Fast Convergence Algorithms for Active Noise Control in Vehicles

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Summary: When reference signal for the FxLMS algorithm is taken from an acoustic sensor convergence can be very slow due to great eigenvalue spread. Using a nonacoustic sensor, such as a tachometer, cancellation of narrow band noise in the sensed fundamental frequency and harmonically related ones can be achieved very fast, although other periodic noises and the underlying broadband noise will remain. Backward prediction errors resulting at the various stages of an Adaptive Lattice Predictor (ALP) represent a time-domain orthogonalization of the input signal. An ALP structure, with the acoustic reference as input signal, before a FxLMS makes up the FxGAL algorithm. Due to orthogonalization, FxGAL can be significantly faster compared to FxLMS with reference from a microphone. When compared to FxLMS with tachometer signal, it is not faster but it can cancel every periodic noise, independently of the harmonical relation between them, as well as the underlying broadband noise. Comparative results between FxLMS (with acoustic and non-acoustic reference) and FxGAL are presented.

I. INTRODUCTION

The Filtered-x Least Mean Square (FxLMS) algorithm (1) is the most widely used in the context of adaptive active control, due to its simplicity as well as robustness. However, the main drawback of this algorithm is its relatively slow and signal-dependent convergence, which is determined by the eigenvalue spread of the underlying correlation matrix of the input signal. When working in nonstationary environments, such as automobiles, slow convergence is a critical problem, since we would desire to cancel transient noise, which occurs at vehicle start-ups, stops, or gearshifts, or with sudden changes of engine speeds or road noise from tyres.

A practical solution to this problem, very commonly used, is to use nonacoustic sensors, such as a tachometer, instead of acoustic ones and artificially generate the signal to use as reference. This way, convergence can be achieved very fast, since it is possible to generate orthogonal references (in-phase and quadrature components). On the other hand, it is only possible to cancel the narrowband noises in the fundamental frequency sensed by the nonacoustic sensor and other harmonically related frequencies, whereas every other periodic or broadband noise will remain uncancelled.

In this paper we introduce an algorithm, the Filtered-x Gradient Adaptive Lattice (FxGAL), that aims to improve the convergence of the whole adaptive system when using acoustic sensors to get the reference signal, at the expense of increased computational complexity. The approach consists in conditioning the FxLMS reference signal by pre-processing it and obtaining a decomposition of the signal in orthogonal (decorrelated) components. With a decorrelated input signal, the convergence modes of the FxLMS system are decoupled, and the whole adaptive filter of order $L$ turns into $L$ independent adaptive filters of just one coefficient. These independent systems can have their own adaptation step size, in order to obtain the same convergence speed for all of them.
II. FXGAL ALGORITHM.

The decorrelating system used in the FxGAL algorithm is an Adaptive Lattice Predictor (ALP) (2, 3). It is a modular structure, in such a way that the predictor of order $L$ consists of $L - 1$ identical cascaded stages. The structure of one of such lattice stages can be seen in Figure 1. The inputs to the stage $l$ are the forward and backward prediction errors of the $(L - 1)$-th order predictor, and the outputs are the forward and backward prediction errors of the $L$-th order predictor. The ALP system itself is an adaptive filter, where a steepest descent method is used to adjust the filter coefficients $\{\kappa[n]\}$ independently at each stage, so as to minimise the mean square of the sum of forward and backward predictor errors at that stage.

The fundamental property of the ALP system that our algorithm relies on is the orthogonality of the backward prediction errors. In every moment, the sequence of backward prediction errors $\{b_0[n], b_1[n], \ldots, b_{L-1}[n]\}$ are mutually uncorrelated and are a transformation of the input sequence $\{x[n], x[n-1], \ldots, x[n-L+1]\}$ without loss of information. Therefore, we can use them as the (decorrelated) input signals to the FxLMS instead of using the reference signal itself, and so, speed up the convergence of the whole system making every mode converge at the same speed.

In Figure 2 we can see the structure of the algorithm we present in this paper. We have called it FxGAL (Filtered-x Gradient Adaptive Lattice) by analogy with the FxLMS, since the LMS block is substituted by a lattice structure where the GAL algorithm is used to adaptively update the filter coefficients. The system can also be seen as a predictive lattice pre-processing stage that decorrelates the reference signal for the transversal LMS filter.

To reduce the computational complexity of the algorithm, sometimes it is feasible to substitute the last $(L_G - L)$ lattice stages of the ALP by a delay line without practically affecting performance, as will be shown in the results obtained. This is possible when the signal can be no longer decorrelated after $L_G$ lattice stages, that is when the optimal lattice coefficients for $l > L_G$ are very close to 0, $\kappa[n] = 0$. Fortunately, this is a very common situation. This way, the filter structure would be a lattice and transversal combination. Another important property of the ALP system is its modularity, that makes possible to dynamically change the order of the prediction filter, adding or removing lattice stages, without having to reassess all the filter coefficients, but just the new ones.

![FIGURE 1. Detailed structure of $l$-th stage of an Adaptive Lattice Predictor.](image1)

![FIGURE 2. Filtered-x Gradient Adaptive Lattice (FxGAL) algorithm block diagram.](image2)
III. RESULTS.

Computer simulations have been performed for artificially generated broadband noise. The reference signal is a pink noise, obtained by low pass filtering of a white noise. The primary noise to cancel is obtained by band pass filtering of the same white noise that generates the reference signal. The secondary path transfer function is taken from the examples in (3) and the FIR model of it, necessary for the adaptive process, was obtained previous to the operation of the system. Both systems, FxLMS and FxGAL, were tested for this arrangement. The parameters of the systems were determined so as to obtain the same steady-state mean square error. The learning curves of the two compared systems are shown in Figure 3. It can be seen the significant speed improvement achieved by the FxGAL system, while keeping the same steady-state solution.

The tests performed have proved that the whole system convergence speed has reduced sensitivity to variations in the ALP adaptation step size. In any case, much lower than the sensitivity to variations in the adaptation step size of the FxLMS block.

Engine noise signals in different points in the inside of a coach were recorded simultaneously for several situations. Two of these signals were used as reference signal and primary noise respectively in the following tests. The secondary path transfer function used was a pure delay.

Figure 4 shows the learning curves obtained for three different systems: FxLMS ($L = 30$), FxGAL ($L_G = L = 30$) and FxGAL combining lattice and transversal structures ($L_G = 10, L = 30$). For viewing purposes, the curves were smoothed by moving average filtering. Adaptation step sizes for the three systems were determined independently so as to maximise the cancellation. As can be seen from the figure, there is little performance difference between the (complete) lattice system and the lattice-transversal combination. Being the signals real
and nonstationary, the FxGAL systems are also able to obtain more cancellation than the FxLMS, due to their faster adaptation to nonstationarities.

Figure 5 shows the error signal spectrums obtained for the same arrangement in three different situations: without active noise control, with the FxLMS system driven with tachometer signal and with the FxGAL system with acoustic reference. It can be seen that the FxLMS is unable to cancel the sinusoidal components in 39 and 59 Hz, since they are not harmonically related to the fundamental noise (50 Hz), whereas the FxGAL cancellation is greater, tending to get a flatter spectrum in the whole frequency band.

In some peculiar cases, where the secondary path transfer function is not flat in magnitude and group delay in the frequency band of interest (s[n] scattered), the orthogonality property of the backward prediction errors of the ALP, \{b_i[n]\}, can be diminished in the filtered signals, \{b'_i[n]\}, that are used to update the filter \( W(z) \), slowing down the algorithm. This problem could be solved orthogonalising the filtered signal, \( x'[n] \), that updates the filter.

**IV. CONCLUSIONS.**

In this paper, we have introduced the FxGAL algorithm, that obtains faster convergence than the classical FxLMS, at the expense of increased computational complexity. In real situations, this faster convergence means also greater noise cancellation. An alternative system that combines the lattice and transversal structures has also been presented. The combined system has approximately the same convergence properties as the FxGAL, but significantly reduced computational complexity.

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**REFERENCES**