mismatch and non-mismatch situations. However, this approach exhibits poor interference rejection.

The aim of this work is to consider the application of support vector machines (SVMs) [31], [32] to robust beamforming. Based on the principle of structural risk minimization, the theory of SVMs was first introduced by Vapnik [31] and has found application in a number of communications problems such as blind equalization/identification [33]–[35] and multiuser detection [36]. Here, we reformulate the minimum variance beamforming problem by incorporating additional inequality constraints that penalize sidelobe levels while, at the same time, allow a certain error in the desired signal direction. The resulting cost function adopts the form of a support vector machine for regression [37]. The proposed SVM-based beamformer is a regularized solution which can be appropriate for rank-deficient scenarios. Unlike [29], which requires a feasible problem to obtain the beamformer coefficients, the proposed SVM-based formulation always provides an approximate solution close to the prescribed sidelobe level. While the SVM solution has traditionally been found by means of quadratic programming (QP) techniques, here we use an iterative reweighted least-squares (IRWLS) procedure, which considerably reduces the complexity of conventional QP techniques. This procedure has been successfully applied to solve SVM problems [38], and it has recently been proven to converge to the SVM solution [39], [40]. Performance simulations of the proposed SVM beamformer solved via IRWLS are presented, including comparisons with the conventional MVDR beamformer and with other robust beamforming techniques. These results indicate that the proposed beamformer shows robust operation in no-mismatch and mismatch scenarios, even when the DOA estimation error is larger than expected.

The organization of this paper is as follows. Section II contains the signal model and presents the MVDR solution. The basic ideas of our SVM-based beamforming approach are introduced in Section III, with emphasis on the formulation of the regularized cost function. The resulting QP problem is derived in Section IV, and the IRWLS minimization procedure is described in Section V. Numerical results under different mismatch scenarios are illustrated in Section VI. Finally, conclusions and possible directions for future work are pointed out in Section VII.

II. BACKGROUND

The output of a narrowband beamformer composed by M sensors is given by

$$y(k) = \mathbf{w}^H \mathbf{x}(k)$$

where k is the time index, $\mathbf{x}(k) = [x_1(k), \ldots, x_M(k)]^T \in \mathbb{C}^{M\times 1}$ is the complex vector of array observations, $\mathbf{w} = [w_1, \ldots, w_M]^T \in \mathbb{C}^{M\times 1}$ is the complex vector of beamformer weights, and $(.)^T$ and $(.)^H$ denote transpose and conjugate transpose, respectively. Regarding the notation of this paper, lower and upper boldface letters are used for vectors and matrices, respectively. The observation (snapshot) vector at time instant k is given by

$$\begin{aligned}\mathbf{x}(k) &= \mathbf{s}(k) + \mathbf{i}(k) + \mathbf{n}(k)\\ &= s(k)\mathbf{a}(\theta_s) + \sum_{j=1}^{N_i} i_j(k)\mathbf{a}(\theta_j) + \mathbf{n}(k)\end{aligned}$$

where N_i is the number of interference signals. Here, $s(k)$ and $i_j(k)$ are the signal and interference symbol samples. The signal and interference directions of arrival (DOA) are θ_s and θ_j, $j = 1, \ldots, N_i$, respectively, with corresponding steering vectors $\mathbf{a}(\theta_s)$ and $\mathbf{a}(\theta_j)$. Let $\mathbf{R}_x$ denote the $M \times M$ theoretical covariance matrix of the array snapshot vector. We assume that $\mathbf{R}_x \succ 0$ is a positive definite matrix with the following form:

$$\mathbf{R}_x = \sigma_s^2 \mathbf{a}(\theta_s)\mathbf{a}(\theta_s)^H + \sum_{j=1}^{N_i} \sigma_j^2 \mathbf{a}(\theta_j)\mathbf{a}(\theta_j)^H + \mathbf{Q}$$

where σ_s^2 and σ_j^2, $j = 1, \ldots, N_i$ are the powers of the uncorrelated impinging signals $s(k)$ and $i_j(k)$, respectively, and $\mathbf{Q}$ is the noise covariance matrix.

The classical formulation for the linearly constrained minimum-variance (LCMV) beamformer [41] is to minimize the array output energy $E\left[|y(k)|^2\right] = \mathbf{w}^H\mathbf{R}_x\mathbf{w}$, subject to a linear constraint on the desired DOA, i.e.,

$$\min_{\mathbf{w}} \quad \mathbf{w}^H\mathbf{R}_x\mathbf{w} \qquad \text{subject to} \qquad \mathbf{w}^H\mathbf{a}(\theta_s) = g. \quad (1)$$

The complex constant g determines the array response at the desired DOA. For the particular case of $g = 1$, this response is maintained constant at the look direction and the LCMV beamformer is commonly denoted as minimum variance distortionless response (MVDR) beamformer. The solution of (1) for this particular case is

$$\mathbf{w}_{\mathrm{MVDR}} = \frac{\mathbf{R}_x^{-1}\mathbf{a}(\theta_s)}{\mathbf{a}(\theta_s)^H\mathbf{R}_x^{-1}\mathbf{a}(\theta_s)}.$$

The idea behind the LCMV beamformer can be generalized by introducing a set of m linear constraints of the form $\mathbf{C}^H\mathbf{w} = \mathbf{g}$. The optimization problem in this case is

$$\min_{\mathbf{w}} \quad \mathbf{w}^H\mathbf{R}_x\mathbf{w} \qquad \text{subject to} \qquad \mathbf{C}^H\mathbf{w} = \mathbf{g}$$

whose solution is given by

$$\mathbf{w}_{\text{LCMV}} = \mathbf{R}_x^{-1}\mathbf{C}(\mathbf{C}^H\mathbf{R}_x^{-1}\mathbf{C})^{-1}\mathbf{g}$$

where $\mathbf{C}$ is the $M \times m$ matrix with the m linear constraints and $\mathbf{g}$ is the $m\times 1$ vector of constraint values. Note that for the LCMV beamformer, the number of linear constraints must be lower than the number of sensors, i.e., $m \leq M$; otherwise, we do not have enough degrees of freedom to minimize the power at the output of the beamformer. Interestingly, it was shown in [42] that this generalization of the LCMV beamformer can be transformed into the so-called generalized sidelobe canceller (GSC). The idea of the GSC consists of decomposing the beamformer in two components: a quiescent vector satisfying the prescribed conditions and an unconstrained vector orthogonal to the subspace of constraints. Some methods for improving the robustness of the GSC have been recently presented in [43] and [44].

In practice, the exact covariance matrix is not available and is replaced by the sample covariance matrix $\hat{\mathbf{R}}_x$

$$\hat{\mathbf{R}}_x = \frac{1}{N}\sum_{k=1}^{N}\mathbf{x}(k)\mathbf{x}(k)^H$$

where N is the number of observed snapshots. A more adequate definition of $\hat{\mathbf{R}}_x$, according to the ratio between the number of sensors and the available observations, can be established based on random matrix analysis [28].

III. Proposed Robust Cost Function With Sidelobe Control

In this section, we modify the conventional minimum variance beamforming problem by incorporating into the cost function additional inequality constraints in order to increase the robustness against mismatches in the SOI steering vector, as well as to control the sidelobe level. The resulting cost function turns out to be equivalent to a SVM for regression.

Let us consider a grid of directions of arrival $\theta_i, i = 1, \ldots, P$, which sample the beampattern in $[-90^\circ, 90^\circ]$. We define an angular mainlobe beamwidth $2\triangle$ centered at the assumed SOI DOA θ_s. P_1 from the total set of angles sample the mainlobe beamwidth, including θ_s. The remaining $P_2 = P - P_1$ angles sample the beampattern outside the mainlobe. Using this sampled grid of DOAs, the following desired or target beamformer response is established

$$d_i = \begin{cases} 0, & \text{if } |\theta_i - \theta_s| > \triangle \\ \text{Re}(g) + j\text{Im}(g), & \text{if } |\theta_i - \theta_s| \leq \triangle \end{cases} \tag{2}$$

which takes into account a possible signal mismatch error up to $\triangle$ degrees. In (2) and from now on, $\text{Re}(\cdot)$ and $\text{Im}(\cdot)$ denote, respectively, the real and imaginary parts of a scalar, vector, or matrix.

In order to apply the proposed procedure, the optimization problem must be rewritten in terms of real variables. To this end, the array output power can be written as

$$\mathbf{w}^H\mathbf{R}_x\mathbf{w} = \widetilde{\mathbf{w}}^T\widetilde{\mathbf{R}}_x\widetilde{\mathbf{w}}$$

where $\widetilde{\mathbf{w}} \in \mathbb{R}^{2M\times 1}$ is

$$\widetilde{\mathbf{w}}^T = \left[\text{Re}\left(\mathbf{w}^T\right) \quad \text{Im}\left(\mathbf{w}^T\right)\right]$$

and $\widetilde{\mathbf{R}}_x \in \mathbb{R}^{2M\times 2M}$ is

$$\widetilde{\mathbf{R}}_x = \begin{bmatrix} \text{Re}\,(\mathbf{R}_x) & -\text{Im}\,(\mathbf{R}_x) \\ \text{Im}\,(\mathbf{R}_x) & \text{Re}\,(\mathbf{R}_x) \end{bmatrix}.$$

Likewise, the beamformer output for each DOA can be written in terms of real variables as

$$\mathbf{w}^H\mathbf{a}(\theta_i) = \widetilde{\mathbf{w}}^T\widetilde{\mathbf{a}}(\theta_i) + j\widetilde{\mathbf{w}}^T\widetilde{\mathbf{a}}'(\theta_i)$$

where $\widetilde{\mathbf{a}}(\theta_i)$ and $\widetilde{\mathbf{a}}'(\theta_i) \in \mathbb{R}^{2M\times 1}$ are given by

$$\begin{aligned} \widetilde{\mathbf{a}}(\theta_i)^T &= \left[\text{Re}\left(\mathbf{a}^T(\theta_i)\right) \quad \text{Im}\left(\mathbf{a}^T(\theta_i)\right)\right] \\ \widetilde{\mathbf{a}}'(\theta_i)^T &= \left[\text{Im}\left(\mathbf{a}^T(\theta_i)\right) \quad -\text{Re}\left(\mathbf{a}^T(\theta_i)\right)\right]. \end{aligned}$$

For notational simplicity, we define the following real variables:

$$\bar{\mathbf{a}}(i) = \begin{cases} \widetilde{\mathbf{a}}(\theta_i), & i = 1, \ldots, P \\ \widetilde{\mathbf{a}}'(\theta_{i-P}), & i = P+1, \ldots, 2P \end{cases}$$

and

$$\widetilde{d}_i = \begin{cases} \text{Re}(d_i), & i = 1, \ldots, P \\ \text{Im}(d_i), & i = P+1, \ldots, 2P. \end{cases}$$

Using these definitions, our goal is to obtain a beamformer that minimizes its output power

$$\min_{\mathbf{w}} \quad \widetilde{\mathbf{w}}^T\widetilde{\mathbf{R}}_x\widetilde{\mathbf{w}}$$

subject to the following set of inequality constraints:

$$\begin{aligned} \widetilde{\mathbf{w}}^T\bar{\mathbf{a}}(i) - \widetilde{d}_i &\leq \epsilon \\ \widetilde{d}_i - \widetilde{\mathbf{w}}^T\bar{\mathbf{a}}(i) &\leq \epsilon \end{aligned}$$

for $i = 1, \ldots, 2P$. The parameter ϵ defines the set of admissible beamformer solutions, i.e., any beamformer whose outputs over the specified grid of DOAs are within an ϵ-band around the target array response is an admissible solution. Among all admissible beamformers, the one with minimum output power would be the solution of the optimization problem. Unfortunately, even for a moderate number of inequality constraints, the set of admissible beamformers is empty, and the previous problem has no solution. To avoid this drawback and in order to relax the constraints, we introduce a set of slack positive variables ξ_i and $\tilde{\xi}_i$ and consider the problem of minimizing

$$L(\widetilde{\mathbf{w}}, \boldsymbol{\xi}, \tilde{\boldsymbol{\xi}}) = \frac{1}{2}\widetilde{\mathbf{w}}^T\widetilde{\mathbf{R}}_x\widetilde{\mathbf{w}} + C\sum_{i=1}^{2P}\left[(\xi_i)^m + (\tilde{\xi}_i)^m\right] \tag{3}$$

subject to

$$\widetilde{\mathbf{w}}^T\bar{\mathbf{a}}(i) - \widetilde{d}_i \le \epsilon + \xi_i \tag{4}$$

$$\widetilde{d}_i - \widetilde{\mathbf{w}}^T\bar{\mathbf{a}}(i) \le \epsilon + \tilde{\xi}_i \tag{5}$$

$$\xi_i, \tilde{\xi}_i \ge 0 \tag{6}$$

for $i = 1, \ldots, 2P$, where $C \ge 0$ is a regularization constant, which sets a tradeoff between the output power term and a term that penalizes mismatches larger than ϵ between the actual and desired array responses for the given angle grid. On the other hand, the exponent m of the slacks allows us to impose a linear ($m = 1$) or a quadratic ($m = 2$) penalty.

The previous optimization problem is equivalent to that obtained for a support vector machine for regression [31], [32]. An alternative formulation of the problem, which will be useful later, is the following:

minimize

$$J(\widetilde{\mathbf{w}}) = \frac{1}{2}\widetilde{\mathbf{w}}^T\widetilde{\mathbf{R}}_x\widetilde{\mathbf{w}} + C\sum_{i=1}^{2P} L_\epsilon^m(u_i) \tag{7}$$

where $u_i = |\widetilde{d}_i - \widetilde{\mathbf{w}}^T\bar{\mathbf{a}}(i)|$, and

$$L_\epsilon^m(u) = \begin{cases} 0, & \text{if } |u| < \epsilon, \\ |u-\epsilon|^m, & \text{if } |u| \ge \epsilon. \end{cases} \tag{8}$$

According to the SVM terminology, the regularization term $L_\epsilon^m(u)$ is the Vapnik's ϵ-insensitive loss function (which can be either linear $m = 1$ or quadratic $m = 2$). To summarize, the procedure can be interpreted as a regression problem for which the parameter ϵ defines the maximum gain level outside the mainlobe beampattern and therefore acts as a sidelobe control parameter. Note also that, unlike the conventional MVDR formulation and the method in [29], the proposed formulation allows errors smaller than ϵ in the array response for the assumed signal arrival angle θ_s. Therefore, strictly speaking, the proposed procedure is not a "distortionless" beamformer.

The optimal values of C and ϵ must be established for each scenario [45], depending on the number of sensors, the noise level, the required sidelobe level and the presumed DOA estimation error. As a rule of thumb, in noisy scenarios, we should decrease the sidelobe level by increasing C and diminishing ϵ. On the contrary, when the presence of jammers is dominant, we must increase the interference rejection capabilities of the method by setting low values of C.

IV. SVM-Based Solution

To solve the above optimization problem (3), we use the method of Lagrange multipliers. For the sake of simplicity in our exposition, we only derive the case corresponding to the Vapnik's linear ϵ-insensitive loss function ($m = 1$). For the quadratic loss function ($m = 2$), we would proceed in a similar way [31].

The solution for the minimization problem (3) subject to (4)–(6) is the saddle point of the following Lagrange functional [31], [32]:

$$\begin{aligned} L(\widetilde{\mathbf{w}}, \boldsymbol{\xi}, \tilde{\boldsymbol{\xi}}, \boldsymbol{\alpha}, \tilde{\boldsymbol{\alpha}}, \boldsymbol{\gamma}, \tilde{\boldsymbol{\gamma}}) &= \frac{1}{2}\widetilde{\mathbf{w}}^T\widetilde{\mathbf{R}}_x\widetilde{\mathbf{w}} - \sum_{i=1}^{2P}\left(\tilde{\gamma}_i\tilde{\xi}_i + \gamma_i\xi_i\right) \\ &\quad + C\sum_{i=1}^{2P}\left(\xi_i + \tilde{\xi}_i\right) - \sum_{i=1}^{2P}\alpha_i\left(\widetilde{d}_i - \widetilde{\mathbf{w}}^T\bar{\mathbf{a}}(i) + \epsilon + \xi_i\right) \\ &\quad - \sum_{i=1}^{2P}\tilde{\alpha}_i\left(\widetilde{\mathbf{w}}^T\bar{\mathbf{a}}(i) - \widetilde{d}_i + \epsilon + \tilde{\xi}_i\right). \end{aligned} \tag{9}$$

Note that a dual set of variables has been introduced to incorporate the constraints. This Lagrange functional must be minimized with respect to the primal variables $\widetilde{\mathbf{w}}$, ξ_i, and $\tilde{\xi}_i$ and maximized with respect to the Lagrange multipliers $\alpha_i \ge 0$, $\tilde{\alpha}_i \ge 0$, $\gamma_i \ge 0$ and $\tilde{\gamma}_i \ge 0$, for $i = 1, \ldots, 2P$.

Differentiating the above Lagrangian with respect to $\widetilde{\mathbf{w}}$, ξ_i, and $\tilde{\xi}_i$ yields

$$\frac{\partial L}{\partial \widetilde{\mathbf{w}}} = 0 \Longrightarrow \widetilde{\mathbf{w}} = \sum_{i=1}^{2P}(\tilde{\alpha}_i - \alpha_i)\widetilde{\mathbf{R}}_x^{-1}\bar{\mathbf{a}}(i) \tag{10}$$

$$\frac{\partial L}{\partial \xi_i} = 0 \Longrightarrow \gamma_i = C - \alpha_i, \quad i = 1, \ldots, 2P \tag{11}$$

$$\frac{\partial L}{\partial \tilde{\xi}_i} = 0 \Longrightarrow \tilde{\gamma}_i = C - \tilde{\alpha}_i, \quad i = 1, \ldots, 2P. \tag{12}$$

Similar to other SVM-based problems, the optimal beamformer given by (10) is expanded in terms of a set of steering vectors (those corresponding to $\tilde{\alpha}_i - \alpha_i \ne 0$), which are the support vectors for the problem [31], [32]. Due to the output energy term in the regularized cost function (3), the support vectors in the expansion (10) are multiplied by the inverse of the covariance matrix. Substituting (10)–(12) into (9), the Lagrange multipliers $\boldsymbol{\alpha}_i$ and $\tilde{\boldsymbol{\alpha}}_i$ are those coefficients maximizing the following quadratic functional:

$$\begin{aligned} W(\boldsymbol{\alpha}, \tilde{\boldsymbol{\alpha}}) = &-\frac{1}{2}\sum_{i=1}^{2P}\sum_{j=1}^{2P}\left[(\tilde{\alpha}_i - \alpha_i)(\tilde{\alpha}_j - \alpha_j)\bar{\mathbf{a}}^T(i)\widetilde{\mathbf{R}}_x^{-1}\bar{\mathbf{a}}(j)\right] \\ &- \epsilon\sum_{i=1}^{2P}(\tilde{\alpha}_i + \alpha_i) + \sum_{i=1}^{2P}(\tilde{\alpha}_i - \alpha_i)\widetilde{d}_i \end{aligned}$$

subject to $0 \le \alpha_i, \tilde{\alpha}_i \le C$.

Once the Lagrange multipliers $\boldsymbol{\alpha}_i$ and $\tilde{\boldsymbol{\alpha}}_i$ are computed, the coefficients $\widetilde{\mathbf{w}}$ can be obtained according to (10). Thus, each complex beamformer coefficient is formed as $w_i = \widetilde{w}(i) + j\widetilde{w}(i+M)$, for $i = 1, \ldots, M$. Typically, the support vectors and their corresponding Lagrange multipliers are found by means of QP techniques. Nevertheless, finding out the QP solution can be computationally expensive when a large number of DOAs is considered in the regularized cost function. In [38], it is shown that the high computational cost of the original procedure can be reduced by transforming the QP problem into an equivalent nonlinear least squares problem. By applying an iterative reweighted least-squares (IRWLS) algorithm the computational requirements of the beamforming algorithm are reduced without any loss of performance [39], [46]: this procedure is described in the next section.

V. IRWLS PROCEDURE

The IRWLS procedure for solving SVMs was first introduced for classification problems in [47] and then extended for regression problems in [38]. Essentially, the IRWLS procedure uses a quadratic approximation of the SVM loss function [39], [40], which has been shown to converge to the true SVM solution. To obtain an IRWLS algorithm, we initially perform a first-order Taylor series expansion of $L_\epsilon^m(u)$ in (7) at the solution obtained after the kth iteration u_i^k, leading to

$$J_1(\tilde{\mathbf{w}}) = \frac{1}{2}\tilde{\mathbf{w}}^T\tilde{\mathbf{R}}_x\tilde{\mathbf{w}} + C\sum_{i=1}^{2P}\left(L_\epsilon^m\left(u_i^k\right) + \left.\frac{dL_\epsilon^m(u)}{du}\right|_{u_i^k}\left[u_i - u_i^k\right]\right)$$

where $u_i^k = |\tilde{d}_i - \tilde{\mathbf{w}}_k^T\bar{\mathbf{a}}(i)|$ and $\tilde{\mathbf{w}}_k$ is the beamforming solution at the kth iteration. Then, a quadratic approximation is constructed by imposing $J(\tilde{\mathbf{w}}_k) = J_2(\tilde{\mathbf{w}}_k)$ and $\nabla_{\tilde{\mathbf{w}}}J(\tilde{\mathbf{w}}_k) = \nabla_{\tilde{\mathbf{w}}}J_2(\tilde{\mathbf{w}}_k)$,[1] where $\nabla_{\tilde{\mathbf{w}}}$ denotes the gradient operator with respect to the vector $\tilde{\mathbf{w}}$

$$\begin{aligned} J_2(\tilde{\mathbf{w}}) &= \frac{1}{2}\tilde{\mathbf{w}}^T\tilde{\mathbf{R}}_x\tilde{\mathbf{w}} + C\sum_{i=1}^{2P}\left(L_\epsilon^m\left(u_i^k\right) + \left.\frac{dL_\epsilon^m(u)}{du}\right|_{u_i^k}\frac{(u_i)^2 - \left(u_i^k\right)^2}{2u_i^k}\right) \\ &= \frac{1}{2}\tilde{\mathbf{w}}^T\tilde{\mathbf{R}}_x\tilde{\mathbf{w}} + \frac{1}{2}\sum_{i=1}^{2P} f_i^m u_i^2 + b_i. \end{aligned} \tag{13}$$

In (13), b_i groups all the terms that do not depend on $\tilde{\mathbf{w}}$. On the other hand, the weights f_i^m depend on the particular penalty function: for the linear ($m = 1$) Vapnik's ϵ-insensitive loss function, they are shown to be

$$f_i^1 = \frac{C}{u_i^k}\frac{dL_\epsilon^1(u)}{du} = \begin{cases} 0, & \text{if } u_i^k < \epsilon \\ \frac{C}{u_i^k}, & \text{if } u_i^k \ge \epsilon \end{cases} \tag{14}$$

and for the quadratic loss function ($m = 2$)

$$f_i^2 = \begin{cases} 0, & \text{if } u_i^k < \epsilon \\ \frac{2C\left(u_i^k - \epsilon\right)}{u_i^k}, & \text{if } u_i^k \ge \epsilon. \end{cases} \tag{15}$$

As can be seen, the new quadratic cost function is a regularized least-squares cost function, whose minimum can be found by equating to zero its gradient with respect to $\tilde{\mathbf{w}}$ [46]

$$\nabla_{\tilde{\mathbf{w}}}J_2(\tilde{\mathbf{w}}) = \tilde{\mathbf{R}}_x\tilde{\mathbf{w}} + \sum_{i=1}^{2P} f_i^m\left(\tilde{d}_i - \tilde{\mathbf{w}}^T\bar{\mathbf{a}}(i)\right)\bar{\mathbf{a}}(i) = 0. \tag{16}$$

[1]These equality assumptions in the definition of the quadratic approximation are needed in order to ensure the convergence of the IRWLS procedure to the SVM solution [39], [48].

TABLE I
IRWLS PSEUDOCODE

```
Initialization: choose ε, C and m ∈ {1,2}.
Initialization: Set k=0, w̃_0=0, u_i^0 = d̃_i and compute f_i^m.
repeat
  Compute w̃_s as (18) and set η_k = 1.
  Calculate w̃_{k+1} = w̃_k + η_k (w̃_s − w̃_k).
  while J(w̃_{k+1}) ≥ J(w̃_k) do
    reduce η_k and calculate again w̃_{k+1}.
  end while
  Set k = k + 1
  Compute u_i^k with the obtained solution.
  Update f_i^m using (14) for m = 1 or (15) for m = 2.
until Convergence when |J(w̃_{k+1}) − J(w̃_k)| ≤ δ
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Equation (16) can be expressed, more conveniently, in matrix form as

$$\nabla_{\tilde{\mathbf{w}}}J_2(\tilde{\mathbf{w}}) = \tilde{\mathbf{R}}_x\tilde{\mathbf{w}} - \mathbf{\Phi}^T\mathbf{D}_f\tilde{\mathbf{d}} + \mathbf{\Phi}^T\mathbf{D}_f\mathbf{\Phi}\tilde{\mathbf{w}} = \mathbf{0} \tag{17}$$

where $\mathbf{\Phi} = [\bar{\mathbf{a}}(1)\ \bar{\mathbf{a}}(2)\ \cdots\ \bar{\mathbf{a}}(2P)]^T$, $\tilde{\mathbf{d}} = \left[\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_{2P}\right]^T$, $\mathbf{D}_f$ is a diagonal matrix with diagonal elements f_i^m, and $\mathbf{0}$ is a column vector of convenient dimension with all zeroes. Finally, the minimum of (13) is given by

$$\tilde{\mathbf{w}}_s = \left[\tilde{\mathbf{R}}_x + \mathbf{\Phi}^T\mathbf{D}_f\mathbf{\Phi}\right]^{-1}\mathbf{\Phi}^T\mathbf{D}_f\tilde{\mathbf{d}}. \tag{18}$$

In order to speed up the convergence of the IRWLS algorithm, we apply a line-search technique [48]. The line-search method finds a descending direction as $\mathbf{p}_k = (\tilde{\mathbf{w}}_s - \tilde{\mathbf{w}}_k)$, with $\tilde{\mathbf{w}}_s$ being the minimum at each iteration of the weighted least-squares problem (18). Then, the coefficients are modified along that direction as

$$\tilde{\mathbf{w}}_{k+1} = \tilde{\mathbf{w}}_k + \eta_k\mathbf{p}_k$$

where $\eta_k \in [0, 1]$ is the step size. Initially, the value of η_k is set equal to 1, but if $J(\tilde{\mathbf{w}}_{k+1}) \ge J(\tilde{\mathbf{w}}_k)$, then it is iteratively reduced until observing a strict decrease in the functional (13). A theoretical analysis about the election of the step size η_k can be found in [39], [40], and [48].

Once the new beamformer $\tilde{\mathbf{w}}_{k+1}$ is obtained, we compute the error terms as $u_i = \left|\tilde{d}_i - \tilde{\mathbf{w}}_{k+1}^T\bar{\mathbf{a}}(i)\right|$ and update the weights f_i^m until the algorithm achieves the prescribed convergence threshold δ. A pseudocode for the IRWLS procedure is shown in Table I.

A final comment is in order here: the IRWLS algorithm presented in this section to obtain the SVM-based beamformer solves the primal problem, whereas the conventional QP optimization problem discussed in Section IV solves the dual problem. It is also possible to obtain an IRWLS algorithm for the dual problem by forcing an expansion of the solution as a linear combination of the prewhitened steering vectors, similarly to (10). For this dual formulation, the number of unknowns would be $2P$ (where P is the number of inequality constraints established by sampling the beampattern),[2] whereas the number of unknowns for the primal problem is $2M$ (where M is the number of beamformer weights). Since typically $P \gg M$, the computational cost of an IRWLS algorithm to solve the dual problem would be much higher. Therefore, in this paper we will only consider the IRWLS solution of the primal problem as given by (18).

VI. Computer Simulations

To evaluate the performance of the proposed SVM-based beamforming technique, some computer simulations have been carried out in an ideal scenario without source steering vector mismatch and more realistic situations with source steering vector mismatches. In the following, we assume a uniform linear array with $M = 10$ sensors and half-wavelength sensor spacing. All signal waveforms are independent and identically distributed (i.i.d.) quadrature phase-shift keying (QPSK). Spatially white Gaussian noise is assumed with unity variance ($\mathbf{Q} = \mathbf{I}$). The signal of interest-to-noise ratio (SNR), i.e., the power of the SOI is set to $\sigma_s^2 = 10$ dB, and the interference-to-noise ratio (INR), i.e., the power of the interferences is $\sigma_j^2 = 30$ dB, $\forall j$. The actual source DOA is $\theta_s = 0^\circ$, and the DOAs of the interferences are $\theta_1 = -30^\circ$, $\theta_2 = 30^\circ$ and $\theta_3 = 70^\circ$. In order to compute the covariance matrix $\hat{\mathbf{R}}_x$, $N = 100$ snapshots are used, and the desired signal is always present in the training data cell. For all scenarios, each point is the average of 5000 independent simulations.

From now on, the SVM-based beamformer with linear and quadratic ϵ-insensitive loss functions will be denoted as SVM-Lin and SVM-Quad, respectively. For comparison purposes, we also illustrate the conventional MVDR beamformer, the *doubly constrained robust Capon beamformer* (DCRCB) presented in [11] and the MVDR Beamformer with sidelobe control based on second-order cone programming (Sidelobe-SOC) presented in [29]. The following figures illustrate the performance of the four aforementioned methods showing: a) a single beampattern realization for SNR equal to 10 dB; b) the output SINR[3] (SINR) versus SNR; c) the SINR versus the number of snapshots; d) the SINR versus the number of existing interferences (for this particular scenario, we simulate interference signals impinging into the array from DOAs $\theta_j = [-75^\circ, 30^\circ, -45^\circ, 60^\circ, 10^\circ, -25^\circ, 35^\circ, -50^\circ, 70^\circ]$). The vertical lines in the beampattern figures indicate the DOAs of the SOI as well as the interferences. Note that the beampatterns have been scaled in order to achieve unity gain, i.e., a distortionless response in the SOI actual direction.

For the DCRCB algorithm, the cost function is [11]

$$\min_{\mathbf{a}(\theta_0)} \quad \mathbf{a}(\theta_0)\hat{\mathbf{R}}_x^{-1}\mathbf{a}(\theta_0) \quad \text{subject to} \quad \|\mathbf{a}(\theta_0) - \mathbf{a}(\theta_s)\|^2 \le e$$
$$\|\mathbf{a}(\theta_0)\|^2 = M$$

where $\mathbf{a}(\theta_s)$ denotes the assumed steering vector of the SOI. Essentially, the above optimization problem tries to find the source steering vector $\mathbf{a}(\theta_0)$ that maximizes the source power estimate assuming that i) $\mathbf{a}(\theta_0)$ belongs to an uncertainty sphere centered on $\mathbf{a}(\theta_s)$ and ii) its Euclidean norm is equal to the number of sensors. Note that an upper bound e on the uncertainty sphere around the assumed steering vector must be specified. Equivalent formulations based on this uncertainty region has been employed in [10], [12], and [13]. In these works, choosing e depends on the number of sensors, the source DOA and the expected accuracy in the SOI DOA estimate, obtained in a previous DOA estimation stage. It should be made as small as possible, since when e is chosen too large the ability to suppress interferences close to the SOI will degrade. On the other hand, very small values of e can result in an inappropriate operation of the beamformer. According to [11], we set $e = 1$ in our simulations. Although this choice could not be the optimal for no-mismatch scenarios, we feel that this beamformer should work in different realistic environments without changing e.

On the other hand, the MVDR beamformer with sidelobe control presented in [29] considers the following optimization problem:

$$\min_{\mathbf{w}} \quad \mathbf{w}^H\hat{\mathbf{R}}_x\mathbf{w} \text{ subject to } \mathbf{w}^H\mathbf{a}(\theta_s) = 1$$
$$|\mathbf{w}^H\mathbf{a}(\theta_i)|^2 \le \epsilon, \quad i = 1, \ldots, P.$$

In this beamforming problem, there is a linear distortionless constraint for the assumed SOI direction of arrival and P quadratic inequality constraints which control the sidelobe level. Unlike the SVM-beamforming technique, the inequality constraints are only imposed over the sidelobe beampattern, and no uncertainty region for mismatch scenarios is considered. To achieve a feasible problem with our basic simulated scenario, the chosen beampattern sidelobe areas were $\Theta = [-90^\circ, -15^\circ] \cup [15^\circ, 90^\circ]$. The number of constraint angles θ_i in that region is $P = 60$, which are uniformly distributed along these sidelobe areas and the sidelobe level to be satisfied is $\epsilon = 0.005$, i.e., we require the sidelobe level to be below -23 dB [29].

For the SVM-based beamformers, the parameter which determines the uncertainty region is $\triangle$, which is set equal to 2°. The angular range $[-90^\circ, 90^\circ]$ is sampled with a uniform grid with $P = P_1 + P_2 = 20 + 40$ angles θ_i, i.e., $P_1 = 20$ angles are uniformly set between $[-90^\circ, 90^\circ]$ and $P_1 = 40$ angles between $[-90^\circ, -2^\circ]$ and $[2^\circ, 90^\circ]$. In addition, the control parameters C and ϵ are equal to 1 and 0.001, respectively.

In our first example, an ideal scenario with exact knowledge of the source steering vector is simulated. Note that even in this case, the presence of the desired signal in the training data cell can deteriorate the performance of the beamformers as compared with the signal-free training data case [1], [12]. From Fig. 1, we observe that, when the signal steering vector is exactly known, all the beampatterns have nulls at the DOAs of the interferences and maintain a distortionless response for the SOI. However, the MVDR response presents high sidelobe levels compared to the other robust methods. This can result in deep degradations in case of unexpected interferences or increase in the noise power. Although the SVM beamformers do not present deeper nulls than the rest, the SVM-Lin beampattern achieves substantially lower sidelobe level than the rest, with only a slight increase in the mainlobe beamwidth. Observe that

[2]Remember that we are considering a problem with real variables.

[3]Here, SINR is defined as $\mathbf{w}^H\mathbf{R}_s\mathbf{w}/\mathbf{w}^H\mathbf{R}_{i+n}\mathbf{w}$. In the particular case of point sources [15], $\mathbf{R}_s = \sigma_s^2\mathbf{a}(\theta_s)\mathbf{a}(\theta_s)^H$ and $\mathbf{R}_{i+n} = \sum_{j=1}^{N_i}\sigma_j^2\mathbf{a}(\theta_j)\mathbf{a}(\theta_j)^H + \mathbf{Q}$.

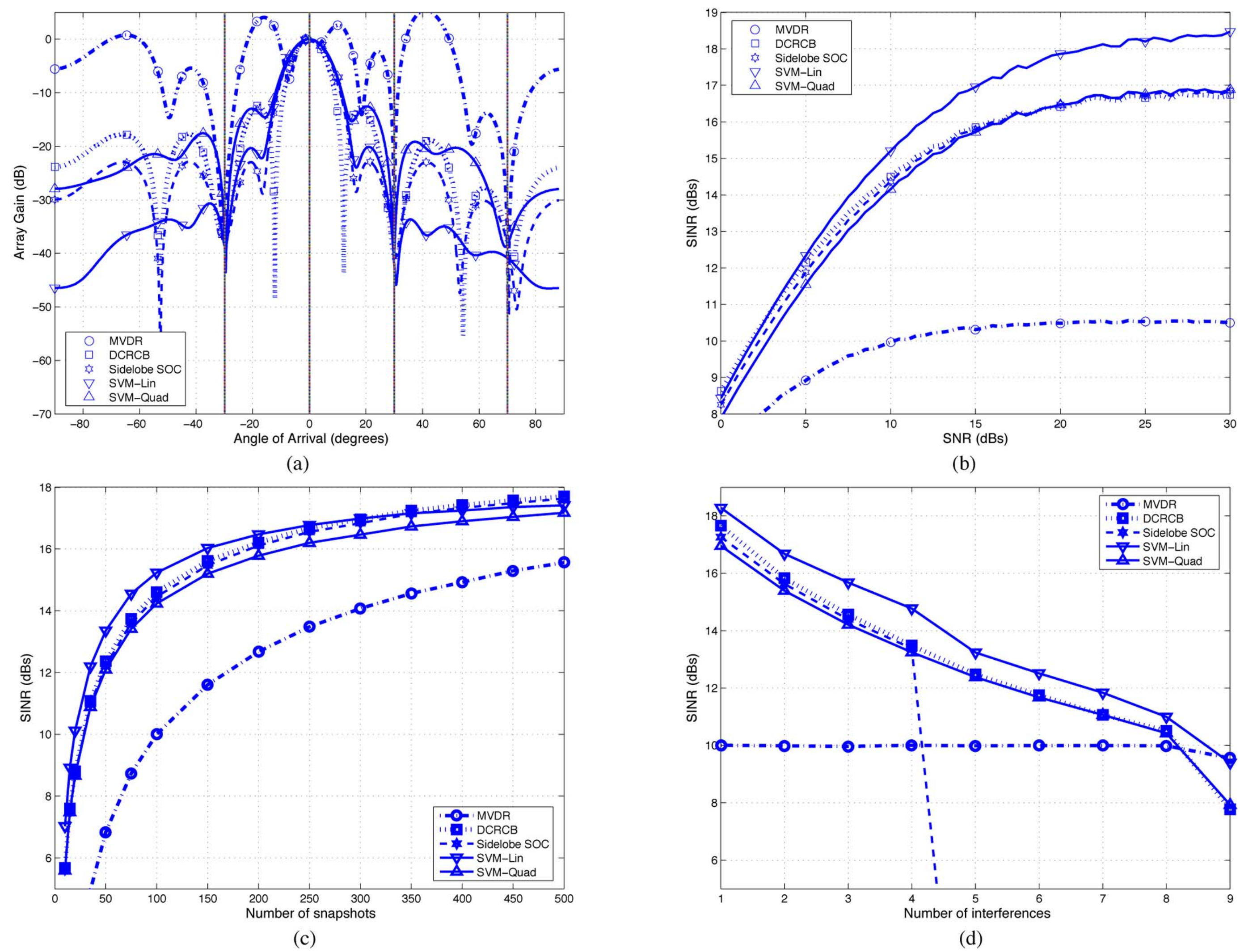

Fig. 1. No-mismatch scenario. MVDR, DCRCB ($e = 1$), MVDR with SOC sidelobe control ($P = 60$, $\epsilon = 0.005$, $\triangle = 15°$), SVM-Lin and SVM-Quad ($P = 60$, $C = 1$, $\epsilon = 0.001$, $\triangle = 2°$): (a) Normalized beampatterns for SNR $= 10$ dB; (b) SINR versus SNR; (c) SINR versus number of snapshots; and (d) SINR versus number of interferences.

the Sidelobe-SOC beamformer fulfills the requested constraint for the sidelobe level. As for the SINR figures, we can state that SVM-Lin method achieves a better performance than the rest of beamformers for SNR values larger than 7 dB and regardless of the number of interferences. For this no-mismatch scenario, the SVM-Quad, DCRCB, and Sidelobe-SOC methods perform similarly in terms of SNR, especially at large SNR values. As illustrated, the SVM-based technique is more convenient than the DCRCB and Sidelobe-SOC approaches when the number of available snapshots is scarce. Finally, notice that when five or more interference signals impinge into the array, the Sidelobe-SOC beamformer breaks down since a feasible solution cannot be achieved with the established sidelobe constraint.

In our second example, a scenario with 2° of mismatch in the SOI DOA is simulated. The rest of parameters are those of the previous scenario. Fig. 2 shows the performance of the tested beamformers. Observe that with a SOI steering vector mismatch, the MVDR fails in its operation, allocating a null in the SOI direction since the source signal is interpreted as an interference. For the sake of a clear illustration, the performance of the MVDR algorithm is not shown in the SINR figures. We can highlight that the SVM-Lin beamformer performs better than the other beamformers for high SNR values or with numerous interference signals. However, the Sidelobe-SOC beamformer works better in low SNR scenarios or with an abundant number of observations. Again, the Sidelobe-SOC operation is unsatisfactory when the number of interferences is larger than four. As in the scenario without mismatch, the SVM-Quad and DCRCB methods operate similarly, with a minor advantage for the SVM-Quad beamformer when the number of snapshots is small, but better performance for the DCRCB with only one or two interferences. Notice that the DCRCB, Sidelobe-SOC and SVM-Quad achieve the same SINR for large SNR values.

The objective in our third scenario is to simulate a breakdown scenario, where the difference between the presumed and actual source DOAs exceeds the expected mismatch. We will assume that the source DOA is 3°: therefore, the actual mismatch is 3°, but the expected mismatch is only 2°. In this scenario, our proposed SVM-based beamforming technique demonstrates an appropriate operation under this unexpected situation. On the other

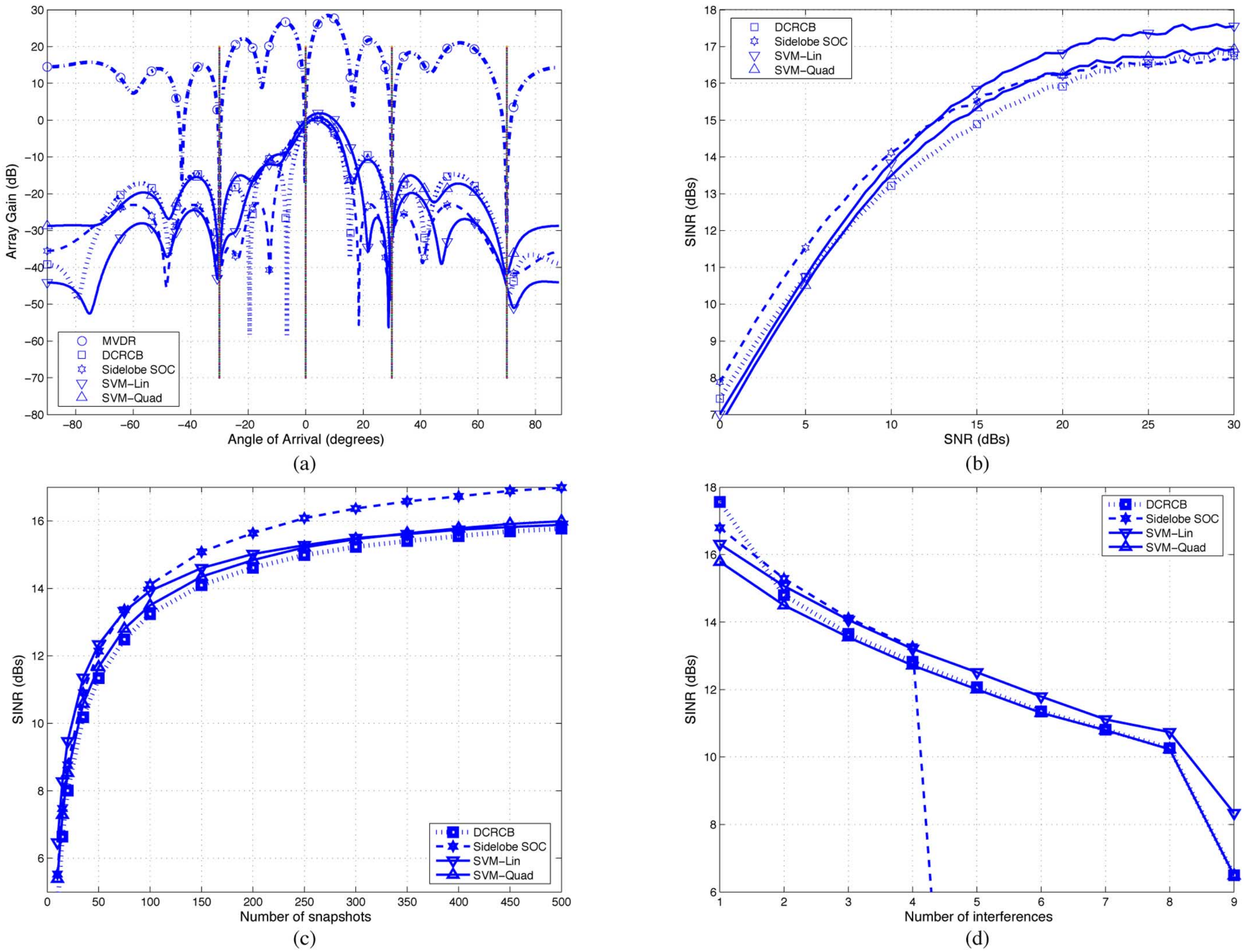

Fig. 2. Scenario with 2° degrees of source DOA mismatch. MVDR, DCRCB ($e = 1$), MVDR with SOC sidelobe control ($P = 60$, $\epsilon = 0.005$, $\triangle = 15°$), SVM-Lin and SVM-Quad ($P = 60, C = 1, \epsilon = 0.001, \triangle = 2°$): (a) Normalized beampatterns for SNR $=$ 10 dB; (b) SINR versus SNR; (c) SINR versus number of snapshots; and (d) SINR versus number of interferences.

hand, as illustrated in Fig. 3(a), the DCRCB array response allocates a deep null for the SOI since it is interpreted as a jamming source. This situation resembles the MVDR beamformer behavior in a mismatch scenario. Hence, the DCRC beamformer demands an accurate definition of the uncertainty region. It is worth mentioning that in Fig. 3(a)–(c), three interferences impinge into the antenna array. According to our experimental results, this inadequate operation of the DCRCB beamformer highly depends on the number of sensors, the number of available degrees of freedom, the angular resolution of the beamformer and interference locations. For instance, Fig. 3(d) shows that the DCRCB beamformer recovers its robustness when more than four jamming signals impinges into the array. With $M = 10$ sensors and for less than five strong interference signals, we have unused degrees of freedom which are devoted to rejecting the desired signal. When more jammers are included, the beamformer concentrates on rejecting the stronger interferences instead of the source signal. The performance of the Sidelobe-SOC is similar to the previous simulated scenarios: it achieves superior performance with abundant snapshots and low SNRs but operates improperly with many interferences since the level of the sidelobes must be increased to obtain a feasible problem. On the contrary, both SVM approaches with linear and quadratic ϵ-insensitive loss functions achieve robust performance at all values of the SNR and independently of the number of available snapshots and interferences.

To fairly assess the performance of the SVM-beamforming approach, its computational complexity must be analyzed. At each iteration, the proposed IRWLS algorithm amounts to solving a linear system of $2M$ equations with $2M$ unknowns, in order to obtain (18). Therefore, the computational cost of the IRWLS procedure is, basically, $Niter \times O(8M^3)$ flops, with $Niter$ being the number of iterations needed for convergence, and M being the number of beamformer weights. Remember that the SVM optimization problem is formulated in terms of real variables. The convergence of the linear and quadratic ϵ-insensitive loss functions for 50 realizations is depicted in Fig. 4. As can be seen, both linear and quadratic cost functions converges quickly and the optimal solution is found after two to four iterations. For comparison, in [11], it is stated that the

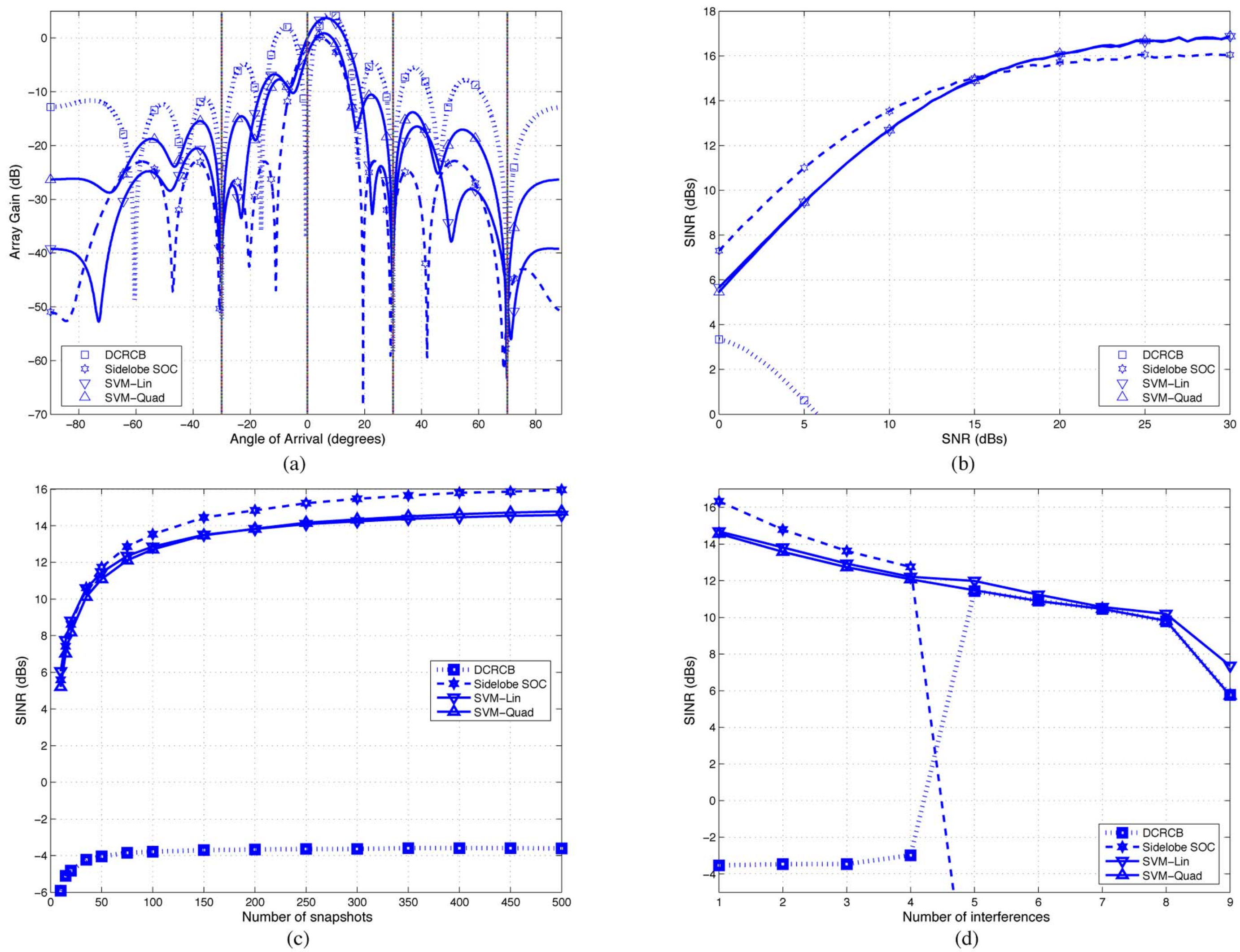

Fig. 3. Scenario with 3° degrees of source DOA mismatch. MVDR, DCRCB ($e = 1$), MVDR with SOC sidelobe control ($P = 60, \epsilon = 0.005, \triangle = 15°$), SVM-Lin and SVM-Quad ($P = 60, C = 1, \epsilon = 0.001, \triangle = 2°$): (a) Normalized beampatterns for SNR = 10 dB; (b) SINR versus SNR; (c) SINR versus number of snapshots; and (d) SINR versus number of interferences.

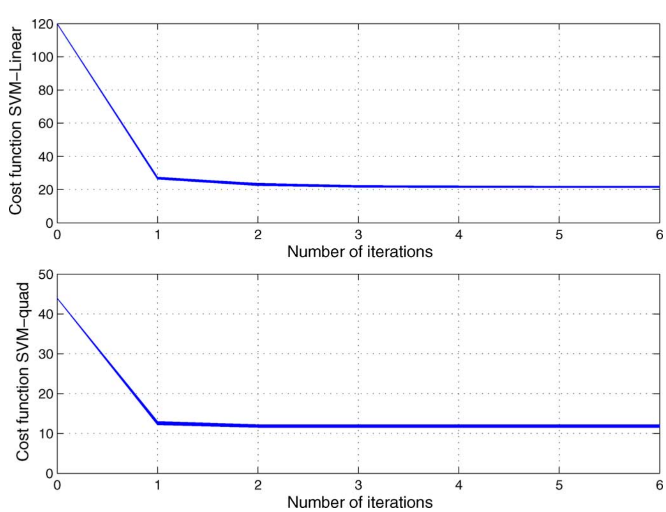

Fig. 4. Convergence of the IRWLS procedure for the linear and quadratic ϵ-insensitive loss functions.

computational complexity of the DCRCB beamformer is comparable to that of the conventional MVDR algorithm, which computes the beamformer weights with $O(M^3)$ flops. On the other hand, the complexity of the Sidelobe-SOC beamformer is $O(M^{3.5} + PM^{2.5})$ flops [29]. Finally, it must be pointed out that employing an IRWLS procedure is not the unique possibility to find the solution of SVM problems. In fact, other alternative algorithms (see [32] and [40] and references therein) can be implemented to solve the SVM regularized cost function, with the purpose of avoiding the high computational cost of quadratic programming techniques.

VII. Conclusion

In this paper, robust beamforming was reformulated as a support vector regression problem. The proposed approach modifies the traditional MVDR beamformer with the goals of: a) increasing the beamformer robustness against errors in the desired signal array response and b) providing some additional control over the sidelobe level. We have presented the SVM beamforming technique considering linear and quadratic ϵ-insensitive loss functions in the regression problem, which ends up in a convex function that can be efficiently minimized. To alleviate the high computational cost of quadratic programming techniques, the optimization was carried out using an IRWLS procedure. The satisfactory and robust performance of the proposed SVM beamformer was demonstrated through computer simulations, both in no-mismatch and mismatch situations, even when this mismatch was larger than expected. Its operation was shown to outperform other robust beamforming techniques, specially for high SNR scenarios and regardless of the number of interference signals.

The authors feel this work is another step forward on the design of robust beamforming solutions. Future work should address the nonlinear beamforming concept, exploiting the nonlinear kernel formulation of support vector machines. As a matter of interest, it is the use of Vapnik's SVM theory for nonlinear regression rooted in Huber's robust statistics [49], which provides the fundamentals for a statistically robust beamforming design.

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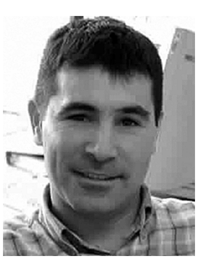
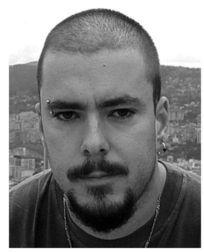


César C. Gaudes (S'06) received the Telecommunication Engineer degree from the University of Zaragoza, Spain, in 2001. Currently, he is working towards the Ph.D. degree at the Communications Technology Group, Aragón Institute for Engineering Research (I3A), University of Zaragoza.

Previously, he worked as a Research Assistant at the Institute of Communications Engineering at the Technology University of Tampere, Finland, dealing with software radio and frequency synthesizer architectures. Since 2004, he has also been a Lecturer at the University of Zaragoza. His current research interests include multiuser multiple-input multiple-output (MIMO) systems and robust array beamforming.

Ignacio Santamaría (M'96–SM'05) was born in Spain in 1967. He received the Telecommunication Engineer degree and the Ph.D. degree in electrical engineering from the Polytechnic University of Madrid, Spain, in 1991 and 1995, respectively.

In 1992, he joined the Departamento de Ingeniería de Comunicaciones, Universidad de Cantabria, Spain, where he is currently an Associate Professor. In 2000 and 2004, he spent periods visiting at the Computational NeuroEngineering Laboratory (CNEL), University of Florida. He has more than 90 publications in refereed journals and international conference papers. His current research interests include nonlinear modeling techniques, machine-learning theories, and their application to digital communication systems, in particular to inverse problems (deconvolution, equalization, and identification problems).



Javier Vía (S'04) was born in Spain in 1979. He received the Telecommunication Engineer degree from the University of Cantabria, Spain, in 2002. Since 2002, he has been working towards the Ph.D. degree at the Communications Engineering Department, University of Cantabria, under the supervision of I. Santamaría.

His current research interests include neural networks, kernel methods, and their application to digital communication problems.

Enrique Masgrau Gómez (S'82–M'86–SM'06) received the M.S. and Ph.D. degrees in electrical engineering from the Polytechnic University of Catalonia (UPC), Spain, in 1977 and 1983, respectively.

From 1978 to 1992, he was an Assistant Professor at UPC. In 1992, he joined the University of Zaragoza, Spain, as a Full Professor with the Department of Electronic Engineering and Communications. He is also a member of the Aragon Institute for Engineering Research (I3A), where he is Manager of the Communication Technologies Group (GTC). His research interest includes speech processing, multiple-input multiple-output (MIMO) and cooperative communications techniques, and ICT applications in automotive (telematics). In these areas, he has published over 100 technical papers in several international conferences and journals. He holds three international patents.

Talesa Sesé Paules was born in Spain in 1980. She received the Telecommunication Engineer degree from the Polytechnic University of Zaragoza, Spain, in 2005.

She is currently working with DMR-Consulting. Her main research topics are robust array beamforming techniques.