Analysis of crosstalk in multi-core polymer optical fibres

A. Berganza, G. Aldabaldetreku, J. Zubia, G. Durana, J. Mateo, and M. A. Illarramendi

***Abstract*—In this article we are going to analyze the crosstalk effect in multi-core polymer optical fibres (MC-POFs), which consist of groups of 127 graded-index cores.**

There are different models available in the literature that describe the crosstalk between optical fibres, even though most of them are only valid for waveguides supporting a few number of modes. For multimode optical fibres, a quasi ray-tracing procedure is proposed, where coupling between cores is calculated using a transmission coefficient, based on the effect of frustrated total internal reflection. However, this model is only valid for meridional rays. It is our purpose to implement such a procedure into our ray-tracing model and extend its applicability to skew rays.

***Index Terms*—Crosstalk, optical fibres, geometrical optics, ray tracing.**

I. Introduction

The aim of this article is to analyze the effect of coupling between the different cores in a MC-POF. In general, MC-POFs consist of groups of individual polymer optical fibre cores arranged in concentric rings around a central core, and they constitute a good alternative to conventional step-index (SI) and graded-index (GI) POFs because of their smaller sensitivity to bending losses [1].

The system we are going to study is shown in Fig. 1. Only the central core is illuminated and, at the end of the fibre, part of the input power will be coupled to the surrounding cores (unilluminated cores).

There are several models describing crosstalk between optical fibres [2, 3, 4]. Our purpose is to analyze the coupling produced between the cores of a highly multimode MC-POF fibre, so that models based on Maxwell's equations are intractable due to the very high number of modes we are dealing with. For this reason, the starting point will be the quasi-ray tracing model developed by Cherin and Murphy [4], where crosstalk between fibres is calculated taking into account only meridional rays. We will use the analytical expressions of this model, which are valid for both plastic optical fibres and glass optical fibres, to implement a ray-tracing method in order to develop a computer model for MC-POF fibres, and we will adapt them in order to take also into account skew rays.

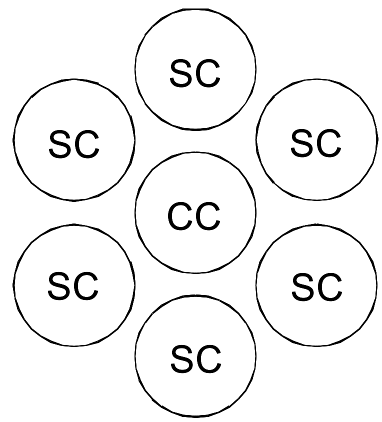

Fig.1 Cross-section of the central core and the surrounding ones of a MC-POF fibre.

The main physical mechanism that describes crosstalk between cores is that of frustrated total internal reflection of waves in a multi-layered medium [4]. Whenever a ray reaches the core-cladding interface, it will give rise to a transmitted ray, which will contribute to crosstalk power between the nearest neighbors, and to a reflected ray that will be confined in the illuminated core. For the sake of simplicity, it is assumed that the propagating modes within a core of a MC-POF are uncoupled.

The structure of the paper is as follows. First of all, we summarize the analysis made by Cherin and Murphy and then show how to adapt it in order to take into account skew rays. Afterwards, the ray-tracing model proposed is explained and validated by comparing the numerical results with those given by the analytical expressions. Next, computer simulations will be carried out with the aim of analyzing the influence of skew rays on crosstalk. Finally, we summarize the main conclusions.

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II. Theory

A. Crosstalk formula adapted to skew rays

In the analysis carried out by Cherin and Murphy, crosstalk is expressed in terms of the FEXT (Far-end and equal-level crosstalk), which is defined as

$$FEXT = 10\log_{10}\left(\frac{P_{OUT}}{P_{XT}}\right) \qquad (1)$$

where P_{XT} and P_{OUT} are the crosstalk power and the output power (not coupled to the surrounding cores) at the end of the fibre respectively, being defined as

$$P_{OUT} = \int_0^{2\pi} d\phi \int_0^{\theta_{\max}} F(\theta,\phi)\sin\theta e^{-\alpha L\sec\theta}\left[1-T(\theta,\phi)\right]^N d\theta \qquad (2)$$

$$P_{XT} = M\int_0^{\theta_{\max}} d\theta \int_{s_1}^{s_2} ds\left(\frac{d\phi}{ds}\right)F(\theta,\phi)\sin\theta e^{-\alpha L\sec\theta}\left\{-\left[1-T(\theta,s)\right]^N\right\} \qquad (3)$$

$$\frac{d\phi}{ds} = \frac{C^2-(d/2)^2-(s+d/2)^2}{(s+d/2)\left\{\left[C^2-s^2\right]\left[(s+d)^2-C^2\right]\right\}^{1/2}} \qquad (4)$$

N is the number of reflections made by a ray along the fibre, s is the distance from the exiting point (A) in the illuminated core to the coupling point (B) in the unilluminated core, _ is the axial angle of the ray, M is two times the number of cores around the illuminated core, T is the transmission coefficient, L is the length of the fibre, _ is the angle that a meridional ray forms with the line that joins the core centers, d is the diameter of the cores and *F(_,_)* is the input power distribution. These parameters are defined in Fig. 2.

In this case, a gaussian input distribution will be used. The expression for *F(_,_)* will be

$$F(\theta,\phi) = e^{(-\theta/K\theta_C)^2} \qquad (5)$$

where kappa (K) is a measure of the width of the gaussian beam and _c is the critical angle of the fibre.

As can be seen in Fig. 2, this analysis is only valid for meridional rays, those rays crossing the center of the illuminated core. For skew rays, the expressions hold no longer, since these expressions depend on _, a parameter that makes only sense for meridional rays.

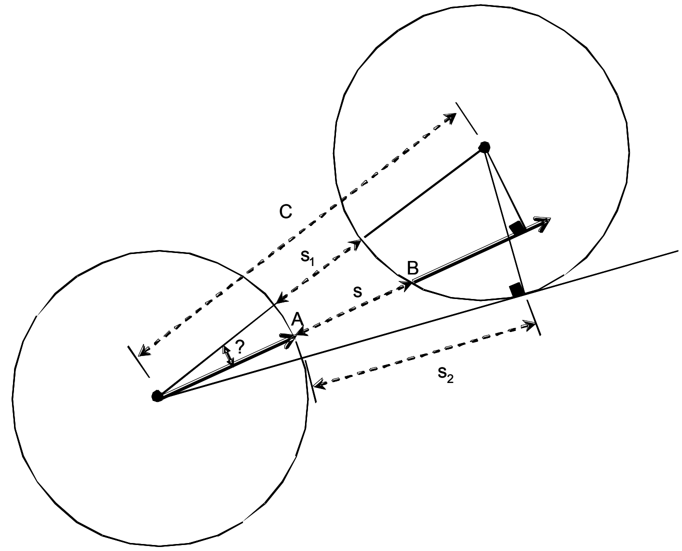

Fig.2 Parameters of the fibre

The expression for the transmission coefficient T is shown in (6). This coefficient has been calculated using wave plane incidence in three homogeneous media (core-cladding-core). The core-cladding interface can be considered locally plane (compared to the wavelength of the light) [5]. Therefore,

$$T(\theta,\phi) = 0.5\left[\frac{1}{\cosh^2\beta + \left[(n_{cl}^2\gamma^2 - n_{co}^2\cos^2\theta_1)/(2n_{co}n_{cl}\gamma\cos\theta_1\right]^2\sinh^2\beta}\right] + 0.5\left[\frac{1}{\cosh^2\beta + \left[(n_{cl}^2\cos^2\theta_1 - \gamma^2 n_{co}^2)/(2n_{co}n_{cl}\gamma\cos\theta_1\right]^2\sinh^2\beta}\right] \qquad (6)$$

$$\gamma = \left[\left(\frac{n_{co}}{n_{cl}}\right)^2 \sin\theta_1{}^2 - 1\right]^{1/2} \qquad (7)$$

$$\beta = \frac{2\pi}{\lambda} n_{cl} s\gamma \qquad (8)$$

n_{co} and n_{cl} are the refractive indexes of the core and the cladding respectively and *_1* is the complementary angle of the axial angle.

In our computational tool, the input distribution is constituted by rays, which will travel along the fibre suffering multiple reflections with the core-cladding interface. In each reflection, part of the power will be transmitted to the surrounding cores and part of the power will be confined to the illuminated core.

Since at each turning point, each ray in the illuminated core generates a new ray in an unilluminated core, we will have 2^N rays at the end of the fibre, being N the number of reflections that the ray suffers along its way (N is in most cases a large number and it can be as high as several millions for a 1km length fibre). The task of including all rays in a computational method is almost intractable; therefore, some approximations must be made. To solve this, a criterion must be stablished in order to keep constant the number of rays along the fibre. The used criterion is the following: if $(1-T)^N > 0.5$, the ray in the central core is chosen and the coupled ray is discarded, otherwise, the coupled ray is selected. The chosen ray will hold the 100% of the power. This way, we have the same number of rays at the input and at the end of the fibre.

A ray in the central core of a MC-POF can generate coupled rays in any of the surrounding cores. The core where the ray will be coupled, if the criterion selects the coupled ray, will be the first one the ray travelling in the central core encounters along its way. As the input ray distribution is homogeneously distributed, the probability of encountering any of the other surrounding cores is practically the same.

With the exposed criterion, when the ray in the illuminated core is chosen, some crosstalk power is neglected and, when the coupled ray is selected, some output power is neglected. If the percentages of these neglected powers are not similar, FEXT will be different of that calculated using (2) and (3). This makes necessary to adapt the power of the output ray distribution at the end of the fibre. For this purpose, we will use two correction factors: F_{XT} and F_{OUT} , one corresponding to the output power (in the illuminated core), and the other one referring to the crosstalk power (in the surrounding cores). If the ray at the end of the fibre is a coupled one, its power must be multiplied by F_{XT} and, if the ray ends in the central core, its power will be multiplied by $F_{OUT.}$ This way, we will obtain P_{XT_TOTAL} and P_{OUT_TOTAL}, being n the number of coupled rays and m the number of rays in the illuminated core at the end of the fibre.

$$F_{XT} = \frac{P_{XT\_NOT\_NEGLECTED} + P_{XT\_NEGLECTED}}{P_{XT\_NOT\_NEGLECTED}} \qquad (9)$$

$$F_{OUT} = \frac{P_{OUT\_NOT\_NEGLECTED} + P_{OUT\_NEGLECTED}}{P_{OUT\_NOT\_NEGLECTED}} \qquad (10)$$

$$P_{XT\_TOTAL} = \sum_{i=1}^{n} P_i F_{XT} =$$
$$= P_{XT\_NOT\_NEGLECTED} \frac{P_{XT\_NOT\_NEGLECTED} + P_{XT\_NEGLECTED}}{P_{XT\_NOT\_NEGLECTED}} \qquad (11)$$

$$P_{OUT\_TOTAL} = \sum_{i=1}^{m} P_i F_{OUT} =$$
$$= P_{OUT\_NOT\_NEGLECTED} \frac{P_{OUT\_NOT\_NEGLECTED} + P_{OUT\_NEGLECTED}}{P_{OUT\_NOT\_NEGLECTED}} \qquad (12)$$

To take into account skew rays, (3) must not be limited to the region between s_1 and s_2. It is possible to calculate the distance s for each ray by geometry, as it is shown in Fig. 3.

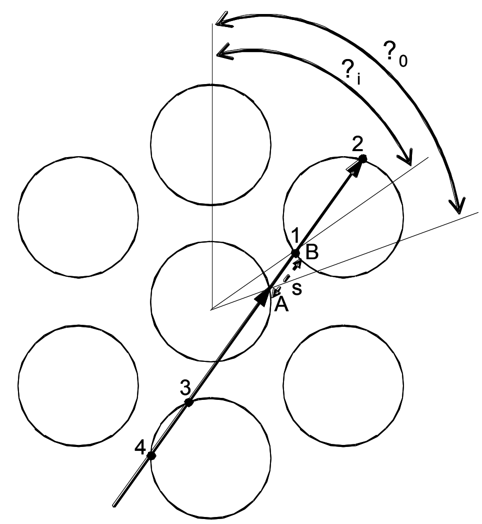

Fig.3 Parameters for calculating s

The intersection points of the straight line that the ray path forms with the surrounding cores are calculated and then, the smallest distance s in the forward sense of the ray is chosen, so s has the following expression

$$s = \frac{-A - \sqrt{A^2 - 4B}}{2} \qquad (13)$$

$$A = 2\lfloor \rho \cos(\theta_{A\varphi} - \varphi_0) - C\cos(\theta_{A\varphi} - \varphi_i) \rfloor \ (14)$$

$$B = C^2 - 2\rho C \cos(\varphi_0 - \varphi_i) \ (15)$$

C is the distance between the core centers, _A_ is the azimuthal angle of the ray, _0 is the polar angle of the exiting point (A) of the ray, _i is the polar angle of the center of the core where the ray is coupled to, and _ is the radius of the cores.

With the value of s, we will be able to calculate the transmission coefficient T and decide which one of the two rays generated in a turning point will be chosen.

In the case of meridional rays, the transmission coefficient is the same at every turning point due to the symmetry of the cross section of the MC-POF (see Fig. 1). This is not the case for skew rays, where the distance s to the next core will be different in each turning point. This has a very negative impact on the computational time, as it is necessary to calculate T at each turning point of each ray. In an attempt to reduce the computational time, the mean value of T is calculated using only the first 200 transmission coefficients (a number which has proven to be enough); using this mean value, the criterion explained above is finally applied.

B. Comparison of the results obtained using the ray-tracing method with the analytical results

In order to validate the ray-tracing method, (2) and (3) have been assessed using Mathematica commercial software [6]. Results are shown in Figs. 4 and 5. For some reason, there are discrepancies between our results and those obtained by Cherin and Murphy; however, since the results obtained using Mathematica are in excellent agreement with those obtained using ray-tracing method we can conclude that the ray-tracing method has been correctly developed.

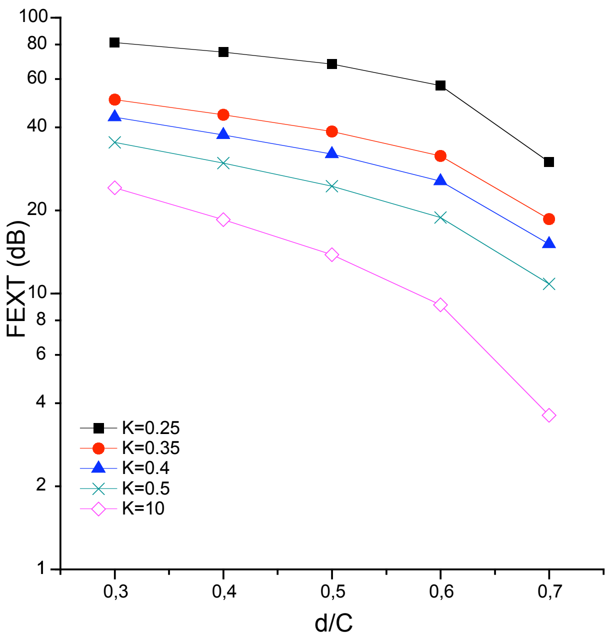

Fig.4 FEXT for meridional rays in a fibre of length 1000m for different input gaussian distributions with Mathematica

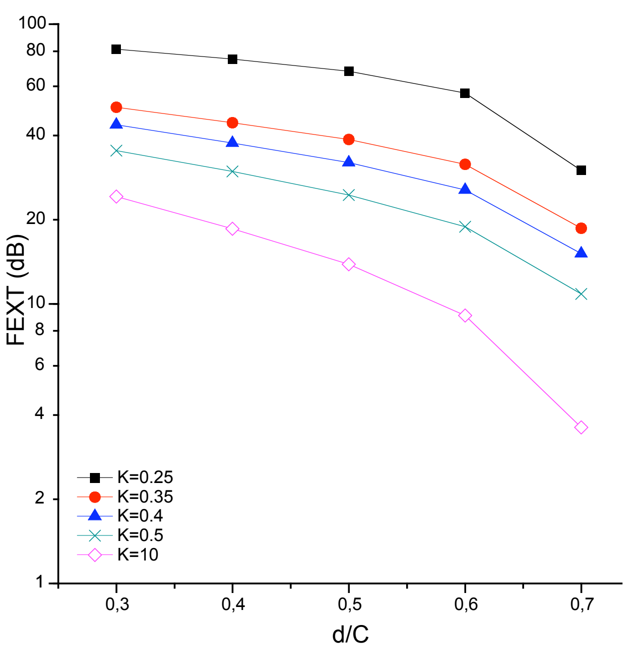

Fig.5 FEXT for meridional rays in a fibre of length 1000m for different input gaussian distributions with the ray-tracing method

The different plots show FEXT against different widths of the input gaussian beam (different values of kappa) and different values of *d/C*.

III. Influence of Skew Rays on Crosstalk

To analyze the crosstalk effect, simulations were carried out by illuminating only the central core and examining the coupled power in the surrounding cores after a fibre length of 1000m.

Adding skew rays to the analysis led to different values for FEXT as can be seen in Fig. 6.

When the centers of the neighboring cores are well separated from the illuminated core (d/C small), the FEXT calculated including only meridional rays is greater than the FEXT calculated for skew rays, because the probability for a skew ray of encountering a neighboring core along its way is greater than that of a meridional ray. That is to say, if a meridional ray does not find a core to couple in the first turning point, it will not find any one until it reaches the end of the fibre. This is not the case of a skew ray, which can encounter a neighboring core at any turning point from the beginning to the end of the fibre.

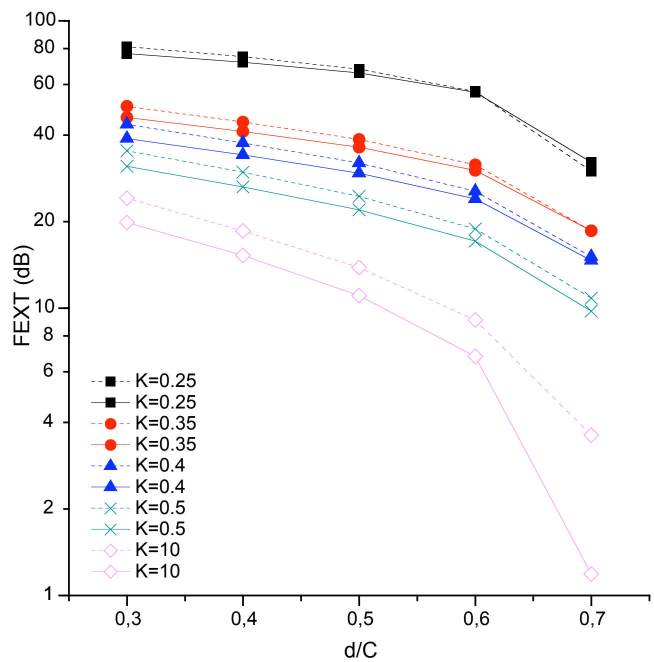

Fig.6 Comparison of FEXT for meridional rays and skew rays in a fibre of length 1000m for different input gaussian distributions with ray-tracing. Solid lines correspond to simulations made taking into account skew rays and dashed lines refer to simulations using only meridional rays.

However, as cores are placed closer to each other, FEXT in both cases becomes more similar, that is, the influence of the skew rays is not so important.

IV. Crosstalk Analysis on a Real MC-POF Prototype

In this section the effect of crosstalk on a real MC-POF is analyzed.

A. Characteristics of the analyzed MC-POF

A new type of MC-POF, made of the perfluorinated polymer called CYTOP, has been recently presented in a configuration of 127 small GI cores grouped together forming rings so that they fill a round cross-section of 350 _m of diameter. Low-cost production is one of the main objectives followed by the manufacturer in order for PF MC-POFs to constitute an attractive and interesting option in next generation Fiber To The Home (FTTH) services in combination with a suitable light source such as a Vertical Cavity Surface-Emitting Laser (VCSEL).[1] This PF MC-POF is a prototype developed by Asahi Glass [7] and is investigated in this paper. Its most important specifications are summarized in Table 1.

The crosstalk in the first ring of this fibre was calculated using the computational model we have developed, even though, as a first approach, we have considered the individual cores as SI (and not GI). The values of the FEXT were obtained for different values of the width of the input gaussian beam, as can be seen in Fig. 8.

TABLE I
SPECIFICATIONS OF THE INVESTIGATED MC-POF

	Quantity	Unit
Number of cores	127	-
Core diameter	25	_m
Cladding diameter	350	_m
Area fraction[1]	64.8	%
Numerical Aperture	0.185	-
Attenuation[2]	45	dB/km

[1] Area fraction = Core area / (Core area + Clad area).
[2] Measured at 850 nm.

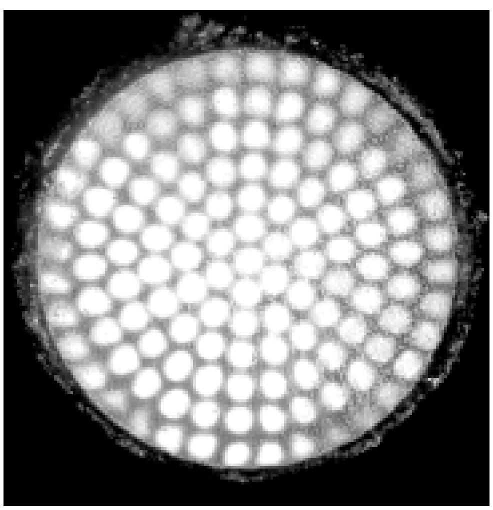

Fig.7 Cross section of the MC-POF fibre

It can be observed that, for low values of kappa, FEXT increases rapidly with the width of the beam (*K*_c), but, for high values of kappa, FEXT increases much slower. This is due to the dependency of the transmission coefficient on the axial angle of the ray and the separation between the exiting point (*A*) and the coupling point (*B*). That is to say, *T* is of a higher value when the axial angle of the ray is closer to the critical angle and the distance *s* is smaller. If the beam is made narrower, the power carried by the rays with axial angles close to the critical one is not very significant, so FEXT present a greater value. On the contrary, if the beam is made wider, those rays that are more likely to be coupled, have more power, so FEXT decreases.

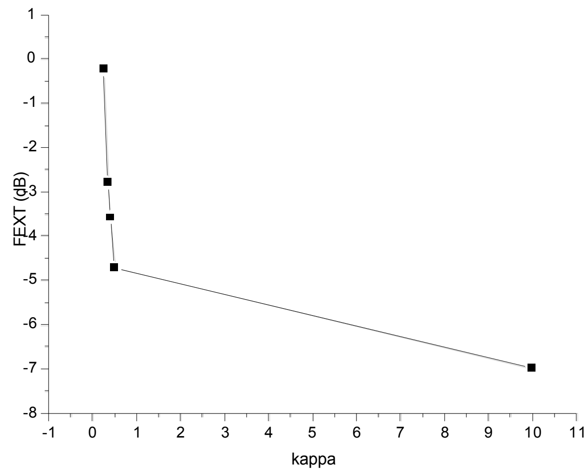

Fig.8 FEXT in the six cores of the first ring of a MC-POF of length 1000m for different input power distributions. The straight lines are just a guide to the eye.

B. Observation of the crosstalk effect on the near-field patterns

Some simulations were carried out in order to examine the near field pattern on the first ring of a MC-POF.

First, a simulation using a gaussian beam of width 0.25_c was made, and, at the end of the fibre, the field obtained was the one showed in Fig. 9.

It is easy to appreciate a more powerful ring inside the central core which indicates that rays suffering more skewness are more likely to be confined inside the illuminated core. This is due to the dependency of the transmission coefficient on the distance *s*. That is, the more skew the ray is, the greater the value of *s* is and, consequently, the lower the value of *T* and the crosstalk power are.

Another simulation was made with a very wide beam, which presents similar characteristics with a uniform distribution. The near field pattern is plotted on Fig. 10. In this case, the ring previously commented is noticeable too, but now, the power in the surrounding cores is slightly greater. This result can be explained by the fact that the rays with an axial angle close to the critical one, are as powerful as those which are close to zero, so rays that couple to the outer cores, will have more power than in the previous case.

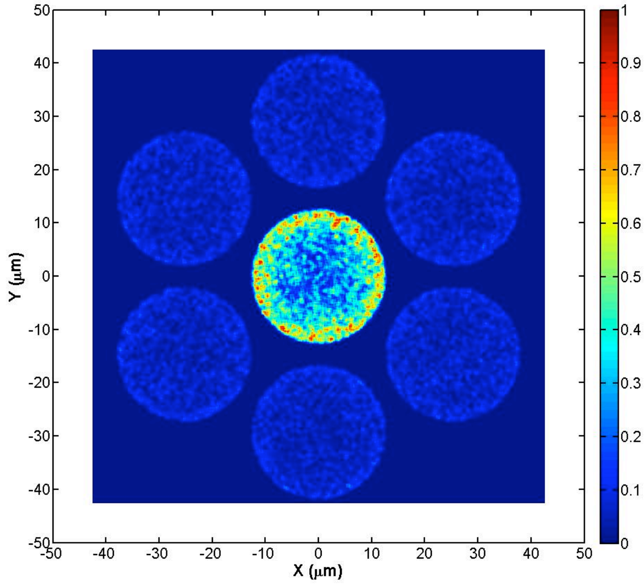

Fig.9 Near field pattern of the first ring of a MC-POF fibre with an input gaussian distribution of width 0.25_C and a fibre length of 1000m.

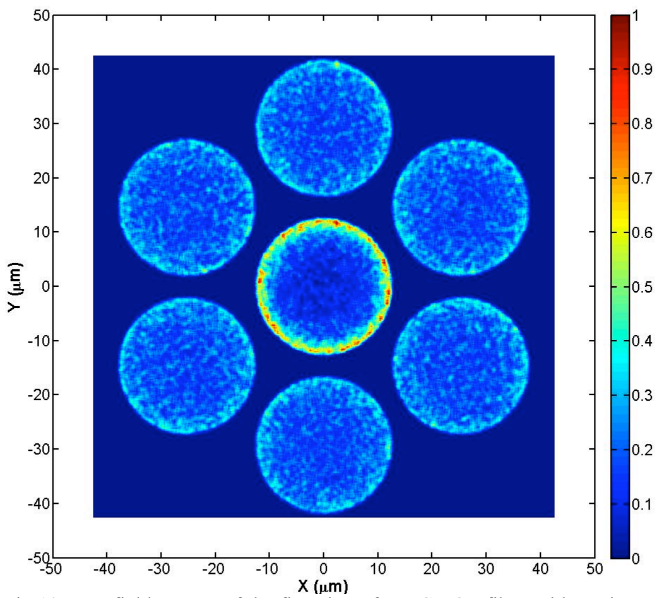

Fig.10 Near field pattern of the first ring of a MC-POF fibre with an input gaussian distribution of width 10_C and a fibre length of 1000m.

V. Future developments

In a further stage, we expect to extend the ray-tracing model to all cores in a SI MC-POF, as well as to GI MC-POFs in order to be able to investigate the crosstalk in real perfluorinated MC-POF prototypes.

Furthermore, coupling among modes within a core will also be taken into account

VI. Conclusion

We have implemented a method based on ray-tracing to analyze the crosstalk in MC-POFs and we have developed a computational tool based on this method.

We have also investigated the influence of skew rays in crosstalk. From the results above, it can be concluded that the crosstalk depends mostly on the separation of the cores and on the input power distribution. Rays that present a higher skewness, are more likely to be confined in the illuminated core and produce no crosstalk power, as the distance from the exiting point to the coupling point is greater in this case. On the other hand, rays with a lower skewness and an axial angle close to the critical one are more likely to couple after a certain fibre length.

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