Chapter 8

ON THE COMPLEXITY-PERFORMANCE TRADEOFF OF TWO ACTIVE NOISE CONTROL SYSTEMS FOR VEHICLES

Pedro Ramos, Luis Vicente, Roberto Torrubia, Ana López, Ana Salinas and Enrique Masgrau.
Communication Technologies Group (GTC). Aragón Institute for Engineering Research (I3A). University of Zaragoza. Spain.

Abstract: The aim of this chapter is to show the experimental results achieved in the attenuation of periodic disturbances inside a vehicle with two Active Noise Control algorithms implemented on the TMS320C6701 DSP and to compare the computational complexity of both strategies:

(1) Modified FxGAL: Modified filtered-x gradient adaptive lattice algorithm. This technique is based on the signal orthogonalization carried out by an adaptive lattice predictor in a previous stage.

(2) Gμ-FxSLMS: Filtered-x sequential least mean square algorithm with step-size gain. This strategy is based on partial updates of the weights of an adaptive filter as well as on the controlled increase in step size of the algorithm.

This work illustrates by means of two different algorithms the tradeoff established among computational costs, convergence rate, stability and mean-square error excess when DSP-based strategies are used in control systems focused on the attenuation of acoustic disturbances.

Key words: Adaptive algorithms; active noise control; gradient adaptive lattice predictor; gain in step size; sequential partial updates.

1. PROPOSED ALGORITHMS

1.1 Modified Filtered-x Gradient Adaptive Lattice (FxGAL) algorithm.

The FxGAL algorithm (Vicente et al., 2003) can be seen as a version of the gradient adaptive lattice (GAL) algorithm (Griffiths, 1978) suitable to be used in the context of active control. The aim of FxGAL and Modified FxGAL algorithms is to obtain faster and much less signal dependent convergence than that of FxLMS systems, while maintaining the numerical stability of stochastic gradient algorithms. Also, better tracking capabilities can be expected in non-stationary environments with the FxGAL algorithms. The price of these improvements is an increase in computational complexity, which can be easily lessened by reducing conveniently the order of the adaptive lattice orthogonalizer.

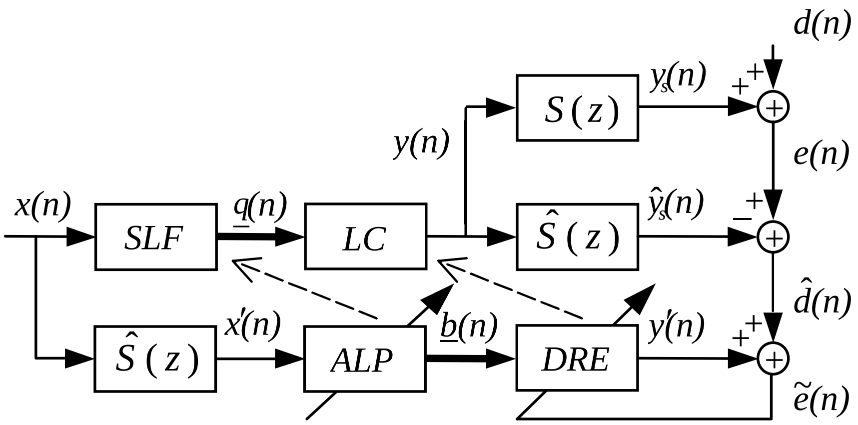

Figure 8-1. Block diagram of the modified FxGAL algorithm.

The modified version of the FxGAL algorithm makes use of the same idea that leads from the standard FxLMS to the Modified FxLMS[1] algorithm (Bjarnason, 1992; Kim et al., 1994): an estimation of the primary noise is used to properly swap the order between secondary path and adaptive control filter and a simultaneous copy of this control filter is used with the reference signal. In this way, the limitations imposed on the step size for the standard version of the algorithm are overcome in the modified one.

[1] This algorithm is also sometimes called Constraint FxLMS (Kim et al., 1994).

The key system in the FxGAL algorithms is an Adaptive Lattice Predictor (ALP), which realizes an approximate time-domain orthogonalization of its input data, without loss of information. The combination of this orthogonalization with independent absolut step sizes for each filter weight in the Desired Response Estimator (DRE) makes possible the expected increase in convergence speed.

The block diagram of the modified FxGAL algorithm is shown in Fig. 8-1. As can be seen, the filtered reference signal is the input to the ALP-DRE combination, while the reference signal goes through a Slave Lattice Filter (SLF) and a Linear Combiner (LC) to yield the control signal $y(n)$. The ALP and DRE blocks are the adaptive ones, while the coefficients of the SLF and LC systems are simply copied from the ALP and DRE, respectively. An iteration of the Modified FxGAL algorithm is given by:

$$t_0(n) = q_0(n) = x(n) \qquad /^*\ \text{Slave Lattice Filter (SLF)}\ ^*/$$

$$for\ \ l = 1\ to\ Lw - 1\ \ do \qquad /^*\ Lw: \text{Length of the filter}\ ^*/$$

$$\quad t_l(n) = t_{l-1}(n) + k_l(n) \cdot q_{l-1}(n-1) \qquad /^*\ \underline{t}(n):\ \text{Forward output of SLF}\ ^*/$$

$$\quad q_l(n) = q_{l-1}(n-1) + k_l(n) \cdot t_{l-1}(n) \qquad /^*\ \underline{q}(n):\ \text{Backward output of SLF}\ ^*/$$

$$end\ of\ for$$

$$y(n) = \underline{w}^T(n) \cdot \underline{q}(n) \qquad /^*\ \underline{w}(n): \text{Linear Combiner (LC) filter}\ ^*/$$

$$\hat{y}_S(n) = \hat{\underline{s}}^T(n) \cdot \underline{y}(n) \qquad /^*\ \hat{\underline{s}}(n): \text{Estimate of the Secondary path}\ ^*/$$

$$\hat{d}(n) = e(n) - \hat{y}_S(n) \qquad /^*\ \hat{d}(n): \text{Estimate of the primary noise}\ ^*/$$

$$x'(n) = \hat{\underline{s}}^T(n) \cdot \underline{x}(n) \qquad /^*\ \text{Filtering of the reference}\ ^*/$$

$$f_0(n) = b_0(n) = x'(n) \qquad /^*\ \text{Adaptive Lattice Predictor (ALP)}\ ^*/$$

$$for\ \ l = 1\ to\ Lw - 1\ \ do$$

$$\quad f_l(n) = f_{l-1}(n) + k_l(n) \cdot b_{l-1}(n-1) \qquad /^*\ \underline{f}(n):\ \text{Forward prediction errors}\ ^*/$$

$$\quad b_l(n) = b_{l-1}(n-1) + k_l(n) \cdot f_{l-1}(n) \qquad /^*\ \underline{b}(n):\ \text{Backward prediction errors}\ ^*/$$

$$\quad /^*\ \text{Recursive power estimate}\ ^*/$$

$$\quad \hat{P}_l(n) = \beta_{ALP} \cdot \hat{P}_l(n-1) + \left(1 - \beta_{ALP}\right) \cdot \left(f_{l-1}^2(n) + b_{l-1}^2(n-1)\right)$$

$$\quad /^*\ \text{Updating reflection coefficients}\ \underline{k}(n)\ ^*/$$

$$\quad k_l(n+1) = k_l(n) - \frac{\alpha_{ALP}}{\hat{P}_l(n)} \cdot \left(f_{l-1}(n) \cdot b_l(n) + b_{l-1}(n-1) \cdot f_l(n)\right)$$

$$end\ of\ for$$

$$y'(n) = \underline{w}^T(n) \cdot \underline{b}(n) \qquad /^*\ \underline{w}(n):\ \text{Desired Response Estimator (DRE)}\ ^*/$$

$$\tilde{e}(n) = \hat{d}(n) + y'(n)$$

$$
\begin{aligned}
&\textit{for } \; l = 0 \textit{ to } Lw-1 \textit{ do}\\
&\quad \text{/* Recursive power estimate */}\\
&\quad \hat{P}_{bl}(n) = \beta_{DRE} \cdot \hat{P}_{bl}(n-1) + (1-\beta_{DRE}) \cdot b_l^2(n)\\
&\quad \text{/* Updating DRE coefficients */}\\
&\quad w_l(n+1) = w_l(n) - \frac{\alpha_{DRE}}{\max\{\hat{P}_{bl}(n), P_{\min}\}} \cdot b_l(n) \cdot \tilde{e}(n)\\
&\textit{end of for}
\end{aligned}
$$

where α_{ALP} and α_{DRE} are, respectively, the step sizes before normalizing of the Adaptive Lattice Predictor and the Desire Response Estimator and β_{ALP} and β_{DRE} are the forgetting factors used to carry out the recursive estimate of the power.

1.2 Filtered-x Sequential Least Mean Square algorithm with Step-Size Gain (Gμ-FxSLMS)

Partial updates algorithms (Douglas, 1997) update only a portion of the filter at each time instant in order to reduce their computational complexity. These algorithms suffer from one drawback: their convergence speeds are also reduced in the same proportion.

The Gμ-FxSLMS algorithm (Ramos et al., 2004) is aimed at reducing the computational costs of the control strategy without either incrementing the final misadjustment or slowing down the convergence speed. The block diagram of the Gμ-FxSLMS is shown in Fig. 8-2.

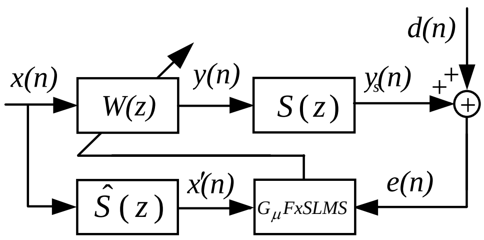

Figure 8-2. Block diagram of the Gμ-FxSLMS

An iteration of the Gμ-FxSLMS algorithm can be expressed as follows:

$y(n) = \underline{w}^T(n) \cdot \underline{x}(n)$ /* Generation of antinoise */
$if \;\; n \bmod N == 0$ /* N : Decimating factor */
  $x'(n) = \underline{\hat{s}}^T(n) \cdot \underline{x}(n)$ /* Filtering with the estimate
end of if   of the Secondary path $\underline{\hat{s}}(n)$*/
$for \;\; l = 0 \; to \; Lw-1 \; do$ /* Lw : Length of the adaptive filter */
  $if \;\; (n\text{-}l) \bmod N == 0$ /* Filter partial updates */
    /* μ : Step size; G_μ : Gain in step size */
    $w_l(n+1) = w_l(n) - \mu \cdot G_\mu(N, Lw, Fs) \cdot e(n) \cdot x'(n-l)$
  end of if
end of for

where the step-size gain Gμ is defined as the ratio between the bounds on the step sizes in two cases: firstly, when the adaptive algorithm uses sequential partial updates and, secondly, when every coefficient is updated at each iteration. In so doing, we obtain the factor by which the step size can be multiplied when the adaptive algorithm uses partial updates. This gain, that is approximately equal to the decimating factor *N* at most frequencies, allows the sequential strategy to achieve the convergence rate of the original FxLMS algorithm.

The theoretical analysis of the strategy prevents from the use of certain frequencies corresponding to notches which appear in the gain in the step size of the adaptive algorithm. Their width and exact location depend on the length of the adaptive filter (*Lw*), the decimating term *(N)* and the sampling frequency. Step-size gains for different values of the length of the adaptive filter and the decimating factor are shown in Fig 8-3. It can be easily derived from the examples given that the number of notches appearing in the gain is *N*-1. As far as the number of taps is concerned, the larger the adaptive filter is, the narrower the notch will be, that is, the narrower the bandwidth at which the gain in step size cannot be applied at its full strength will be.

To sum up, the step size can by multiplied by *N* in order to compensate the inherently slower convergence rate of the sequential adaptive algorithm as long as the regressor signal has no components at the notch frequencies.

2. COMPUTATIONAL COSTS

Table 8-1 compares the computational complexity of the Modified FxGAL and Gμ-FxSLMS algorithms when both strategies are used in the context of a two independent channel implementation of a feedforward ANC system:

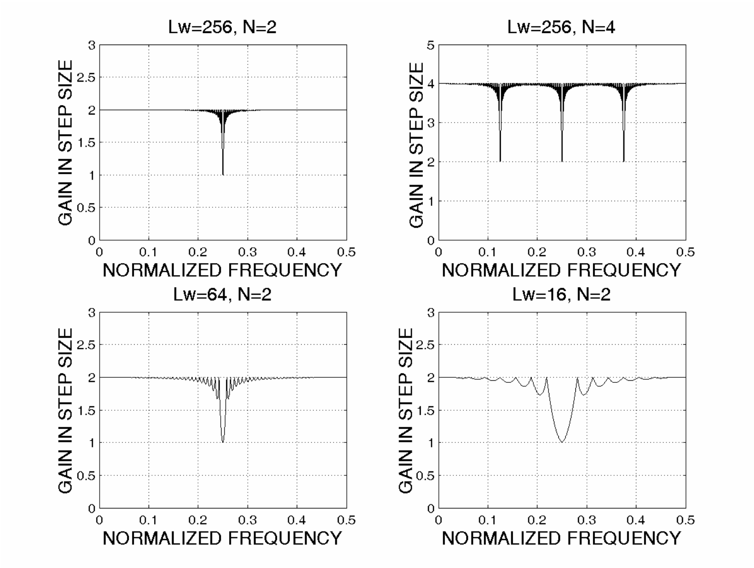

Figure 8-3. Gain in step size for different values of the length of the adaptive filter *Lw* and the decimating factor *N*.

Table 8-1. Computational complexity of the Modified FxGAL and the G*μ*-FxSLMS algorithms in terms of the average number of additions, multiplies and divisions required for each per iteration.

Algorithm	# Additions	# Multiplications	# Divisions
Mod. FxGAL	$2 \cdot (12 \cdot Lw + 2 \cdot Ls - 10)$	$2 \cdot (17 \cdot Lw + 2 \cdot Ls - 10)$	$2 \cdot (2 \cdot Lw - 1)$
Gμ-FxSLMS	$2 \cdot \left(\left(1 + \frac{1}{N} \right) \cdot Lw + \frac{Ls - 1}{N} \right)$	$2 \cdot \left(\left(1 + \frac{1}{N} \right) \cdot Lw + 1 + \frac{Ls}{N} \right)$	*None*

where *Lw* is the length of the adaptive filter, *Ls* is the length of the off-line estimate of the secondary path and *N* is the decimating factor used in the partial updates of the second algorithm proposed.

It should be noticed that the sampling frequencies chosen for the practical implementation of both strategies were not the same. While for the Modified FxGAL a sampling frequency of 1000 samples/s was considered to be enough to deal with low frequency noise, in the case of the G*μ*-FxSLMS algorithm, the sampling frequency was set to a value 8 times higher, that is, 8000 samples/s is order to broaden the bandwidth free of notches in the step-size gain. As a result of that, the comparison between both strategies should be carried out on the basis of the number of operations required per second, instead of the number of operations per iteration.

3. EXPERIMENTAL RESULTS

3.1 Laboratory test set-up

The physical arrangement of the electro-acoustic elements used in the implementation of the 1x2x2 Active Noise Control system placed at the front seats of a Nissan Vanette is depicted in Fig. 8-4.

The main Digital Signal Processor board employed to develop both strategies is the PCI/C6600, based on the DSP TMS320C6701. The Input/Output board is the PMCQ20DS that disposes of 4 A/D and 4 D/A converters.

The control strategy implemented was either the Modified FxGAL algorithm or the Gμ-FxSLMS algorithm.

In order to carry out a performance comparison of different control strategies it is essential to repeat the experiment in the same conditions. So as to avoid fluctuations in level and frequency of the undesired disturbance, instead of starting the engine, we have previously recorded a signal consisting of two harmonics (150 and 450 Hz). The omnidirectional source Brüel & Kjaer Omnipower 4296 placed inside the van is fed with this signal and acts as the source of the primary noise.

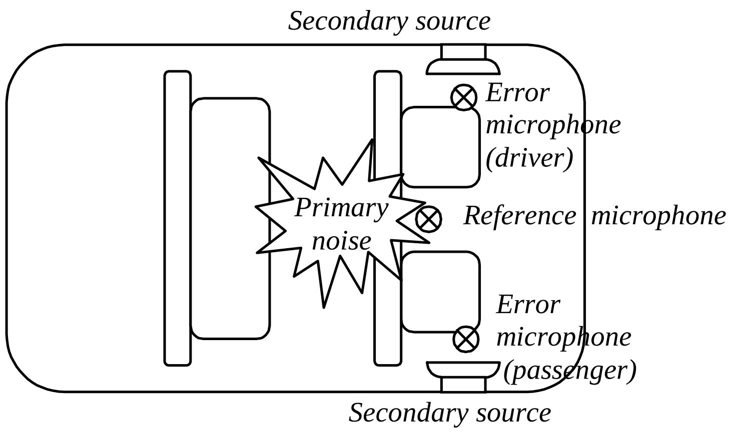

Figure 8-4. Physical disposal of the electro-acoustic elements inside the van.

3.2 Sets of parameters chosen

In order to obtain similar results with both algorithms in the attenuation of the undesired disturbance, the parameters are set to the following values:

- Modified FxGAL algorithm
  - Number of weights of the adaptive filter, Lw = 8.
  - Number of weights of the estimate of the secondary path, Ls = 200.
  - Sampling Frequency, Fs = 1000.
  - Normalized step size for the ALP stages is set to 0.06.
  - Forgetting factor, β = 0.97.
  - Normalized step size for the FxLMS algorithm is set to 0.08.
- Gμ-FxSLMS algorithm
  - Number of weights of the adaptive filter, Lw = 128.
  - Number of weights of the estimate of the secondary path, Ls = 200.
  - Sampling Frequency, Fs = 8000.
  - Decimating factor, N = 8.
  - Gain in step size, Gμ = 8;
  - Step size of the adaptive algorithm, μ = 0.1.

So as to carry out a comparison of the computational requirements, it is assumed that the DSP can deal with 1 MAC operation -multiplication & accumulation- per DSP cycle whereas needs 40 cycles to perform a division (Poland, 1999). According to the parameters chosen and taking into account the complexity expressed in Table 8-1, the number of clock cycles required between two consecutive samples is 2252 for the Modified FxGAL algorithm and 340 for the Gμ-FxSLMS algorithm. Considering that the sampling frequency is 8 times higher in the latter case, the cycles required per millisecond are 2252 and 2720, respectively. Thus, not only the performance achieved but also the computational costs of both strategies are quite similar despite being based on opposite underlying ideas.

3.3 Analysis in the time domain

Figures 8-5 and 8-6 show, respectively, the learning curves of the Modified FxGAL and the Gμ-FxSLMS algorithms, when the error signals are measured at the microphones located near to the head of the driver and the passenger.

In both cases the two-harmonic signal is effectively attenuated by more than 20 dB within relatively short time.

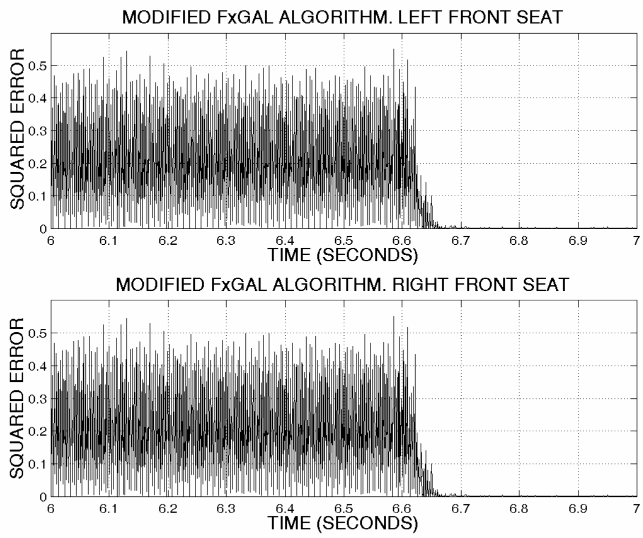

Figure 8-5. Evolution of the squared error when the ANC system based on the Modified FxGAL algorithm is switched on. Upper: left front seat, lower: right front seat.

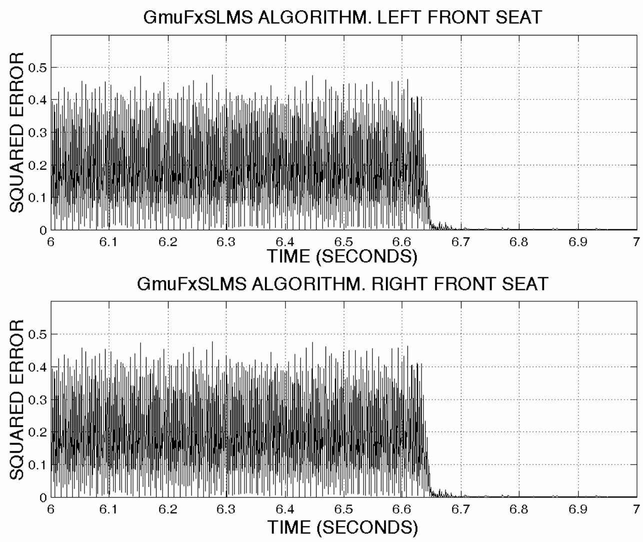

Figure 8-6. Evolution of the squared error when the ANC system based on the Gμ-FxSLMS algorithm is switched on. Upper: left front seat, lower: right front seat.

3.4 Analysis in the frequency domain

The frequency response functions measured at the error sensors located at the front seats of the van are shown in Fig. 8-7 -Modified FxGAL algorithm- and Fig. 8-8 -Gμ-FxSLMS algorithm-. The signal before control is shown in a dotted line whereas the signal after control is shown in a solid line. As far as the attenuation achieved is concerned, more than 25 dB of peak reduction are obtained at the main harmonics with both ANC algorithms. Nonetheless, very little off-peak reduction was obtained.

Power spectral density of the undesired noise depicted in Fig. 8-7 and Fig. 8-8 consists of two harmonics at 150 and 450 Hz. Looking carefully into the graphs, it can be noticed that an unexpected noise component appears at a narrow frequency band between 15 and 35 Hz. Provided that this component was not present in the two-harmonic signal when it was generated, we can conclude that it corresponds to a mode imposed by the geometry of the van. In fact, we have verified that this low frequency noise vanishes as soon as the microphone is located outside the van.

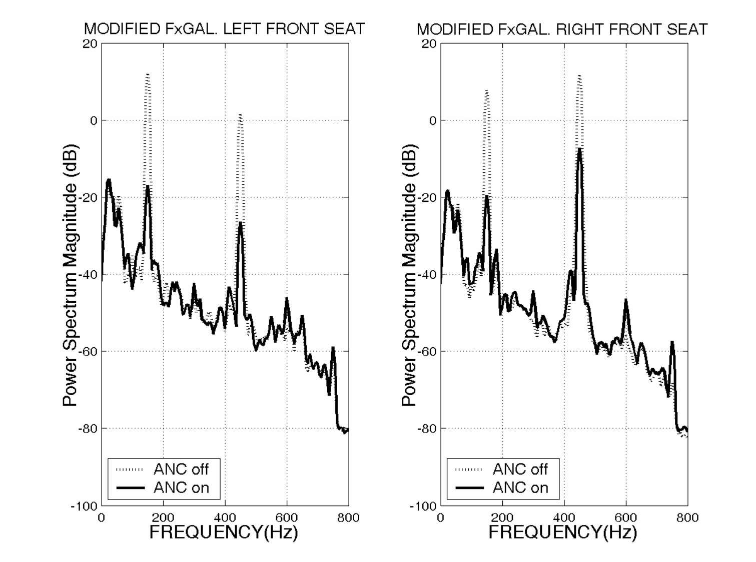

Figure 8-7. Experimental control results for the ANC system based on the Modified FxGAL algorithm in the frequency domain. Left: left front seat, right: right front seat.

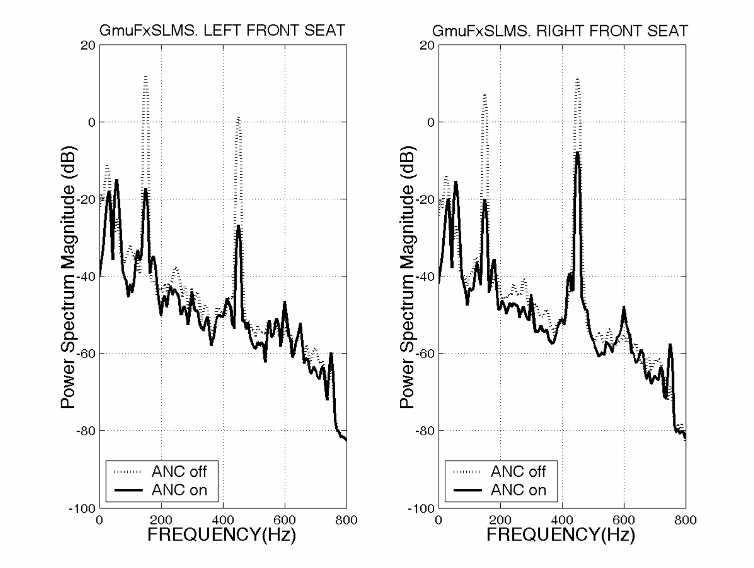

Figure 8-8. Experimental control results for the ANC system based on the Gμ-FxSLMS algorithm in the frequency domain. Left: left front seat, right: right front seat.

4. CONCLUSIONS

This chapter presents the results for applying two different control algorithms -Modified FxGAL and Gμ-FxSLMS- to actively attenuate periodic noise in a van.

The former strategy is aimed at speeding up the convergence rate at the expense of increasing the computational requirements whereas the latter puts forward a computationally less intensive solution without slowing down the convergence rate.

In spite of the fact that the underlying proposals of both algorithms are based on opposite control strategies -higher complexity and faster convergence rate versus lower complexity-, the subsequent choice of the parameters allows Modified FxGAL and Gμ-FxSLMS algorithms to achieve similar performance in terms of convergence speed, residual error and degree of attenuation with a computational complexity of the same order.

It has been experimentally shown that periodic noise may be substantially attenuated by Active Noise Control systems based on both algorithms.

With a sampling frequency of 1000 samples/s -for the Modified FxGAL algorithm- or 8000 samples/s -for the Gμ-FxSLMS-, the effective ANC system bandwidth is approximately 500 Hz, and usually produces more than 20 dB of reduction within about 0.05 seconds of the starting.

ACKNOWLEDGEMENTS

These researches are supported by CICYT (Comisión Interministerial de Ciencia y Tecnología) of Spanish Government under grants TIC-2002-04103-C03-01 and TIN-2005-08660-C04-01.

REFERENCES

Bjarnason, E., 1992, Active noise cancellation using a modified form of the filtered-x LMS algorithm, in: *Proceedings of Eusipco-92*, Brussels, pp. 1053–1056.

Douglas, S.C., 1997, Adaptive filters employing partial updates, *IEEE Trans. Circuits and Systems II: Analog and Digital Signal Processing* **44** (3): 209–216.

Griffiths, L. J., 1978, An adaptive lattice structure for noise-cancelling applications, in: *Proceedings of ICASSP 1978*, Tulsa, pp. 87–90.

Kim, I.-S., Na, H.-S., Kim, K.-J., and Park, Y., 1994, Constraint filtered-x and filtered-u least-mean-square algorithms for the active control of noise in ducts, *J. Acoust. Soc. Am.* **95** (6): 3379–3389.

Poland, S., 1999, TMS320C67xx Divide and Square Root Floating-Point Functions, Texas Instruments, App. Rep, SPRA516.

Ramos, P., Torrubia, R., López, A., Salinas, A., and Masgrau, E., 2004, Computationally efficient implementation of an active noise control system based on partial updates, in: *Proceedings of ACTIVE 2004*, Williamsburg.

Vicente, L., Masgrau, E., and Sebastián, J.M., 2003, Active noise control experimental results with FxGAL algorithm, in: *Proceedings of Internoise 2003*, Jeju.