Respiratory Frequency Estimation From Heart Rate Variability Signals in Non-Stationary Conditions Based on the Wigner-Ville Distribution

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Abstract

A method for respiratory frequency estimation from the high frequency (HF) component of heart rate variability (HRV) by means of the smoothed pseudo Wigner–Ville (SPWVD) distribution is presented. The method is based on maxima SPWVD detection with time-varying frequency smoothing window length, which reduces the estimation error, specially when the respiratory frequency is a nonlinear function of time.

Evaluation is performed over HRV simulated signals with time-varying amplitude, nonlinear HF frequency, and 20dB SNR, obtaining a mean frequency estimation error of 0.22±2.04% (0.10±5.96 mHz). The method has been tested on a database of ECG and respiratory signals simultaneously recorded during the listening of different musical stimuli, obtaining a median respiratory frequency estimation error of 0.02±1.90% (0.00±0.98 mHz) during musical stimuli and of 1.98±7.21% (35.41±33.20 mHz) during transitions between stimuli.

1. Introduction

Respiratory sinus arrhythmia (RSA) is a modulation of heart rate synchronous with respiration, and allows the estimation of respiratory frequency from the high frequency (HF) component of the heart rate variability (HRV) signal. In non–stationary conditions the respiratory frequency can be estimated from the maximum peak of the smoothed pseudo Wigner–Ville distribution (SPWVD) of the HRV signal in the HF band.

The extraction of the respiratory frequency from the maxima of the SPWVD is challenging, since the time-frequency (TF) smoothing used to suppress the interference terms of the Wigner–Ville distribution introduces a frequency estimation error which can be high in both, mean and standard deviation [1].

A method for estimating the instantaneous frequency (IF) of a frequency modulated (FM) signal based on the SPWVD is presented in [1]. This method uses a frequency smoothing window with time-varying length to resolve the bias–variance tradeoff that appears, specially when the IF varies nonlinearly. For each time instant the optimal frequency smoothing window length depends on the IF trend as well as on the signal amplitude and noise variance. The assumption of constant signal amplitude made in [1] is not suitable for respiratory frequency estimation from the HRV signal in situations where the RSA amplitude varies in time, such as during stress testing, tilt testing or induced emotion experiments.

In this paper an extension of the method in [1] is presented so as to estimate the respiratory frequency from the HRV signal, which accounts for time-varying amplitudes.

2. Methods and Materials

2.1. Instantaneous frequency estimation

It is assumed that the discrete analytic version of the HF component of the HRV signal can be modeled as [2]:

$$z(n) = A_{\text{HF}}(n)e^{j\phi_{\text{HF}}(n)} + v(n) \tag{1}$$

where $A_{\text{HF}}(n)$ and $\phi_{\text{HF}}(n)$ are the instantaneous amplitude and phase of the HF component, and $v(n)$ complex additive white gaussian noise.

The IF is estimated from the maxima of the SPWVD at each time instant by:

$$\hat{F}_{\text{HF}}(n) = \frac{F_s}{4M} \arg\max_{m} \{W_z(n,m)\}\,, \tag{2}$$

where F_s is the sampling frequency of $z(n)$ and $W_z(n,m)$ represents the SPWVD calculated as in [2]

$$W_z(n,m) = 2\sum_{k=-K+1}^{K-1} |h(k)|^2 \left[\sum_{p=-N+1}^{N-1} g(p) r_z(n+p,k)\right] e^{-j2\pi\frac{m}{M}k}$$
$$m = -M+1, ..., M \tag{3}$$

where n and m denote time and frequency indexes, respectively, $r_z(n,k) = z(n+k)z^*(n-k)$, $g(n)$ and $|h(k)|^2$

the time and frequency smoothing windows with lengths $2N$-1 and $2K$-1, respectively.

The asymptotic formula for the variance, σ^2_{L}, and bias, θ_{L}, of the estimation error of the IF, are extended here to the case in which $A_{\text{HF}}(n)$ in (1) is time-varying

$$\sigma^2_L(n) = \frac{3\sigma^2_v}{2\pi^2 A^2_{\text{HF}}(n)}\left[1+\frac{\sigma^2_v}{2A^2_{\text{HF}}(n)}\right]\frac{T_s}{(LT_s)^3} \tag{4}$$
$$\theta_L \le \frac{1}{80}\sup_n\left\{\left|F^{(2)}_{\text{HF}}(n)\right|\right\}(LT_s)^2\,,$$

where L=2K-1 is the frequency smoothing window length, σ^2_v is the noise variance, $T_s = \frac{1}{F_s}$, and $F^{(2)}_{\text{HF}}(n)$ represents the second derivative of $F_{\text{HF}}(n)$. From (4) it can be seen that by increasing L the bias increases and the variance decreases. The idea is to find for each time instant n the optimal L which resolve the bias-variance tradeoff minimizing the frequency mean squared error (MSE).

In [1] a suboptimal approach for estimating the optimal L without needing "a priori" information about the IF derivatives is proposed. An increasing sequence of L, $L_1 < L_2 < \cdots < L_j$ is considered, and for each L_i the IF estimate $\hat{F}_{\text{HF},\text{L}_i}(n)$ as well as the variance $\sigma^2_{\text{L}_i}(n)$ are computed. Assuming that L_i is small enough so that $|\theta_{\text{L}_i}| < \kappa\sigma_{\text{L}_i}(n)$, the following confidence interval is defined

$$D_{L_i}(n) = \left\{\hat{F}_{\text{HF},L_i}(n) - 2\kappa\sigma_{L_i}(n), \hat{F}_{\text{HF},L_i}(n) + 2\kappa\sigma_{L_i}(n)\right\} \tag{5}$$

The largest L for which $D_{\text{L}_{i-1}}$ and D_{L_i} have at least one point in common is chosen as the optimal L, for which θ_{L_i} and $\sigma_{\text{L}_i}(n)$ have the same order. The IF estimate is initialized to the shortest length L_1 estimate, and then corrected with the optimal length L_i estimate. In this work a value of κ =2 is used [1].

2.2. Instantaneous amplitude and noise estimation

In order to estimate $\sigma^2_{\text{L}_i}(n)$, the instantaneous amplitude $A_{\text{HF}}(n)$ and noise variance σ^2_v need to be estimated.

The method proposed in this paper to estimate $A_{\text{HF}}(n)$ from the SPWVD comprises integration of $W_z(n,m)$ over a suited band and correction with a time-varying factor depending on $|h(k)|^2$.

Let us define $\hat{P}_w(n)$ as the instantaneous power obtained by the integration of $W_z(n,m)$ over a band [m_1, m_2], where m_1 and m_2 are the discrete frequency indexes corresponding to the minimum and maximum frequency of $\hat{F}_{\text{HF}}(n) \pm \frac{\Delta f}{2}$, and Δf is the frequency smoothing window bandwidth estimated from $H(m) = DFT_{2M}\left\{|h(k)|^2\right\}$ as the frequency distance between the first zero crossing at each side of the its maximum peak.

The instantaneous power of the HF component $P_{\text{HF}}(n)$ can be computed from the SPWVD as [3]

$$\hat{P}_{\text{HF}}(n) = \hat{P}_w(n) f_c(n) \tag{6}$$

where $f_c(n)$ is a correcting factor computed as

$$f_c(n) = \frac{\sum_{m=-M+1}^{M} H(m)}{\sum_{m=m_1}^{m_2} H(m - m_{\text{HF}}(n))} \tag{7}$$

being $m_{\text{HF}}(n)$ the discrete frequency index corresponding to $F_{\text{HF}}(n)$. Finally, the instantaneous amplitude is computed as $\hat{A}_{\text{HF}}(n) = \hat{P}^{\frac{1}{2}}_{\text{HF}}(n)$.

The noise present in the signal is estimated subtracting from $z(n)$ the estimated HF component, $\hat{z}(n)$ with amplitude $\hat{A}_{\text{HF}}(n)$ and frequency $\hat{F}_{\text{HF}}(n)$, so that the estimated noise signal $\hat{v}(n)$ accounts also for the amplitude and frequency estimation errors. Finally, noise variance is computed as the mean of $\hat{\sigma}^2_v(n) = \frac{1}{2}\hat{v}(n)\hat{v}^*(n)$.

2.3. Simulation study

A simulation study has been designed in order to evaluate the proposed method. The analytic version of HRV signals have been simulated according to

$$z(n) = A_{\text{LF}}(n)e^{j\phi_{\text{LF}}(n)} + A_{\text{HF}}(n)e^{j\phi_{\text{HF}}(n)} + v(n) \tag{8}$$

where $A_{\text{LF}}(n)$ and $\phi_{\text{LF}}(n)$ are the instantaneous amplitude and phase of the LF component. The frequency of the LF component is considered constant and equal to 0.1 Hz. The $A_{\text{HF}}(n)$ and $F_{\text{HF}}(n)$ vary as shown in Fig.2(a) and Fig.2(c), respectively, $A_{\text{LF}}(n)$ is defined to have a constant sympathovagal balance $B_{sv} = A^2_{\text{LF}}(n)/A^2_{\text{HF}}(n)$ of 0.5. The noise $v(n)$ is set to have a SNR of 20 dB at the instant of maximum instantaneous power. Since the model in (1) assumes monocomponent signals the simulated signals are filtered by a 9^{th} order Butterworth band-pass filter with bandpass [0.1–0.65] Hz.

2.4. Database

A database containing simultaneous ECG and respiratory signals of 58 subjects during the listening of different musical stimuli is analyzed [4]. The database is characterized by the non-stationarity of both respiration and HRV signals, as well as by nonlinear IF variations specially in the transitions between different musical stimuli. The ECG and respiration signals are sampled at 1000 Hz.

The HRV signal is estimated from the ECG by an algorithm based on the integral pulse frequency modulation (IPFM) model, which accounts for the presence of ectopic beats [5]. Respiratory signals baseline wander is removed by means of a 3^{rd} order Butterworth high-pass filter with cut-off frequency 0.1 Hz. Both, respiratory and HRV signals are resampled at 4Hz and bandpass filtered, as in the simulation study. The IF estimation on the respiratory signal is used as the reference IF for the evaluation over real signals.

2.5. Evaluation

Evaluation over the simulated signals is done in terms of mean and standard deviation of the $F_{\mathrm{HF}}(n)$ and $A_{\mathrm{HF}}(n)$ estimation errors while over real signals it is done in terms of median and median absolute deviation (MAD) in order to minimize the effect of outlier estimates.

Relative values of the estimation errors are obtained normalizing instantaneous estimation errors by the corresponding instantaneous reference values.

3. Results

3.1. Simulation study

A total of 100 realizations were generated. The minimum root mean squared error (RMSE) is obtained with a Hamming window for time smoothing and an exponential window for frequency smoothing with the same area as a rectangular window of $2N$-1=51 and L=31 samples, respectively. The algorithm estimates the instantaneous amplitude with a mean estimation error ($e_{\mathrm{A}} \pm \sigma_{\mathrm{A}}$) of 0.08±1.90%. Fig.1 shows the instantaneous amplitude estimation error over a single realization, note that larger errors appear at the edges, where the autocorrelation fuctions lacks of enough samples and on the instants where $F_{\mathrm{HF}}(n)$ and $A_{\mathrm{HF}}(n)$ present higher degree of variation (see Fig.2(a) and Fig.2(c)).

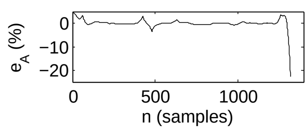

Figure 1. A_{HF} estimation error on one realization of $z(n)$.

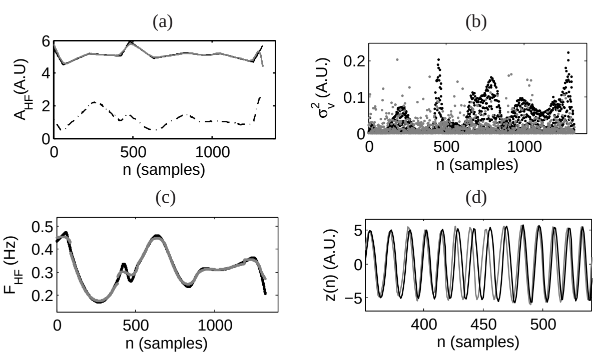

Figure 2. (a) Different estimates $\hat{A}_{\mathrm{HF}}(n)$, simulated (black solid line), estimated from (6) (gray solid line) and by integration over the classical HF band (dash-dot line), (b) comparison between $\hat{\sigma}_v^2(n)$ (gray) and $\sigma_v^2(n)$ (black), (c) comparison between $F_{\mathrm{HF}}(n)$ (black) and $\hat{F}_{\mathrm{HF}}(n)$ (gray) and (d) comparison between $z(n)$ (black) and $\hat{z}(n)$ (gray).

Fig.2(a) presents different estimates $\hat{A}_{\mathrm{HF}}$. The method proposed in this paper estimates $A_{\mathrm{HF}}(n)$ quite accurately, improving the classical estimation obtained by integrating the SPWVD in the clasical HF band. Fig.2(b) shows a comparison between $\sigma_v^2(n)$ and $\hat{\sigma}_v^2(n)$, where it can be appreciated a central part with high estimation error due to estimation errors in both $\hat{A}_{\mathrm{HF}}(n)$ and $\hat{F}_{\mathrm{HF}}(n)$, which introduce a phase shift into $\hat{z}(n)$ as it can be appreciated in Fig.2(d).

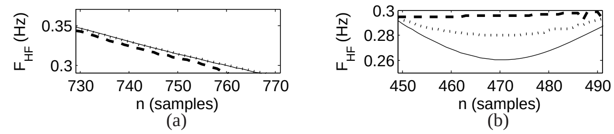

Figure 3. IF comparison on (a) linear IF trend and (b) nonlinear IF trend between F_{HF} (solid line), $\hat{F}_{\mathrm{HF}}$ with instantaneous $\hat{A}_{\mathrm{HF}}$ (dotted line) and with method in [1] (dashed line), $2N$-1 = 51.

The IF of the simulated HRV signals is estimated with a mean estimation error of 0.22±2.04% (0.10±5.96 mHz), using a Hamming window for time smoothing and rectangular window for frequency smoothing. The IF estimates were compared to those obtained with method in [1] based on constant amplitude estimation. Our method performs better in both types of segment, linear and nonlinear IF trends (see Fig.3), being more noticeable during nonlinear IF trends.

The IF estimates were also compared to those obtained with constant frequency smoothing window lengths (see Fig.4, where the distribution of the $F_{\mathrm{HF}}(n)$ estimation error obtained with different methods is displayed). It can be appreciated that introducing instantaneous amplitude estimation increases the performance of the method in [1] and of constant window lengths, since even the median error is approximately the same for all methods, variability is increased when using constant window lengths larger than L=15 samples, and larger interquartilic ranges (IQR) are found for lengths shorter than L=15.

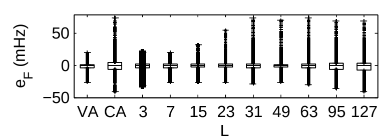

Figure 4. Distribution of $F_{\mathrm{HF}}(n)$ estimation error on 100 realizations. VA refers to the estimation algorithm presented in this paper, CA refers to [1].

3.2. Database

The algorithm proposed in this paper, using a Hamming window for time smoothing and rectangular window for frequency smoothing, allows the respiratory frequency estimation from the HF component of HRV with a median error of 0.02±1.90% (0±0.98 mHz) during musical stimuli and of 1.98±7.21% (35.41±33.20 mHz) during transitions between stimuli, which are highly non–stationary and nonlinear.

Fig.5 shows the estimate $\hat{F}_{\text{HF}}(n)$ during 4 musical stimuli with the method proposed in this paper, as well as the reference respiratory frequency derived from the respiratory signal.

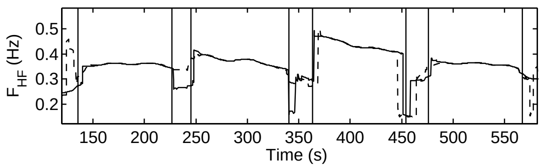

Figure 5. Comparison between the reference respiratory frequency (solid line) and $\hat{F}_{\text{HF}}(n)$ (dashed line).

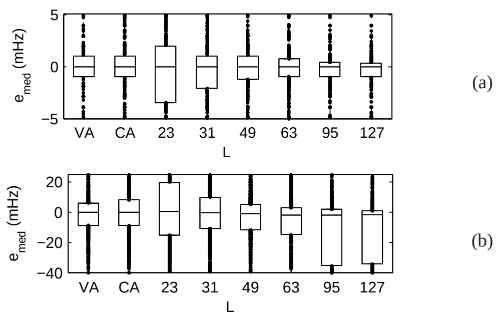

Figure 6. IQR of $F_{\text{HF}}(n)$ median estimation error distribution on real signals during (a) musical stimuli, (b) transition between stimuli, VA refers to the method presented here and CA refers to the method in [1], $2N$-1 = 51.

Fig.6 shows the IQR of $F_{\text{HF}}(n)$ median estimation error distribution during musical stimuly and transitions between stimuly, shorter L values have been discarded as they did not provide sufficient smoothing. Results are similar to those obtained in the simulation study. Using method in [1] or constant window lengths for frequency smoothing provides IF estimation with larger IQR than using the method proposed in this paper, being more noticeable during transitions between stimuly where the IF variations are highly nonlinear.

4. Discussion and conclusions

In this paper a method for the estimation of the respiratory frequency from the HF component of HRV signal in non–stationary conditions has been presented. The method is based on maximum peak detection of the SPWVD and includes time-varying frequency smoothing window length to reduce the MSE of the estimation error, specially when the IF variations are nonlinear, which makes the bias high. It is based on the method proposed in [1] but includes instantaneous amplitude estimates, reducing the MSE of the frequency estimation errors, specially for large or nonlinear variations of the IF.

Evaluation over HRV simulated signals with time-varying amplitude, nonlinear frequency trend, and an HRV SNR of 20dB, yielded a mean frequency estimation error of 0.22±2.04% (0.10±5.96 mHz). Over the database the method obtained a median respiratory frequency estimation error of 0.02±1.90% (0.00±0.977 mHz) during musical stimuli and of 1.98±7.21% (35.41±33.20 mHz) during transitions between stimuli.

In the simulation study the IF estimation algorithm presented in this paper showed a better performance than the method proposed in [1] or the use of constant window lengths, even though the simulated signals used in this paper presented sharper IF trends than those on [1], this results are supported in the database results. Although with some constant lengths results are similar, we have observed that the constant length which achieves similar results as those obtained by the algorithm proposed in this paper, is not known “a priori” and depends on the IF variations and the time smoothing of the SPWVD.

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